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NATURAL SERIES.

SECOND BOOK.

FELTER'S
NEW
INTERMEDIATE ARITHMETIC,

CONTAINING

ORAL AND WRITTEN PROBLEMS,

AND

DRILL CARD EXERCISES.

6919

BY

S. A. FELTER, A.M.,
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RAND, AVERY, &

PREFACE.

THIS book is designed to make the pupil quick and accurate in calculation, and to give him a knowledge of those principles and processes of arithmetic which are needed in the ordinary transactions of life, together with skill in their application.

To accomplish this, the DRILL CARD EXERCISES are arranged to furnish any desired amount of practice in computation; while the processes and analyses leading directly to the rule, together with the number, gradation, and character of the practical examples, give the knowledge of necessary principles, and skill in their use.

All unnecessary distinctions, processes, and classifications have been omitted, and those only retained which are required in the preparation for ordinary business.

Business-men generally and justly complain of the inability of pupils fresh from school to perform accurately the calculations required in ordinary transactions. This inaccuracy arises from want of suitable and sufficient practice. The Drill Card Exercises furnish an unlimited amount of graded work, from which the teacher, without labor, may select such examples as any pupil or class may require.

The combination of oral and written work will commend itself to the intelligent teacher.

The author wishes to acknowledge his indebtedness to S. A. Farrand, A.M., Principal of Newark Academy, whose name is associated on the title-page. His scholarship and extended schoolroom experience have contributed much to the excellence of this work.

S. A. F.

JULY, 1875.

CONTENTS.

	PAGE.
Definitions	1
Roman Notation	2
Arabic Notation	4
Numeration	4
Addition	10
Subtraction	37
Multiplication	59
Division	75
United-States Currency	102
Greatest Common Divisor	113
Least Common Multiple	116
Cancellation	118
Fractions	120
Addition of Fractions	137
Subtraction of Fractions	141
Multiplication of Fractions	146
Division of Fractions	154
Decimals	173
Addition of Decimals	177
Subtraction of Decimals	178
Multiplication of Decimals	182
Division of Decimals	185
Compound Denominate Numbers	201
Linear Measures	202
Square Measures	212
Solid or Cubic Measure	221
Liquid Measure	227
Dry Measure	228
Apothecaries' Weight and Measure	232
Troy Weight	233
Measure of Time	236
Circular Measure	238
Stationers' Table	240
English Money	240
Addition of Denominate Numbers	241
Subtraction of Denominate Numbers	242
Multiplication of Compound Numbers	244
Division of Compound Numbers	[illegible]
Denominate Fractions	[illegible]

INTERMEDIATE ARITHMETIC.

SECTION I.

DEFINITIONS, NOTATION AND NUMERATION.

IN writing numbers we use ten characters, called *figures.* The first figure, 0, is called a *cipher*, or *nought*, and denotes *nothing*, or the absence of number.

Each of the other figures represents a number of *single things*, or units.

1. A Unit* is a single thing; as, 1 horse, 1 dog, 1 apple, 1 dozen, 1 fourth.

---

* NOTE FOR THE TEACHER. — Let this be fully explained, that the pupil may observe the distinction between a hundred as one unit, illustrated by a hundred-dollar-note, and *as many units*, illustrated by one hundred one-dollar-notes.

The teacher should question the class thoroughly on each subject; he should

2. *A Number* is a unit, or collection of units; as, 2 cows, 1 house, 5 sheep, 1, 3, 4, 9.

3. *A Concrete Number* is one which indicates some definite kind of thing; as, 1 bird, 7 sheep, 2 ounces.

4. *A Denominate Number* is one in which the kind of thing named is a unit of weight, measure, or currency; as, 5 pounds, 2 yards, 6 dimes.

5. *An Abstract Number* is one which does not indicate any definite kind of thing; 1, 5, 7, 6, 9, 14.

6. *Arithmetic* treats of numbers, and the art of using them.

NOTATION.

7. *Notation* is a method of expressing numbers by words, letters, or figures. The methods most used are the **Roman**, which employs letters, and the **Arabic**, which employs figures.

8. *The Roman Notation* uses the letters I, V, X, L, C, D, M. The numbers of sections, chapters, and

---

strive by illustrations and suggestions to develop in the mind of the pupil a clear conception of the idea contained in the definition, proposition, or problem, before requiring the one to be learned, or the other to be solved. The following notes and hints are merely suggestive, and are not intended to be either systematic or exhaustive. No book can take the place of a living teacher; neither can any teacher follow any prescribed method of instruction, for by so doing he sacrifices that individuality which is essential to success.

REVIEW QUESTIONS.* — What is a unit? (1.) What is a number? (2.) What is a concrete number? (3.) What is a denominate number? (4.) What is an abstract number? (5.) Of what does arithmetic treat? (6.)

* NOTE FOR THE TEACHER. — The numbers at the end of the questions refer *to the paragraphs containing* the answers.

N.B. — These review questions should by no means be omitted.

other divisions of books, are usually written by this method.

TABLE.

I.	denotes *one*,	1.	C.	denotes *one hundred*,	100.
V.	" *five*,	5.	D.	" *five hundred*,	500.
X.	" *ten*,	10.	M.	" *one thousand*,	1,000.
L.	" *fifty*,	50.	$\overline{\text{M}}$.	" *one million*,	1,000,000.

All other numbers are expressed by the use of these letters repeated or combined; as,—

I.	one.
II.	two.
III.	three.
IV.	four.
V.	five.
VI.	six.
VII.	seven.
VIII.	eight.
IX.	nine.
X.	ten.
XX.	twenty.
XXX.	thirty.
XL.	forty.
L.	fifty.
LX.	sixty.
LXX.	seventy.
LXXX.	eighty
XC.	ninety.
C.	one hundred.
CC.	two hundred.
CCC.	three hundred.
CCCC.	four hundred.
D.	five hundred.
DC.	six hundred.
DCC.	seven hundred.
DCCC.	eight hundred.
DCCCC.	nine hundred.
M.	one thousand.
MD.	fifteen hundred.
MM.	two thousand.
$\overline{\text{X}}$.	ten thousand.
$\overline{\text{M}}$.	one million.

OBSERVATIONS.—From the above it will be noticed, that

1. The value of a letter is repeated as many times as the letter itself is repeated; as, XXX. *thirty*, CC. *two hundred.*

2. If a letter of *less value* stands *before* a letter of *greater value*, the value of the less is *subtracted* from that of the greater; as, X. *ten*, IX. *nine*, L. *fifty*, XL. *forty*.

3. If a letter of *less value* stands *after* one of *greater value*, the value of the *less* is *added* to that of the greater; as, V. *five*, VI. *six*, M. *one thousand*, MD. *fifteen hundred.*

4. A dash (—) placed over a letter, or a combination of letters, repeats the value a thousand times; as X. *ten*, $\overline{\text{X}}$. *ten thousand*, XII. *twelve*, $\overline{\text{XII}}$. *twelve thousand.*

NOTE FOR THE TEACHER.—Slate and blackboard drill in writing and reading numbers in Roman notation should be furnished by the teacher.

REVIEW QUESTIONS.—What is the Roman notation? (7.) Why so called? What is its use? How many letters does it use? (8.) Name them. Which letter has the *greatest value?* Which the least? Which letter stands for fifty? *Which for ten?* Which for one hundred? Which for five hundred? What *effect has the repetition of a letter?*

ARABIC NOTATION.

9. *The Arabic Notation* uses ten characters, or figures; as,

0,	*cipher.* *nought.* *zero.*
1,	*one.*
2,	*two.*
3,	*three.*
4,	*four.*
5,	*five.*
6,	*six.*
7,	*seven.*
8,	*eight.*
9,	*nine.*

1, 2, 3, 4, 5, 6, 7, 8, and 9 are often called *digits.*

When we count ten things, as we have no figure to represent the number *ten,* we unite the ten into a group,* and call it *ten,* or, 1 *ten,* and represent it by writing the 1 in the second place, thus, 10; the cipher showing, that, while we have 1 *ten,* we have no additional units.

When we count twelve things, we have one ten and two single things, represented thus; 12.

When we count ten *tens,* as we have no figure to represent the number, we unite the *tens* into a group, and call it 1 hundred, and write the 1 in the third place; thus, 100; the ciphers showing that we have no additional *tens* or *units.*

10. *The Simple Value* of a figure is the one expressed by it when standing alone, or in the unit's place.

11. *The Local Value* of a figure is the one it acquires by standing in a particular place in a number; as, 3 in 30, where it represents three *tens,* or thirty; 3 in 364, where it represents three *hundreds;* &c.

12. *Numeration* is the art of reading numbers expressed by figures.

---

* *The different orders of* units can be very clearly illustrated by tying ten *sticks into a bundle to* represent 1 *ten,* then a package of ten such bundles to represent *1 hundred, &c.*

NUMERATION TABLE.

6th Period.*			5th Period.			4th Period.			3d Period.			2d Period.			1st Period.		
Quadrillions.			Trillions.			Billions.			Millions.			Thousands.			Units.		
Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Every place in a number not occupied by digits must be filled with a cipher.

The periods of a number are often separated by commas.

EXERCISES.

1. Write 4 hundreds and 2 units, commencing at the left hand, and read the number.

Write 3 tens and 7 units in the same manner, and read it.

Write 5 hundreds, 3 tens, and 2 units in the same manner.

Write 4 tens, and 6 units.

Write 5 hundreds and 4 units.

Write 7 hundreds and 2 tens.

2. Write 5 hundreds, 7 tens, and 2 units, and read the number.

Write 3 tens and 1 unit.

Write 5 hundreds and 6 units.

Units' Period.		
H.	T.	U.
4	0	2
	3	7
5	3	2
	4	6
5	0	4
7	2	0

---

* From quadrillions, the table may be extended to quintillions, sextillions, septillions, octillions, nonillions, decillions, undecillions, duodecillions, &c.

3. Write 6 units.
   9 tens and 4 units.
   6 hundreds and 7 units.
   5 hundreds and 6 tens.
   9 hundreds and 5 tens.
4. Write 7 hundreds and 6 tens.
   7 tens and 3 units.
   5 hundreds and 8 tens.
   6 hundreds and 5 units.
   3 tens and 7 units.
5. Write 8 tens and 3 units.
   7 hundreds and 6 tens.
   3 tens and 4 units.
   8 tens and 3 units.
   4 hundreds.
6. Write 3 tens and 8 units.
   6 tens and 3 units.
   5 hundreds and 4 tens.
   9 hundreds and 3 tens.
   7 tens and 8 units.

7. Write 2 in each period, commencing at the left hand, and read the number.

Write 22 in each period, commencing at the left hand, and read the number.

Write 222 in each period, commencing at the left hand, and read the number.

Note for the Teacher. — The pupil should be required to prepare the following exercises thus arranged, on slate or paper, to be read as a class-exercise. The mechanical execution of the work should be carefully inspected.

MODEL OPERATION.

Tr.			Bill.			Mill.			Th.			U.		
H.	T.	U.	H.	T.	U.	H.	T.	U.	H.	T.	U.	H.	T.	U.
		2	0	0	2	0	0	2	0	0	2	0	0	2
	2	2	0	2	2	0	2	2	0	2	2	0	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2

To be read. — Two *trillions*, two *billions*, two *millions*, two *thousand* and two.

Twenty-two *trillions*, twenty-two *billions*, twenty-two *millions*, twenty-two *thousand* and twenty-two.

Two hundred and twenty-two *trillions*, two hundred and twenty-two *billions*, two hundred and twenty-two *millions*, two hundred and twenty-two *thousand*, two hundred and twenty-two.

8. Vary example 7, by writing 3 wherever 2 was used; vary again by using 4, and thus by using each of the digits in succession. Read the numbers.

9. Write in each period the numbers from 11 to 20, *and read.*

10. Let each pupil dictate an example, to be written and read by the class.

The teacher should require the pupils, in the same manner, to write and read, as a class-exercise, series of numbers; as, from 1 to 20, from 100 to 130, &c., until promptness and accuracy are secured.

The exercise may be varied by dictating from the drill-card, Tables I. and II. (see page 17), as well as reading from the same. The teacher should write in figures, on the blackboard, numbers, to be copied in words by the pupils on their slates. The pupils' work should be criticised in spelling, &c.: numbers should also be copied in Roman notation, to be read as a class-exercise.

13. *Observe,* that, in the Arabic notation, —

1. Ten units of any order make one unit of the next higher order.

2. A digit in any place has a value ten times as large as that of the same digit one place to the right: hence,

3. The value of any figure is increased tenfold by removing it one place to the left. It is decreased tenfold by removing it one place to the right; thus tens will become UNITS; UNITS will become *tenths; tenths* become *hundredths*, &c.

REDUCTION.

Extend this table to millions, and present it as an exercise: —

10 *thousandths* = 1 *hundredth.*
10 *hundredths* = 1 *tenth.*
10 *tenths* = 1 UNIT.
10 UNITS = 1 ten, &c.

ANALYSIS OF NUMBERS.

It is required to analyze the number 3,917,893.

ANALYSIS. — 9 tens are equal to 90 units; 8 hundreds are equal to 80 tens, or 800 units; in the thousands' period 7 thousands are equal to 70 hundreds, or 700 tens, or 7,000 units; 1 ten of thousands is equal to 10 thousands, or 100 hundreds, or 1,000 tens, or 10,000 units: 9 hundreds of thousands are equal to 90 tens of thousands, or 900 thousands, or 9,000 hundreds, or 90,000 tens, &c.

After the model, analyze the following numbers: —

1. One million three hundred and eighty-five thousand six hundred and seventy-four.
2. 19 millions 713 thousand 486.
3. 186 millions 247 thousand 391.
4. 421 millions 223 thousand 86.
5. 5 trillions 876 millions 876 thousand.
6. 189 quadrillions 317 thousand 346.
7. 896 millions 348 thousand.
8. 876 millions 384 thousand.

14. ***A Rule*** is a concise direction for solving problems.

15. RULE FOR NOTATION. — *Begin at the left hand, and write in the place of each successive order the figure belonging to that order, filling with ciphers the places otherwise vacant.*

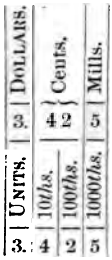
Indicate the place of units by a decimal point whenever an order is written which has a lower value than units.

16. RULE FOR NUMERATION. — *Begin at units' place, and separate the number into periods of three figures each.*

Beginning at the highest period, read each separately, giving the name of each period except that of units.

DECIMAL SCALE. — U. S. CURRENCY.

U. S. Currency.



Decimal Scale.

1. It will be seen by the accompanying scale that the United States currency is based upon the decimal scale of notation; the "dollars" corresponding to the *units*, the "cents" to the *tenths* and *hundredths*, and the "mills" to the *thousandths*.

2. Teachers who may desire can easily dictate numbers like the following: —

1. Write 3 and 4 *tenths.*	2. Write 3 dollars and 40 cents.
1 and 42 *hundredths.*	1 dollar and 42 cents.
16 and 327 *thousandths.*	16 dollars, 32 cents, and 7 mills.
5 and 41 *thousandths.*	5 dollars, 4 cents, and 1 mill.

For exercise in reading decimals, or United States Currency, see Drill Card, Table III. (page 17).

ANALYSIS OF SECTION I.*

I. Number.	**II. Notation.**	**III. Numeration.**
1. CONCRETE.	1. ROMAN.	ORDERS.
2. DENOMINATE.	*Letters.*	(*a.*) *Ratio of increase.*
3. ABSTRACT.	2. ARABIC.	(*b.*) *Names.*
	Figures.	
	Simple value.	
	Local value.	

---

* The "analysis" at the close of each section should form the outline for a topical review, which may be either oral or written, or both.

SECTION II.

ADDITION.

In this picture some boys and girls have been gathering peaches.

Arthur has 1 peach in each hand.

Ezra has 2 peaches in one hand, and 1 in the other.

Sophie has 3 peaches in a little basket, and 1 in her hand.

Selina has 4 peaches in her lap, and 1 in her hand.

How many has each?

If Sophie gives her peaches to Selina, how many will *she* have?

If all put their peaches into Selina's lap, how many *will she have?*

This putting of things together to find the entire number is called, in arithmetic, *addition;* and the number found is called the *sum*, or *amount:* hence,

17. *Addition* is the process of finding the sum of two or more numbers.

18. *The Sum* of two or more numbers is the number which contains all their units.

19. *The Sign of Addition* is a vertical cross; thus, +. It is called *plus*, and, when placed between two numbers, it denotes that they are to be added together; as, 4 + 6 equals 10.

20. *The Sign of Equality* is two short horizontal parallel lines; thus, =. When placed between two numbers or quantities, it denotes that they are equal; as, 4 + 6 = 3 + 7.

21. *The Dollar Sign* ($) is placed before dollars, and the decimal point after them; thus, $86.42, to be read eighty-six dollars and forty-two cents.

22. *An Axiom* is a self-evident truth. The following is used in addition:—

AXIOM.—*Things can be added to like things only.*

When it is required to find how many peaches Selina and Ezra have together, the question, in arithmetic, is called a *problem;* and the explanation of the act, or process of putting them together is called a *solution*, or an *analysis*. A problem having but one such question is called a *simple problem;* but one that has two or more questions, requiring an answer to each, is called a *complex problem*. Each question in a complex problem indicates an *analytical step:* hence,

---

REVIEW.—What is the Arabic notation? (9.) How many figures, or characters, are used? What is the simple value of a figure? (10.) What is the local value? *(11.) What is numeration?* (12.) Repeat the numeration table by *periods from right to left.* What is the use of the cipher?

'. ***A Problem,*** in arithmetic, is a question which ires solution.

4. ***A Simple Problem*** contains but one ele- tary question.

5. ***A Complex Problem*** contains more than elementary question.

26. ***An Analytical Step*** is one of the steps of process of solving a complex problem.

27. ***Order of Solving Problems:***—

1. State the problem.
2. Give necessary definitions.
3. Analyze.
4. Give conclusions.
5. Prove.

28. ***Analysis of Addition.****

PROBLEM.—How many are 389 + 486 + 394 + 437?

PROCESS.

389
486
394
437

1706

1. STATEMENT.—We are to find the sum of 389, 486, 394, and 437.

2. DEFINITION.—The sum of two or more numbers is the number which contains all their units.

3. ANALYSIS.—(1.) For convenience, the numbers are so written that units shall stand in the column of units, tens in the column of tens, &c.

(2.) For convenience, add the column of the lowest order; thus, 7 11, 17, 26: 26 units = 2 tens and 6 units. Write the units in th column of units, and add the tens with the column of tens; thus,

(3.) 2, 5, 14, 22, 30: 30 tens = 3 hundreds. Write 0 in the colum of tens, and add the hundreds with the column of hundreds; thus,

(4.) 3, 7, 10, 14, 17: 17 hundreds = 1 thousand and 7 hundre

---

* *The analysis of addition* may be omitted until a final review of the boo

Write the hundreds in the column of hundreds, and the 1 thousand in the place of thousands.

4. CONCLUSION.—Therefore the sum of 389, 486, 394, and 437, is 1 thousand 7 hundred and 6.

Hence we have the

5. RULE.—I. *Write the numbers so that units shall stand in the column of units, tens in the column of tens, &c.*

II. *Beginning with the lowest order, add each column separately: if the sum be less than ten, place it under the column added.*

III. *If the sum of any column be ten or more, write the units' figure of that order under the column added, and add the others with those of the next column.*

6. PROOF.—*Repetition is the only practical proof of addition. Add the numbers from the top downward, and, if the sum agrees with that previously obtained, it is presumed to be right.*

29. *Blackboard Exercises.*

The teacher should require the class, department, or entire school, to spend at least five minutes daily for ONE YEAR, in concert drill on the following column exercises in addition. In order to prevent habits of hesitation and inaccuracy, each column exercise should be THOROUGHLY MASTERED before attempting the next.

REVIEW.—What should be done with the vacant places of a period? (12.) What with vacant periods? How are the periods of a number usually separated? What is a rule? (14.) What is the rule for notation? (15.) Repeat the rule for numeration. (16.) What is addition? (17.) What is the sum of two or more numbers? (18.) Give the description and use of the sign of addition? (19.) Of the sign of equality? (20.) Of the dollar sign? (21.) What is an axiom? (22.) Repeat the axiom used in addition. What is a problem? (23.) What is a simple problem? (24.) Give an example. What is a complex problem? (25.) What is an analytical step? (26.) Give the order of solving problems. (27.)

The following table gives the key to the secret of rapid addition. It contains all the different changes in the combination of the digits by addition.

TABLE.

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)	(9.)	(10.)
&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.
2	2	1	3	1	4	4	1	5	3
2	2	2	3	2	4	4	4	5	2
2	2	1	3	1	4	4	3	5	1
2	2	1	3	2	4	4	1	5	2
2	2	2	3	3	4	4	2	5	4
2	2	2	3	2	4	4	1	5	3
2	2	1	3	1	4	4	4	5	1
2	2	2	3	3	4	4	2	5	2
2	2	1	3	1	4	4	3	5	3
2	2	1	3	2	4	4	4	5	1
2	2	2	3	3	4	4	3	5	4
2	1	1	3	1	4	1	1	5	5

(11.)	(12.)	(13.)	(14.)	(15.)	(16.)	(17.)	(18.)	(19.)	(20.)
&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.
6	6	3	7	3	8	8	1	9	3
6	6	4	7	4	8	8	7	9	1
6	6	3	7	3	8	8	8	9	4
6	6	6	7	1	8	8	3	9	2
6	6	4	7	2	8	8	4	9	3
6	6	6	7	3	8	8	3	9	1
6	6	5	7	6	8	8	1	9	4
6	6	3	7	7	8	8	7	9	3
6	6	6	7	1	8	8	8	9	6
6	6	3	7	3	8	8	7	9	7
6	6	5	7	1	8	8	3	9	1
6	1	1	7	7	8	1	1	9	9

METHOD OF RECITATION.

Copy the exercise on the blackboard, and let a pupil, or class in concert, commence at the bottom, and add, repeating only the result of each addition: thus, (Exercise 1) 2, 4, 6, 8, 10, 12, 14, &c., to 50, or more, as the teacher may judge best. When the first exercise is THOROUGHLY mastered, proceed with the second in the same manner, MASTERING each exercise successively, until all are finished. The exercises in this lesson need not be completed before the pupil proceeds; but they should form a part of each recitation until finished.

DRILL CARD EXERCISES.

N.B. — The problems in the following card exercises are graded by digits and steps, in the same way as those in the Primary Arithmetic. Such exercises should be selected as are best suited to the necessities of the class, as few if any classes will need them all.

The problems may be assigned for home work, seat or rapid class drill; but in no case should the pupils be allowed the answers while preparing lessons.

The *answers*, *explanations*, and *methods* are published in a KEY for the use of teachers only, and may be obtained through the mail by addressing the *publishers*.

TABLE I., *Units' Period.*	TABLE I., *Thousands.*	TABLE II., *Units.*
Exercise I. — 3 numbers.	Exer. VIII. — 3 numbers.	Exer. XV. — 2 numbers.
II. — 4 "	IX. — 4 "	XVI. — 3 "
III. — 5 "	X. — 5 "	XVII. — 4 "
IV. — 6 "	XI. — 6 "	XVIII. — 5 "
V. — 7 "	XII. — 7 "	XIX. — 6 "
VI. — 8 "	XIII. — 8 "	XX. — 7 "
VII. — 9 "	XIV. — 9 "	XXI. — 8 "
		XXII. — 9 "

TABLE II., *Thousands.**	TABLE II., *Billions.*	*U. S. Currency*, TABLE III.
Exer. XXIII. — 2 numbers.	Exer. XXXI. — 10 numbers.	Exer. I. — 3 numbers.
XXIV. — 3 "		II. — 4 "
XXV. — 4 "		III. — 5 "
XXVI — 5 "		IV. — 6 "
XXVII. — 6 "		V. — 7 "
XXVIII. — 7 "		VI. — 8 "
XXIX. — 8 "		VII. — 9 "
XXX. — 9 "		VIII. — 10 "

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* *Includes* units' and thousands' period.

DIRECTIONS FOR USING FELTER'S DRILL CARDS.

EXERCISE I.

PREPARATION: 1ST SERIES OF TWENTY PROBLEMS. 1st Example. — Copy three numbers from the card on a slate or paper, commencing with the number opposite the letter A (Table I., Units' Period).

2d Example. — Copy three numbers, commencing with the number opposite the letter B, as before.

3d Example. — Copy three numbers, commencing with the letter C.

4th Example. — Copy three numbers, commencing with the number opposite the letter D: thus,

(1.)	(2.)	(3.)	(4.)
A. 022	B. 202	C. 200	D. 220
202	200	220	222
200	220	222	110
Ans. 424	Ans. 622	Ans. 642	Ans. 552

In the same manner, commence with the number opposite each letter in the column between the tables. If there are not numbers enough in the column of figures to finish the example, commence again at the top, and take a sufficient number.

EXERCISE II.

SECOND SERIES: FOUR NUMBERS. 1st Example. — Copy (Table I., Units' Period) four numbers, commencing with the number opposite the letter A.

2d Example. — Copy four numbers, commencing with the letter B.

3d Example. — Copy as before, &c.

EXERCISE III.

THIRD SERIES. — Five numbers, copy as before.

REMARKS. — Pupils should not be allowed to copy results from each other. The careful teacher can easily detect and prevent it. No work should be accepted, unless neatly and carefully done. Once or twice a week, the lesson should be copied from dictation. A committee of one or two pupils may be appointed to examine the slates, and report on the neatness and accuracy of the work, and thereby save the teacher's time.

ANSWERS. — The answers in the key will be found to correspond with the letters opposite the first number in each problem.

RECITATION. — 1. Let the teacher mention the letter opposite the first number of each example; at which, let the class rise.

2. Let the teacher name one of the class to read his answer.
3. Let the pupils who have the same result sit.
4. If any remain standing, let the teacher name another pupil, as before.
5. Let the teacher read the correct answer; at which, let those who are correct rise, those who are wrong remain seated. Draw a line distinctly across the incorrect answers.
6. At the close, let the teacher make a record of the recitation. Those who are wrong correct their work at their seats.

FELTER'S

ARITHMETICAL DRILL CARDS.

TABLE I.		TABLE II.		TABLE III.
Thou. Units.		B. Mill. Thou. Units.		Dollars. Cts. m.
579,438		9,381,908,191		5,319.43 4
242,022	A	8,448,646,446	A	3,402 cts.
442,202	B	6,688,286,466	B	1,811 m.
442,200	C	868,424,246	C	60 dol.
240,220	D	606,486,644	D	403 cts.
402,222	E	428,664,246	E	10 dol.
331,110	F	75,351,535	F	121 m.
303,211	G	73,735,353	G	8 cts.
311,121	H	53,533,353	H	195 dol.
342,221	I	53,131,533	I	2,000 m.
221,121	K	33,713,555	K	715 dol.
550,112	L	29,036,632	L	86 cts.
404,201	M	19,241,747	M	7 dol.
040,121	N	9,866,474	N	9 m.
443,121	O	9,673,342	O	8,189 cts.
552,232	P	9,477,773	P	908 m.
243,110	Q	8,347,544	Q	4,003 cts.
152,231	R	887,744	R	500 dol.
435,112	S	347,654	S	6,001 cts.
543,121	T	281,263	T	2,100 m.
402,301	U	187,602	U	5,000 cts.

N.B.—A ***Key*** *containing* the answers to the problems on the *above card may be obtained* by *mail*, by addressing the publishers.

EXERCISES IN NOTATION AND ADDITION.

The following problems should be prepared from the book WITHOUT ANSWERS, on slates, or paper, by the pupils at their seats. They may be recited as follows:—

Those who are prepared on the first problem rise. One pupil is required to read *his* answer; and those whose answers agree with it then sit down. If any remain standing, they, also, are required to read, after which the teacher announces the correct result from the key. Those who are wrong draw a line across the incorrect answer, and report at the close of the recitation. The other problems may be recited in the same way. The recitation of ten problems should only take from *five to seven* minutes.

1. Add,	MODEL PROCESS.
1 thousand,	1000
1 thousand, 280,	1280
10 thousand, 50,	10050
50 thousand, 200,	50200
529 thousand, 625.	529625
	592155

2. Add,

80 thousand, 7,
40 thousand, 40,
70 thousand, 20,
50 thousand, 5,
90 thousand, 90,
4 thousand, 4.

3. Add,

300 thousand, 6,
400 thousand, 60,
900 thousand, 600,
11 thousand, 11,
14 thousand, 9,
700 thousand, 683.

4. Add,

500 thousand, 32,
600 thousand, 438,
200 thousand, 29,
900 thousand, 30,
800 thousand, 284,
20 thousand, 384.

5. Add,

538 thousand, 224,
829 thousand, 720,
80 thousand, 9,
400 thousand, 560,
940 thousand, 12,
82 thousand, 36.

REVIEW.—What is a unit? (1.) A number? (2.) A concrete number? (3.) *A denominate number?* (4.) An abstract number? (5.) Of what does arithmetic *eat?* (6.)

6. Add,

750 thousand, 20,
870 thousand, 496,
100 thousand, 20,
99 thousand, 50,
30 thousand, 98,
700 thousand, 561.

7. Add,

350 thousand, 880,
60 thousand, 56,
499 thousand,
22 thousand, 85,
456 thousand, 790,
600 thousand, 70.

8. Add,

3 millions, 387 thousand, 24,
4 millions, 46 thousand, 4,
9 millions, 8 thousand, 6,
20 millions, 7 thousand, 720,
10 millions, 900 thousand,
5 millions, 248 thousand, 6.

9. Add,

14 millions, 14 thousand, 14,
3 millions, 760 thousand, 480,
8 millions, 20 thousand, 2,
50 millions, 640 thousand,
22 millions, 321 thousand, 56,
8 millions, 777 thousand, 988.

10. Add,

25 millions, 927 thousand,
14 millions, 26 thousand, 327,
32 millions, 591 thousand, 33,
40 millions, 800 thousand,
8 millions. 700 thousand, 7,
11 millions,256 thousand,277.

11. Add,

65 millions, 999 thousand, 80,
8 millions, 200 thousand,
38 millions, 45 thousand, 240,
17 millions, 590 thousand,
25 millions, 25 thousand, 25,
20 millions, 444 thousand, 960.

12. Add,

400 millions, 900 thousand,
100 millions, 450,
700 millions, 7 thousand, 9,
900 millions, 489 thousand,
300 millions, 7 thousand, 60,
800 millions, 700 thousand, 19.

13. Add,

1100 millions, 920,
1800 millions,
300 millions, 4 thousand, 50,
500 millions, 200 thousand,
800 millions, 800 thousand,
100 millions, 567

14. Add,

400 millions, 9 thousand, 30,
900 millions, 99,
600 millions, 24 thousand,
1200 millions, 884,
300 millions, 350 thousand,
700 millions, 256.

15. Add,

1200 millions, 96 thousand,
1800 millions, 27,
200 millions, 426thousand, 200
500 millions, 956 thousand,
900 millions,
300 millions, 50.

16. Add,
5 hundred, 20,
99,
3 thousand, 96,
9 hundred,
26 thousand, 780,
1 million, 480.

17. Add,
36,
9 thousand, 900,
228 thousand,
4 millions, 26 thousand, 580,
500 millions, 4 thousand, 10,
4 hundred, 65.

18. Add,
1100 millions,
98,
24 thousand, 420,
7 millions, 20 thousand,
58,
200 millions, 4 thousand, 50.

19. Add,
10 thousand, 10,
11 millions, 726 thousand,
5 hundred, 90,
78,
300 millions, 989,
66 thousand, 251.

20. Add,
4 hundred, 50,
720 thousand, 787,
500 millions, 66 thousand,
69,
400 millions, 228 thousand, 90,
26 thousand, 10.

21. Add,
6 millions, 84 thousand, 291,
900 millions,
29,
650,
3 thousand, 80,
400 millions, 86.

22. Add,
790,
800 millions, 596 thousand,
500 thousand, 970,
88,
3 millions, 700,
26 thousand, 777.

23. Add,
900 thousand,
47,
19 millions, 19 thousand,
7 thousand, 47,
600 millions, 59,
9 hundred, 60.

Note to Teacher. — Examples *ad libitum* may be dictated from the drill cards, answers to which will be found in the key.

24. Add,

	PROCESS.
3 dollars, 20 cents, 1 mill,	$3.201
23 dollars, 40 cents, 2 mills,	23.402
63 cents, 2 mills,	.632
4 mills,	.004
4 cents, 4 mills.	.044
	$27.283

25. Add,
5 dollars, 33 cents, 4 mills,
2 dollars, 3 cents, 1 mill,
4 dollars, 5 cents, 3 mills,
6 dollars, 1 cent, 2 mills,
8 dollars, 5 cents, 1 mill.

26. Add,
4 dollars, 18 cents, 3 mills,
6 dollars, 5 cents, 2 mills,
46 dollars, 14 cents, 3 mills,
14 dollars, 21 cents, 1 mill,
142 dollars, 16 cents, 3 mills.

27. Add,
9 dollars, 49 cents, 7 mills,
5 dollars, 16 cents, 2 mills,
37 dollars, 22 cents, 4 mills,
1 dollar, 1 cent, 5 mills,
2 dollars, 0 cents, 4 mills.

28. Add,
13 dollars, 80 cents, 2 mills,
16 dollars, 3 cents, 3 mills,
15 dollars, 0 cents, 0 mills,
0 dollars, 0 cents, 8 mills,
0 dollars, 10 cents, 0 mills,

29. Add,
85 dollars, 85 cents, 3 mills,
1 dollar, 1 cent, 1 mill,
34 dollars, 14 cents, 0 mills,
58 dollars, 13 cents,
43 dollars.

30. Add,
58 dollars, 16 cents, 2 mills,
41 dollars, 38 cents, 4 mills,
4 cents,
4 dollars,
43 dollars, 23 cents.

31. Add,
48 cents,
5 mills,
10 dollars,
6 cents,
50 dollars.

32. Add,
32 dollars,
46 cents,
5 mills,
3 dollars,
2 cents.

Note to Teacher.—For additional examples, dictate from drill cards, Table III., in the same manner as in addition of simple numbers.

ORAL PROBLEMS CONTAINING BUT ONE ELEMENTARY QUESTION.

SIMPLE NUMBERS.

Mary has 30 books, and Margaret has 40: how many have both?

Formula.—30 books + 40 books = 70 books.

Analysis.—If Mary has 30 books, and Margaret has 40, they both *have the sum of 30 books* and 40 books, which is 70 books.

1. A man bought a cow for $30, and a horse for $80: what was the cost of both?

2. A farmer sold 60 sheep, and kept 40: how many had he at first?

3. Susan is 20 years older than John, and John is 40 years old: how old is Susan?

4. Some silk cost 60 cents, some calico 80 cents: what is the cost of both?

5. Edward has 60 nuts, which is 50 less than the number Charles has: how many nuts has Charles?

At the option of the teacher, additional oral examples may be formed by substituting the following numbers in the corresponding examples; as, 40 and 85 in the place of 30 and 80 in the first example; then, again, 26 and 82 in the place of 30 and 80.

1. 40 and 85, 26 and 82, 96 and 47.
2. 50 and 67, 93 and 72, 47 and 96.
3. 53 and 70, 67 and 21, 87 and 37.
4. 63 and 80, 57 and 62, 29 and 67.
5. 49 and 60, 23 and 72, 67 and 95.

UNITED STATES CURRENCY.

The examples of United States currency in the fundamental rules may be omitted until a review, if thought best by the teacher. We think, however, with the proper explanation, and sufficient card drill work, no difficulty will be experienced by the pupils.

6. A boy bought a knife for 30 cents, and a book for 80: how many cents did both cost?

FORMULA. — 30 cents + 80 cents = $1.10.

ANALYSIS. — They both cost the sum of 30 cents and 80 cents, which is (110 cents) 1 dollar and 10 cents.

7. A geography costs 80 cents more than an arithmetic: the arithmetic costs 60 cents. What is the cost of the geography? How many dollars and cents?

8. A pair of skates is worth 90 cents, which is ten cents less than the cost of a new sled: what is the cost of the sled? How many dollars and cents?

9. Henry sold some chestnuts for 80 cents, and some filberts for 90 cents: how much did he receive? How many dollars and cents?

10. If a pig cost 3 dollars, and a pair of chickens 90 cents, what is the cost of both? How many cents?

If desired, additional examples may be formed by substituting the following numbers in place of the numbers in the corresponding examples:—

6. 60 and 75, 83 and 72, 96 and 84.
7. 76 and 50, 64 and 63, 87 and 96.
8. 87 and 20, 63 and 22, 47 and 16.
9. 40 and 64, 78 and 31, 95 and 18.
10. 4 and 36, 3 and 94, 5 and 50.

WRITTEN PROBLEMS.

33. Henry had 15 marbles, James 36, and John 49: how many marbles had all?

Formula.—15 marbles + 36 marbles + 49 marbles = 100 marbles.

Analysis.—All must have had the sum of 15 marbles, 36 marbles, and 49 marbles, which is 100 marbles.

PROCESS.
15 marbles.
36 "
49 "
100 "

34. James gave 125 cents for a geography, 98 cents for a reader, and 75 cents for an arithmetic: what did all cost him?

The answers should be given in dollars and cents.

Review.—What is notation? (7.) Give the method of Roman notation. (7.) What is the difference between the Roman and Arabic notation? (8, 9.) What is the difference between the simple and the local value of a figure? (10, 11.)

35. Mary gave 137 cents for some calico, 39 cents for some braid, and 230 cents for some silk: how much did she pay for all?

36. A builder paid $610 for a city lot, and built upon it a house which cost him 2,085 dollars: how much did the house and lot cost him?

37. In January a laborer deposited in the savings bank 12 dollars, in February 30 dollars, in March $13, in April $21, in May 17 dollars, and in June $16: how much did he deposit in the six months?

38. A fruit-grower sold 123 barrels of apples to one man, 31 barrels of pears to another, 210 barrels of apples to another, and 30 barrels of pears to a fourth: how many barrels of fruit did he sell?

39. James paid 12 cents for a slate, 97 cents for a geography, and 113 cents for a reader: how much did he pay out?

40. Long Island is divided into three counties. King's County contains 72 square miles; Queen's County, 410 square miles; and Suffolk County, 1,200 square miles: how many square miles in the island?

UNITED STATES CURRENCY.

Additional work in United States currency may be given by substituting the following numbers in place of those in the corresponding examples above, the analyses and methods of recitation to be the same as in simple numbers.

35. $2.14, 86 cents, and $1.89. $1.67, $1.24, and $2.14.
36. $412.30 and $2,564.89, $512.67 and $1,234.73.
37. $13.13, $28, $15.17, $25, $31.17, and $3.427.
39. 25 cents, $1.19, and $1.125, 17 cents, 69 cents, and 99 cents.

41. The mason work of a new schoolhouse cost $1,220; the carpenter work, 1,037 dollars; and the painting and glazing, $427: how much did the schoolhouse cost?

42. A farmer sold his milk for one month for 387 dollars, the next month for $320, and the third month for $129: how much did he receive for the three months?

43. I had 103 sheep, my father gave me 444, I bought 321 more: how many sheep have I in all?

44. If I have 413 peach-trees in one orchard, 34 in another, and 212 in another, how many have I in all?

45. If I have $141 in one bank, and $636 in another, how many dollars have I in both?

46. If I sell 3 horses for $430, 9 cows for $334, and 55 sheep for $223, for how many dollars do I sell them all?

47. A drover sold 123 sheep, and kept 197: how many had he at first?

48. A farmer paid 5,750 dollars for his farm, and $375 for his tools: what was the cost of the whole?

UNITED STATES CURRENCY.

Form examples by substituting the following numbers in the corresponding examples which precede.

41. $1,371.20, $903.64, and $572.893.
42. $267.30, $241.37, $176.03.
45. $243.80 and $593.80, $972.46 and $913.80.
46. $398.67, $256.40, and $189.36.
48. $2,673.20 and $974.30, $1,867.60 and $213.
49. $9,167.60 and $8,913.23, $6,934 and $941.30.

REVIEW.—Repeat the numeration table. (12.) How do the places of a number increase or *decrease? (13.) In* which direction do they increase? What is a *rule? (14.) Repeat the rule for* notation. (15). Repeat the rule for numeration. (16.)

49. The purser of a steamer received $9,346 from th
passengers for fare, and $31,426 for carrying goods: ho
many dollars did he receive in all?

50. A man travelled 476 miles by railroad, 390 mile
by steamboat, and 120 by stage: how many miles in a
did he travel?

51. In one school are 516 pupils; in another, 314
and in another, 215: how many pupils in all?

52. A farmer sold 3 loads of hay. There were 1,78
pounds in the first load, 2,976 pounds in the secon
and in the third 2,967 pounds: how many pounds i
the three loads?

53. If I buy 3 horses for $430, 8 cows for $334, an
55 sheep for $321: how much will they cost me?

54. In the year 1869, I bought 1,472 sheep; in t
year 1870, 1,943 sheep; in the year 1871, 1,473 shee
how many did I buy in all?

55. In 1873, I sold 1,870 sheep; in 1874, 1,674 shee
in 1875, 1,864 sheep: how many did I sell in all?

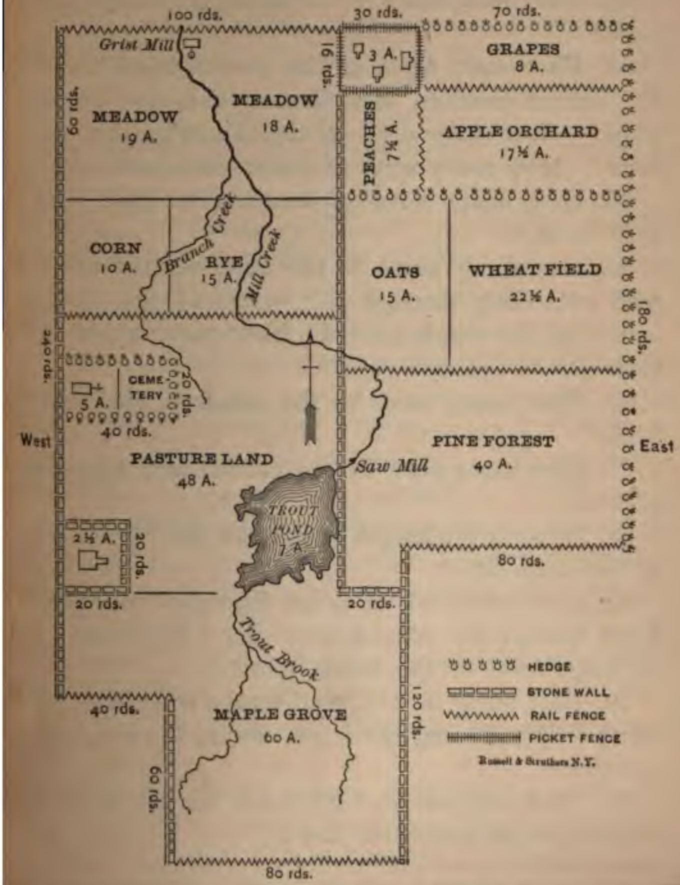
56. I bought 4,134 sheep, and sold 3,172; I the
bought 2,134 more, and sold 1,413: how many sheep d
I sell in all?

57. I bought 913 sheep, and sold 613; I then boug
217, and sold 188; after which I bought 691: ho
many sheep did I buy in all?

58. I had two fields of melons. In the first I ha
347, from which 189 were stolen; in the second I ha
439, from which 159 were stolen: how many melo
were stolen from the fields?

59. A man by his will gave his wife $1,234, to h
daughter 124 dollars, to his son $937: what was t
amount of his bequests?

60. If I owe A 347 cents, B 5,674 cents, C 3,413 cents, how many cents do I owe them all?



The above is the map of the Maple Grove Farm, where I live. you tell me how many acres are contained in each of the foll *portions:—*

61. How many acres in the pine-forest and the mapl grove?

62. How many acres in the maple-grove and pastur land?

63. How many acres in the pasture-land, includi the church-grounds, schoolhouse-yard, and the trou pond? How many acres of apples and peaches on t farm? How many acres of wheat, oats, corn, and rye

64. How many acres of meadow on both sides o Mill Creek?

65. How many acres in the farm west of the ston wall extending through the centre of the farm, no including the maple-grove? How many acres in th farm east of the same wall?*

66. How many acres in the meadow, corn, and ry fields?

67. How many acres in the church and schoolhous grounds?

68. What is the length of the east line? The lengt of the south line?

69. How many rods of hedge fence, not including th fence around the church-grounds? How many rod of hedge fence on the entire farm?

70. How many rods of wall on the west line? Ho many rods including the fence around the schoolhous grounds?

71. How many rods of rail fence on the south line On both north and south line?

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* The teacher will find little difficulty in adding the fractions, if first illu trated by halves of an apple.

REVIEW.—What is addition? (17.) What is the sum of two or more nu *bers? (18.)* Describe and give the use of the sign plus. (19.) Of the sign *ality. (20.) Of* the dollar sign. (21.)

72. How many rods of rail fence around the pine-forest? How many around the maple-grove and pine-forest?

73. I sold from my farm $124.30 worth of grapes, and $434.72 worth of apples: how much did I receive for both?

74. I received $53.70 for 105 bushels of oats, and $73.20 for 112 bushels of corn: how much did I receive? How many bushels did I sell?

75. I sold at one time 78 bushels of wheat for $93.20; at another time, 43 bushels for $75.23; and at another 67 bushels for $125: how many bushels did I sell? How much did I receive for it?

76. I cut from the meadow west of Mill Creek 64 tons of hay; from the east of the creek, 51 tons: how many tons were cut from the entire meadow? The hay on the west side was worth $150.50; on the east side, $100: how much was my hay worth?

77. The first week in April, there were sawed at the mill, 126,474 feet of maple lumber, and 623,478 feet of pine lumber: what number of feet were sawed during the week?

78. There are 40 acres in the pine-forest: how many acres in four such fields of equal size?

79. How many acres of land in the entire Maple Grove Farm?

80. There are three barrels of eggs. The first contains 675; the second, 1,276; the third, 1,575: how many eggs do these barrels contain?

81. A farmer sold three car-loads of potatoes. The first contained 174 barrels; the second, 259 barrels; the third, 199: how many barrels did the cars contain?

82. Gave 86 cents for a kite, 91 cents for a ball and bat, 138 cents for a wagon: what was given for all?

83. In an orchard there are 137 cherry-trees, 634 apple-trees, 72 pear-trees, 65 plum-trees: how many trees in the orchard?

84. The owner sold the cherries for $86, the apples for 387 dollars, the pears for $76, and the plums for 197 dollars: how many dollars did he receive for the fruit?

85. The hind-quarters of an ox weighed 375 pounds each; the two fore-quarters weighed 621 pounds; the hide and tallow, 112 pounds: what was the entire weight?

86. A merchant bought three pieces of cloth. The first measured 37 yards; the second, 41 yards; the third, 29 yards: how many yards in all the pieces?

87. He sold the first piece for 29 dollars, the second for 37 dollars, the third for $19: how many dollars did he receive for all the pieces?

88. Henry and James were playing marbles. Henry won in the first game 43, in the second, 57, and in the third, 156: how many did he have when he finished?

89. A father divided his money among his four sons, giving each 2,087 dollars: how much was he worth?

90. By selling a farm for $1,384, I lost $976: what was the cost of the farm?

91. Of three flocks of blackbirds, the first contained 3,467; the second, 4,171; the third, 3,741. A boy fired into the flocks, and frightened away 3,414: how many did the flocks contain before they were fired into?

Review. — What is an axiom? (22.) Repeat the one used in addition. Define *a problem.* (23.) What is the difference between a simple and a complex prob-
n? (24, 25.)

92. A man deposited $341 in a certain bank on Monday, and drew out $314; on Tuesday he deposited $693, and drew out $574; on Wednesday he deposited $674, and drew out $513: how many dollars did he deposit in the three days?

93. How many dollars did he draw out in the three days?

94. If Henry can count 5,389 in one hour, how many can he count in nine hours?

95. I paid $1,655 for flour, $1,473 for pork, $1,342 for beef, $187 for sugar, $134 for cheese: how much did I pay out in all?

96. I paid away $396 in gold, $574 in silver, and $413 in currency, and had $413 left: how many dollars had I at first?

97. I bought 730 bushels of potatoes, of which I sold 396 bushels; I then bought 1,341 bushels, of which I sold 387 bushels: how many bushels did I buy in all?

98. My father gave me $347, and I earned $395; he then gave me an amount equal to the sum I earned; afterward I earned $345. How much did my father give me? How much did I earn? How much did I have n all?

99. My father had $2,380. He gave me $278, and y sister $467; he then bought me a farm for ,008, and my sister a horse, for which he paid $347. w much cash did he give us? How much prop- ?

00. With my money I bought a cow for $45, five

VIEW.—*Define an analytical step.* (26.) Give the order of solving a prob- .) *Give the analysis of* an example in addition. (28.)

sheep for $33, and had $200 left: how much did my father give me?

101. By selling a farm for $6,479, the owner lost $2,734: what did it cost?

102. A man sold 327 bushels of corn for 296 dollars, 486 bushels for $329, and 320 for $211; this amount lacked $437 of what it cost him: what did it cost?

103. A lady went shopping, and bought some calico for 96 cents, some lace for 70 cents, some pins for 20 cents, and had 380 cents left: how much had she when she started?

104. Two boys started from the schoolhouse; one went north 76 rods, and the other went south 125 rods: how far were they then apart?

105. John and Henry started from the same place. John ran north 25 rods the first minute, and 27 rods the next minute; Henry ran south 23 rods the first minute, and 24 rods the next minute. How far did John run north? How far did Henry run south? How far were they apart at the end of the second minute?

106. John stood in the schoolhouse door, and shot an arrow from his cross-bow 20 yards up the road; a little boy from the schoolhouse brought it to him: how far did the boy run to bring it to John?

107. He then shot two arrows; one up the road 17 yards, and the other down the road 22 yards: how far must the little boy go to bring them to John?

REVIEW.—What is the difference between a unit and a number? (1, 2.) Between a concrete and an abstract number? (3, 5.) Between a concrete and a denominate number? (3, 4.) What is the use of arithmetic? (6.) What is the difference between the Arabic and the Roman notation? (8, 9.) Between the simple and the local value of a figure? (10, 11.)

108. Henry lends $913 to Thomas, $473 to Samuel, $576 to Theodore, and has $576 left: how many dollars had he at first?

109. A man was 37 years old when his son was born: how old will he be when his son is 59 years old?

110. Marcus had $347, and Oren had $576 more than Marcus: how many dollars had Oren?

111. Lillie had 17 roses; Laura has 11 more than Lillie, Charles has 18 dahlias more than Laura has roses, and Jennie has 14 dahlias more than Charles: how many dahlias has Jennie?

112. Henry's purse contains 329 cents; Edward's contains 43 more than Henry's, and Henry's contains as many cents as Sarah's, lacking 94 cents: how many cents does Sarah's purse contain?

113. Mary bought a pencil for which she gave 95 cents; Maude bought one for which she gave 13 cents more than Mary; and Maude's pencil cost as much as Jane's, lacking 23 cents: what was the cost of Jane's pencil?

UNITED STATES CURRENCY.

114. Add $3.14, $2.17, $9.84, and $1.37.

115. Add $3.14, $0.39, $0.67, and $1.67.

116. Add $14.132, $31.47, $56.213, and $21.

117. Add $46.141, $2.13, $6.17, $0.234, and $31.64.

118. Add $21, $0.21, $0.021, $0.002, and $0.02.

119. Add $30, 50 cents, 40 dollars, and 6 mills.

120. Add 30 cents, 2 dollars and 2 cents, 2 cents and 2 mills.

121. Add 50 cents, 50 dollars, 3 dollars and 16 cents.

sheep for $33, and had $200 left: how much did m father give me?

101. By selling a farm for $6,479, the owner los $2,734: what did it cost?

102. A man sold 327 bushels of corn for 296 dollar 486 bushels for $329, and 320 for $211; this amour lacked $437 of what it cost him: what did it cost?

103. A lady went shopping, and bought some calic for 96 cents, some lace for 70 cents, some pins for 2 cents, and had 380 cents left: how much had she whe she started?

104. Two boys started from the schoolhouse; or went north 76 rods, and the other went south 125 rods how far were they then apart?

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REVIEW.—What is the difference between a unit and a number? (1, Between a concrete and an abstract number? (3, 5.) Between a concrete and denominate number? (3, 4.) What is the use of [illegible] [illegible]; is t difference between the Arabic and the Roman [illegible] [illegible] t simple and the local value of a figure? [illegible]

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118. Add $21, $0.21, $0.021, $0.002, and $0.02.

119. Add $30, 50 cents, 40 dollars, and 6 mills.

120. Add 30 cents, 2 dollars and 2 cents, 2 cents and mills.

121. Add 50 cents, 50 dollars, 3 dollars and 16 cents.

122. Add $1.14, 5 dollars and 5 cents, and 13 cents and 2 mills.

123. Add 49 cents, 4 cents and 9 mills, and 9 mills.

124. Add 6 dollars and 6 mills, 4 cents, 2 mills.

125. $43 + $25 + $1.30 + $21.303 + $0.004 = how much?

126. $37 + $41.30 + $13.13 + $2.46 + $0.13 + $1,913 = how much?

127. Add 5 dollars, 50 cents, 5 cents, and 5 mills.

128. Add 30 dollars, 30 dollars and 30 cents, 30 cents and 3 mills.

129. Add $50, 25 cents, 1 dime, 1 cent, and 1 mill.

130. Add 1 eagle, 1 dollar, 1 dime, 1 cent, and 1 mill.

131. What is the cost of the following items: 9 wagons for 312 dollars and 6 cents, 4 horses for 987 dollars, 1 wheelbarrow for 3 dollars, 9 ploughs for $35 and 4 cents, 2 pigs for $2.25.

132. What is the cost of the following items: 1 gallon of molasses for 38 cents, 1 pound of cheese for 13 cents, 3 pounds of butter for 1 dollar and 30 cents, 41 pounds of coffee for 25 dollars, 19 gallons of vinegar for 4 dollars and 30 cents.

133. A farmer bought a cow for which he paid 30 dollars, a horse for which he paid $104.31, a wagon for which he paid 85 dollars: how much did the whole cost?

134. Mr. D. Spring sold farmer Mahan 1 caddy of tea for $7.38, 9 yards of cloth for $27.16, a plough for

---

REVIEW. — Give the rule for notation. (15.) For numeration. (16.) Give rule for addition. (28.) How is addition proved? Place on the blackboard, and show the use of, the sign of addition. (19.) The sign of equality. (20) The dollar sign. (21).

9 dollars and 16 cents, a harness for 21 dollars, 16 cents and 5 mills :, what was the amount of the bill?

135. A man is indebted to A $3.41, to B $6.14, to C $21.39, and to D 13 dollars and 5 cents: how much does he owe them?

136. Paid $4.37 for a hat, $13.13 for a coat, $5.62 for pair of boots, and 98 cents for an umbrella: what was paid for the whole?

137. One day a toll-gate keeper received $17.16; the next two days, $13.27 each: how much did he receive for the three days?

138. A furniture-dealer sold a washstand for $6.50, a bureau for 11 dollars and 13 cents, and 3 rocking-chairs for $4.37 each: for how much did he sell them all?

139. A man received $39.40. He paid $1.342 for flour, $3.171 for sugar, $0.124 for meal, and $0.137 for salt: how much did he pay out in all?

140. A man bought a village lot for 325 dollars; he sold it so as to gain $123.416: for how much did he sell it?

141. I bought a hat for $3.28, a coat for 11 dollars, and a pair of boots for 13 dollars: how much did I pay for them?

142. I bought 340 bushels of potatoes for $75, 68 bushels of corn for $39.20, and 112 bushels of wheat for $137.243: how much did all cost? How many bushels did I buy?

143. Bought 16 lbs. of sugar for 187 cents, 17 lbs. of coffee for 273 cents, and 34 lbs. of rice for 357 cents: how many cents did I give for all? How many dollars and cents? How many pounds did I buy?

144. A boy bought 13 oranges for 67 cents, 24 lemons for 87 cents, 19 apples for 16 cents: how many cents did all cost? What number of oranges, lemons, and apples had he, counted together? How many oranges and lemons, counted together?

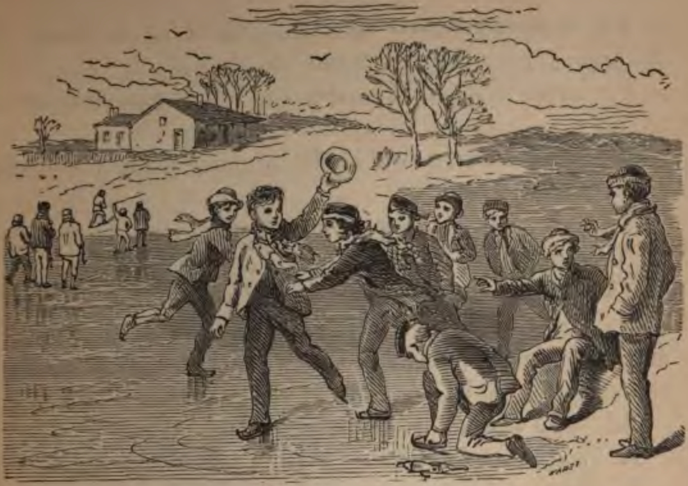
145. Thomas W. Scudder bought a city lot for $125. He built a house worth $538.76, he spent $38.26 for fencing, he then sold it for $189.72 more than the whole cost: for how much did he sell it?

146. 43 + 971 + 1,893 + 12 + 4 + 8 = how many?
147. $27 + $36.48 + $3.946 + $4.21 = how much?
148. $0.27 + $3.648 + $39.46 + $0.421 = how much?
149. $0.027 + $0.364 + $3946 + $0.042 = how much?
150. $0.002 + $3648 + $3946 + $421 = how much?

ANALYSIS OF SECTION II.

Addition.

1. ILLUSTRATION.
2. DEFINITION.
3. SIGNS.
4. AXIOM.
5. PROBLEM.
    - (a.) *Abstract Problem.*
        - Process.
        - Analysis.
        - Rule.
    - (b.) *Concrete Problem.*
        - Simple.
        - Complex.
        - Formula.
        - Analytical Step.
        - Analysis.



SECTION III.

SUBTRACTION.

IN this picture the boys have a skating-party. There were at first 15 boys; but 6 of them, becoming tired of the fun, have started for home: how many remain?

Of those who remain, 2 have no skates: how many have skates?

Of those who have skates, 1 is just putting on his skates: how many have their skates on?

Of the boys who remain, all but 2 are standing: how many are on their feet?

Six of the boys started for home, leaving 9 on the ice: how many more stay than go?

This taking of one number from another, or taking away a part to find how *many are left,* is called *subtraction*, and is just the opposite *of putting things together,* or addition.

31. *Subtraction* is the process of finding the difference between two numbers.

32. *The Subtrahend* is the number or part taken away.

33. *The Minuend* is the number from which the *subtrahend* is taken.

34. *The Remainder, or Difference,* is the part of the minuend left after the subtrahend is taken away.

35. *The Sign of Subtraction* is a short horizontal line; thus, —. It is called ***minus***, meaning ***less;*** and, when placed between two numbers, it denotes that the second is to be taken from the first; as, $6 - 4 = 2$.

36. The axioms used in subtraction are, —

1. ***Things can be subtracted from like things only.***
2. ***If two numbers are equally increased or diminished, their difference will not be changed.***

37. ANALYSIS OF SUBTRACTION.

1. PROBLEM. — Find the difference between 3027 and 1538.

PROCESS.

Subtraction.		*Proof.*	
3027	minuend.	1538	subtrahend.
1538	subtrahend.	1489	difference.
1489	difference	3027	sum = minuend.

2. ANALYSIS. — (1.) For convenience, write the subtrahend under the minuend, placing units under units, tens under tens, &c.

(2.) For convenience, begin to subtract at units; thus,

(3.) 8 units cannot be taken from 7 units; but one of the 2 tens can be changed to units (1 ten = 10 units), and 10 units and 7 units are 17 units. 8 units from 17 units leave 0 units, to be written in the place of units.

(4.) As one of the 2 tens has been used, only 1 ten remains, from *which 8 tens cannot* be taken. Take 1 from the next significant figure of the minuend (which, in this case, is 3 thousands), 1 thousand = 10 hundreds; of the 10 hundreds take 1 hundred = 10 tens: 10 tens and 1 ten are 11 tens; 3 tens from 11 tens leave 8 tens, to be written in the place of tens.

(5.) 5 hundreds from 9 hundreds leave 4 hundreds.

(6.) 1 thousand from 2 thousands leaves 1 thousand.

3. Conclusion. — Therefore, 1538 subtracted from 3027 leaves 1489 as a remainder.

4. Rule. — I. *Write the numbers so that the units of the subtrahend shall stand under the units of the minuend, the tens under the tens, &c.*

II. *Begin at the right hand, and, if it can be done, subtract each figure of the subtrahend from the figure above it, and write the result underneath.*

III. *If any figure of the subtrahend is greater than the corresponding figure of the minuend, add ten to the figure of the minuend before subtracting. In this case, make the next left-hand figure in the minuend* ONE *less, (or the next left-hand figure in the subtrahend* ONE *greater.)*

5. Proof. *Add the difference to the subtrahend; and, if the sum equals the minuend, the work is correct.*

38. Some prefer the following analysis: —

3897
2938
——
959

Analysis. — (1.) As 8 units cannot be taken from 7 units, add 10 units to the 7, making 17 units: 8 units from 17 units leave 9 units, to be written in the place of units.

(2.) Since 10 units have been added to the minuend, add their equal, 1 ten (Axiom 2, **36**), to the subtrahend. 1 ten and 3 tens are 4 tens: 4 tens from 9 tens leave 5 tens, to be written in the place of tens.

(3.) As 9 hundreds cannot be taken from 8 hundreds, add 10 hundreds to the 8 hundreds, making 18 hundreds: 9 hundreds from 18 hundreds leave 9 hundreds, to be written in the place of hundreds.

(4.) Since 10 hundreds have been added to the minuend, add their equal, 1 thousand, to the subtrahend: 2 thousands and 1 thousand are 3 thousand; 3 thousands from 3 thousands leave nothing.

DRILL CARD EXERCISES.

TABLE I., *Thousands.*

Exercise I. — 579438 minuend.
II. — 885973 “
III. — 868943 “
IV. — 998998 “
V. — 687789 “
VI. — 889787 “

TABLE II., *Thousands.*

Exer. XIII. — 944444 minuend.
XIV. — 955555 “
XV. — 966666 “
XVI. — 977777 “
XVII. — 988888 “
XVIII. — 999999 “

TABLE II., *Thousands.*

Exer. VII. — 908191 minuend.
VIII. — 809081 “
IX. — 908080 “
X. — 911111 “
XI. — 922222 “
XII. — 933333 “

TABLE II., *Thousands.*

Exer. XIX. — 1000000 minuend
XX. — 1020401 “
XXI. — 2413621 “
XXII. — 1010201 “

TABLE II., *Billions.*

XXIII. — 9381908191 “
XXIV. — 9503403916 “

TABLE II., *Billions.*

Exer. XXV. — Find difference of A and B, B and C, C and D, &c

TABLE I., *Thousands.*

Exer. XXVI. — Find difference of A and B, B and C, C and D, &

SUBTRACTION. — UNITED STATES CURRENCY. — TABLE III.

Exer. I. — $973.864 minuend.
II. — $734.001 “
III. — $846 minuen
IV. — $800 “
V. — Find difference of A and B, B and C, C and D, &c.
VI. — Find difference of A and C, B and D, C and E, &c.

DIRECTIONS FOR USE OF DRILL CARDS.

EXERCISE 1. — FIRST SERIES OF TWENTY PROBLEMS (TABLE I UNITS' AND THOUSANDS' PERIOD). Copy and subtract the numbe opposite each letter from 579438; thus,

Example (1).	Example (2).	Example (3).
579438	579438	579438
A. 242022	B. 442202	C. 442200
Ans. 337416	Ans. 137236	Ans. 137238

N.B. — *See Exercises* above for *minuends.* For answers, see Key.

FELTER'S

ARITHMETICAL DRILL CARDS.

TABLE I.		TABLE II.		TABLE III.
Thou. Units. 579,438		B. Mill. Thou. Units. 9,381,908,191		Dollars. Cts. m. 5,319.43 4
242,022	A	8,448,646,446	A	3,402 cts.
442,202	B	6,688,286,466	B	1,811 m.
442,200	C	868,424,246	C	60 dol.
240,220	D	606,486,644	D	403 cts.
402,222	E	428,664,246	E	10 dol.
331,110	F	75,351,535	F	121 m.
303,211	G	73,735,353	G	8 cts.
311,121	H	53,533,353	H	195 dol.
342,221	I	53,131,533	I	2,000 m.
221,121	K	33,713,555	K	715 dol.
550,112	L	29,036,632	L	86 cts.
404,201	M	19,241,747	M	7 dol.
040,121	N	9,866,474	N	9 m.
443,121	O	9,673,342	O	8,189 cts.
552,232	P	9,477,773	P	908 m.
243,110	Q	8,347,544	Q	4,003 cts.
152,231	R	887,744	R	500 dol.
435,112	S	347,654	S	6,001 cts.
543,121	T	281,263	T	2,100 m.
402,301	U	187,602	U	5,000 cts.

N.B. — A KEY *containing* the answers to the problems on the *above card may be obtained* by *mail*, by addressing the publishers.

ORAL PROBLEMS CONTAINING BUT ONE ELEMENTA QUESTION.

1. Henry had 50 apples, he gave 30 of them Charles: how many had he left?

FORMULA. — 50 apples – 30 apples = 20 apples.

ANALYSIS. — Henry must have had left as many apples as difference between 50 apples and 30 apples, or 20 apples.

2. James is 80 years old, his son is 50: how m older is James than his son?

3. One bag contains 80 pounds of wheat, anot bag contains 60 pounds: how many more pounds d one bag contain than the other?

4. There are 109 scholars in a school; 81 of them girls: how many boys in the school?

5. A farm is worth $896, and a house is worth $1 how much more is the farm worth than the house?

6. A man travelled 890 miles; in returning, he tr elled the first day 60 miles: how far had he yet to trav

7. Mary went shopping with 120 cents, and, w she returned, she had only 80 cents left; how m cents had she spent?

8. A cistern which holds 110 gallons was full in morning, but at night there were but 30 gallons rema ing: how many gallons had leaked out?

Substitute the following numbers in the above examples: —

Example 2. 93 and 42, 76 and 54, 84 and 65.
" 3. 74 and 33, 120 and 60, 121 and 75.
" 4. 108 and 76, 97 and 55, 101 and 47.
" 5. $483 and $62, $372 and $49, $100 and $76.
" 6. 89 and 43, 78 and 69, 146 and 97.
" 7. 133 and 28, 67 and 34, 158 and 73.
" 8. 150 and 73, 100 and 84, 150 and 34.

UNITED STATES CURRENCY.

1. Charles had $2; he gave 97 cents for a knife: how many cents had he left?

ANALYSIS. — $2 equal 200 cents: he must have had left the difference between 200 cents and 97 cents, or 103 cents = $1.03.

2. Susan had 200 cents; she spent 46 cents: how many cents had she remaining? How many dollars and cents?

3. A boy had 100 dollars; he gave away 97 cents: how much money had he left? How many dollars and cents?

4. A man had 10 dollars; he spent 6 cents: how much had he left?

5. A little girl had $1.50; she bought a book for 80 cents: how much money had she left?

6. A woman went shopping with $3.30 in her purse; on her return she had only 80 cents left: how much did she spend?

7. A man went to market with a 5-dollar bill; on his return he had a 2-dollar bill, and 54 cents in change: how much had he spent?

8. A boy bought a sled for $1.45; he sold it so as to lose 86 cents: for how much did he sell it?

Substitute the following numbers in the above examples: —

Example 1. $3 and 55, $2 and 37, $2.50 and 99.
" 2. 108 and 54, 79 and 39, 185 and 77.
" 3. 100 and 67, 100 and 100, 10 and 10.
" 4. 40 and 40, 20 and 20, 5 and 5.
" 5. $2.20 and 30, $6 and 60, $1.10 and 39.
" 6. 4 dollars and 40 cents, $3.56 and 97 cents.
" 7. *10-dollar bill* and 3-dollar bill, 60 cents.
" 8. $2.30 and 75, $1.67 and 58, $3.80 and 94.

WRITTEN EXERCISES.

1. A man who had a farm of 623 acres gave his son 186 acres: how much land had he left?

2. In a certain school-district there are 389 children of school age; of these only 297 attend school: how many do not attend school?

3. A grocer receiving an order for 386 barrels of beef has only 275 barrels on hand: how many more barrels will he require to fill the order?

4. In a certain town, 435 men were drafted for the army, but 167 were rejected by the examining surgeon: how many passed examination?

5. A market-gardener in one year received $3,730 for fruits and vegetables, and his expenses were 2,134 dollars: how much were his profits?

6. I have $374 in the savings bank: if I draw out 89 dollars, how much money shall I have left on deposit?

7. A merchant sold for $1,034 goods which cost him 989 dollars: how much did he gain by the transaction?

8. Henry has 197 marbles less than John, and John has 200: how many has Henry?

9. The sum of two numbers is 25, one of the numbers is 17: what is the other?

10. The difference of two numbers is 34, the greater is 97: what is the less?

11. A farmer has 7,012 sheep, and as many lambs, lacking 749: how many lambs has he?

REVIEW.—What is a unit? (1.) Name a number. (2.) When is a concrete *number called denominate?* (4.) What is the difference between a concrete and *an abstract number? (3, 5.).* rithmetic treat? (6.)

12. Joseph has 513 cents, and Jerrie as many, lacking 213: how many cents has Jerrie?

13. The sum of two numbers is 13,416, and one of the numbers is 4,137: what is the other?

14. The sum of two numbers is 5,416, the greater of the numbers is 4,134: what is the less?

15. The sum of two numbers is 13,416, the less is 3,147: what is the greater?

16. Our national independence was declared in the year 1776: how many years from that time to the close of the war in 1815?

17. The war of the Revolution broke out in the year 1775: how many years to the breaking-out of the war with Mexico, in 1846?

18. America was discovered in the year 1492: how many years from that period to the landing of the pilgrims, in 1620?

19. The mariner's compass was invented in Naples, 190 years before America was discovered, in 1492: in what year was the compass invented?

20. Bonaparte was declared emperor in the year 1804, at which time he was 35 years of age: in what year was he born?

21. Sir Isaac Newton was born in 1642, and died in 1727: how old was he when he died?

22. Gunpowder was invented by Schwartz, in the year 1330: how long was this before the invention of printing, which took place in the year 1441?

23. What number is that which, taken from 2579, will leave 1397?

REVIEW.—How many methods of notation in common use? (7.) Name the letters used in Roman notation, and give the value of each. (8.) When is a figure said to have a simple value? (10.) When a local value? (11.) Recite the numeration-table by periods. (12.) In what ratio do the orders in a number increase? (13.)

24. If the remainder is 15, the subtrahend 29, what is the minuend?

25. Supposing a man to have been 80 years old in 1854, in what year was he born?

26. There are two numbers whose difference is 876, the greater number is 1673: what is the less?

27. What number is that, which, taken from 3,795, will leave 473?

28. How long time will elapse between the years 1874 and 1892?

29. What number is that which, if added to 789, will make 1637.

30. One of two numbers is 387, the other is 673: what is their sum?

31. One of two numbers is 387, the other is 673: what is their difference?

32. The minuend is 3,917, the subtrahend is 413: what is the remainder?

33. The subtrahend is 4,167, the remainder is 413: what is the minuend?

34. 38 thousand 24 *less* 3 thousand 40 are how many?

35. 3 thousand 47 *less* 583 equals what?

36. 46 thousand and 6 *less* 3,413 equals what?

37. 25 thousand *less* 3 thousand and 3 = what?

38. 163 thousand *less* 46 thousand 213 = what?

39. 327 thousand 147 *less* 25 thousand 25 = what?

40. 439 thousand less 4,712 equals what?

REVIEW.—What is a rule? (14.) Recite the rule for notation. (15.) For numeration. (16.) Define addition. (17.) What is meant by the sum of two or more numbers. (18.) Illustrate the sign of addition. (19.) Of equality. (20.) Illustrate the use of the dollar-sign. (21.) What is an axiom? (22.) Give the axioms used in addition; in subtraction. (22, 36.) Define a problem. (23.) A simple problem. (24.) A complex problem. (25.)

UNITED STATES CURRENCY.

41. From $896 take $41.30.
42. From $100 take $0.57.
43. From $389 take $0.643.
44. From $52.60 take $1.234.
45. From $327 take $21.34.
46. From $345 take $62.13.
47. From 95 dollars take 95 cents.
48. From 100 dollars take 100 cents.
49. From 10 dollars take 10 cents.
50. From 10 cents take 10 mills.
51. From 1 cent take 1 mill.
52. From $3.20 take 1 cent and 1 mill.
53. From $2.413 take 20 cents and 2 mills.
54. From $6.134 take $3.37.
55. From 58 dollars take $1.334.
56. From 97 dollars take $21.213.
57. From 12 dollars take $3.713.

58. A man purchased a yoke of oxen for $78, a cow for 26 dollars and 50 cents: how much more did he give for the oxen than for the cow?

59. A man buys a horse for $97.50, and gives a hundred-dollar-bill: how much change should he receive?

60. How much must be added to $60.039 to make the sum 1 thousand and 5 dollars?

61. How much must be added to 38 dollars 79 cents to make the sum 100 dollars and 3 mills?

62. A man bought a pair of oxen for $100, and sold them for $75.375: did he make, or lose? and how much?

REVIEW.—*Define an analytical step.* (26.) Give the analysis of an example *in addition.* (28.) *How is addition proved?* What is the answer called? (18.)

63. A man starts on a journey with a hundred dollars; he spends 87 dollars, 87 cents and 5 mills: how much has he left?

64. How much must be added to $40.173 to make 100 dollars?

PROBLEMS CONTAINING NOT MORE THAN TWO ELEMENTARY QUESTIONS.

PROBLEM. — Sold a cow for $20, a calf for $12; received in pay $19; how much was still due?

FORMULA. — 1. $20 + $12 = $32.
2. $32 — $19 = $13. Ans.

ANALYTICAL STEPS. — 1. *Find the amount for which the cow and calf were sold.*
2. *Find the amount due.*

The analytical steps should be given in the analysis of every problem.

65. A farmer bought a horse for $87, and a wagon for $50; he exchanged both for two yoke of oxen worth $200: how much did he gain?

66. A man bought a yoke of oxen for $97; their keeping cost $13; he sold them for $125: did he gain, or lose? and how much?

67. A box of raisins was bought for $7, a firkin of butter for $28; both were sold for $40: how much was gained?

68. A jeweller bought a watch for $86, a chain for $36; he sold both for $175: how much did he gain by the bargain?

69. Henry and George started from the same place, and travelled in the same direction. Henry travelled 160 miles, and George 78: how far apart were they?

70. Two boats sail from the same place, and in the

same direction. One sails at the rate of 190 miles a day; the other, at the rate of 97 miles: how far apart will they be at the end of the day?

71. A father is 90 years of age, and is 37 years older than his eldest son: how old is his son?

72. James had 40 cents; he paid 12 cents for a whistle, and 25 cents for a knife: how many had he left?

73. Arthur had 97 nuts in a basket; he put 12 of them in one pocket, and 13 in the other: how many were left in the basket?

74. A schoolboy paid 56 cents for an atlas, and 30 cents for an arithmetic; he gave the shopkeeper a dollar-bill: how much change should he receive?

75. A drover bought 190 sheep of three farmers; he bought 27 of the first, 28 of the second: how many did he buy of the third?

76. Lillie had 9 oranges, her mother gave her 11 more, and her father enough to make 40: how many did her father give her?

77. A man had 88 dollars, and gave 28 dollars for a cow, and 14 dollars for a calf: how much money had he left?

78. A man bought one barrel of pork for $18, and another for $17; he sold both for $46: how much did he gain?

79. George spent 19 cents for candy, and 17 cents for fruit: how much more will he have to spend to make 50 cents.

80. Two men bought a horse; the one paying 110 dollars, and the other 25 dollars less: how much did *both pay?*

81. A man sold a firkin of butter for 17 dollars, a cheese for \$19; he received in payment 4 barrels of flour worth \$36: how much remains due?

82. Solomon gave 13 cents for some oranges, 27 cents for some cake, and had 47 cents remaining: how much had he at first?

83. Stoddard at a game of marbles won 13, and then lost 25, after which he had 17 remaining: how many had he at first?

84. A farmer bought a horse for \$67, and exchanged it for a yoke of oxen, which he sold for \$87: how much did he gain by the transaction?

85. A box of raisins was bought for 5 dollars, a firkin of butter for \$18; both were sold for 20 dollars: how much was lost?

86. A drover bought some sheep for \$40, some cattle for \$130; he sold them all for \$200: what was his gain?

87. A jeweller bought a watch for \$83, and a chain for 15 dollars; he sold them both for \$105: what was his gain?

88. A merchant bought a hogshead of molasses for \$53; he paid \$5 cartage: for how much must he sell it to gain \$23?

89. I bought a horse for 90 dollars. a carriage for \$75; I sold them both at a loss of \$37: how much did I receive for them?

39. A Parenthesis () denotes that all the numbers enclosed are to be taken as a whole, and that the same process is to apply to them all. Thus $14 - (3 + 4)$ signifies that the sum of 3 and 4 is to be taken from 14: $(13 - 4) - (5 + 2)$ signifies that the sum of 5 and 2 is *to be taken from* the difference of 13 and 4.

COMPLEX PROBLEMS REQUIRING NOT MORE THAN TWO ANALYTICAL STEPS.

PROBLEM. — Of a salary of \$2,447, a man spent \$650.40 for rent, and \$834.17 for family expenses: how much remained?

PROCESSES.

(*a.*) \$2,447 — \$650.40 — \$834.17 = \$962.43.
(*b.*) \$2,447 — (\$650.40 + \$834.17) = \$962.43.

ANAL. STEPS. — (*a.*) 1. *Find what was left after paying rent; then,*
2. *Find what was left after paying family expenses.*

(*b.*) 1. *Find what was paid for rent and expenses.*
2. *Find what was left of the salary.*

NOTE FOR TEACHER. — The attention of the pupil should be called to the two methods of solving problems of this class.

90. A boy had 378 cents; he gave 127 cents for a book, and 40 cents for a slate: how many cents had he remaining?

91. A drover bought a drove of cattle for \$6,843; he paid at one time \$462, at another time \$413: how much did he still owe?

92. A boy had 100 cents; he bought an arithmetic costing 63 cents, and a slate which cost 30 cents, and gave the remainder for a sponge: what did the sponge cost him?

93. I lent a man \$13.74; he paid me at one time \$3.87, at another \$4.76: how much does he still owe me?

94. A merchant bought 946 yards of calico; he sold

REVIEW. — Define subtraction. (31.) Subtrahend. (32.) Minuend. (33.) *Remainder. (34.) Difference.* (34.) Describe the sign of subtraction. (35.)

137 yards to one person, and 673 to another: how many yards had he remaining?

95. A boy gave some nuts to his little brother and two sisters. To his brother he gave 125; to his elder sister, 13 less than to his brother; and to his younger, 24 less than to the elder: how many nuts did the younger receive?

96. A man had 4,832 sheep; on Monday he sold 1,199, on Tuesday he sold 2,259 more: how many had he then remaining?

97. A man had $2,986 in cash. He bought a building-lot, for which he paid $341; a house, for which he paid $1,418: how much cash had he left?

98. A boy went to a store with a ten-dollar bill; he bought an algebra, a history, and a chemistry for $3.83; on his way home he met a boy with a little carriage; he immediately gave for it all the money he had left, excepting $2: how much did the carriage cost him?

99. A gentleman, having $7,564, deposited $4,319 in a bank, and spent $86 of the remainder: how much then remained?

100. A boy had 374 cents; he gave 37 cents for a dove, 90 cents for a rabbit, and 114 cents for a pair of pigeons: how many cents had he left?

FORMULA.—374 — (37 + 90 + 114) = $1.33. Ans.

ANAL. STEPS.—1. *Find the total amount expended.*
2. *Find the amount remaining.*

101. Suppose a gentleman has an income of $1,597: if he pays $150 for taxes, and for his other expenses $873, how much of his income does he save?

102. A farmer, having 554 sheep, sold 168, and then bought 896 more: how many had he then?

103. In a certain army there were 3,512 men; in a battle, 513 of them were killed, 627 were wounded, and 97 of them deserted: how many remained?

104. A farmer, having 625 bushels of grain, sold to A 97 bushels, to B 127, to C 197, and gave 110 bushels to the poor: how many bushels had he remaining?

105. A man travelled by railroad 372 miles, and started to return on foot. On the first day, he travelled 35 miles; on the second day, 43; on the third, 44; on the fourth day, he took the cars and came home: how many miles did he travel the last day?

106. Since there are 60 minutes in an hour, how many minutes are there between 2 o'clock and 4 o'clock?

107. How many minutes between 11 minutes before 8 o'clock and 10 o'clock?

108. How many minutes between 25 minutes before 8 o'clock and 13 minutes before 12 o'clock?

109. How many minutes between 28 minutes after 12, M., and 26 minutes after 1, P.M.?

110. How many minutes between 47 minutes before 11, A.M., and 32 minutes after 2 o'clock, P.M.?

111. How many minutes between 39 minutes after 4 o'clock, A.M., and 42 minutes before 4 o'clock, P.M.?

112. How long a time will elapse between 3 minutes before 9 o'clock, A.M., and 3 minutes after 3 o'clock, P.M.?

REVIEW.—Name the axioms used in subtraction. (36.) Analyze by the first *method an example in subtraction.* (37.) By the second method. (38.) Illustrate *the use of the parenthesis.* (39.)

UNITED STATES CURRENCY.

113. If a man's income is $175 per month, and he pays $25.38 for rent, $44 for provisions, and $18.25 for other expenses, how much will he have left?

114. A man gave his note for $32.45; he paid at one time $7.80, and at another time $4.84: how much remained unpaid?

115. A man paid $140.36 for a horse, and 165 dollars for a carriage; he afterwards sold them for $300: did he gain, or lose, and how much?

116. A flour-merchant, having 700 barrels of flour worth $3,500, sold 278 barrels to one man for $1,078, and 142 barrels to another for $542: how many barrels had he left? How much had the remainder cost him?

117. Three men bought a farm for 9,840 dollars. The first paid $2,672.31; the second paid $3,089.40; and the third, the remainder: how much did the third pay?

118. A man bought a house for $1,500, and, having expended $315.46 for repairs, sold it for $2,000: how much was his gain?

119. A man deposited 5,000 dollars in a bank. He drew at one time, $23.50; at another, $12.40; and, at another, 737 dollars: how much had he remaining in the bank?

120. A farmer bought a piece of land for $122.50; he bought a second piece for $153.42, lacking $63.40: how much more did he pay for the first than for the second?

121. A drover bought a drove of cattle of one farmer for $861.11, a drove of a second farmer for $987.63, lacking $189.90: how much more did he pay for the first than for the second?

122. A boy had 33 cents given him to buy a slate and sponge. He paid 9 cents for his sponge; and his slate cost 30 cents: how much did he owe for his slate?

123. If it costs a man $2 per day to support his family, and he receives $5 per day for his work, how much can he save in one week?

124. A merchant bought 500 barrels of flour for $3,500; he sold 250 barrels for $2,000: how many barrels had he on hand?

125. Since there are 30 days in June, how many days will elapse between the 7th of June and the 23d of the same month?

126. Since there are 31 days in May, how many days will elapse between the 9th of May and the 28th of the same month?

127. Having $4,937.60 in my possession, I wish to know how much I must add to this amount to be able to purchase a farm valued at $6,741, and yet save $576.34 for other purposes.

128. A boy bought a sled for $1.48, and gave 87 cents for having it repaired; he sold it for $1.49: did he gain, or lose, by the transaction? and how much?

129. One man travels 67 miles in a day, another man follows at the rate of 42 miles in a day: if they both start from the same place at the same time, how far apart will they be at the close of the day?

130. Bought 14 yards of cloth for $13.27, 5 yards of calico for $1.36, 12 pounds of sugar for $1.972, 4 pounds of tea for $1.75: what is the amount of the bill? Paid 8 dollars, 75 cents, and 3 mills: how much is still owed?

131. Bought 5 yards of ribbon for 79 cents, 6 yards of tape for 13 cents, 41 yards of calico for 5 dollars,

23 yards of cloth for 18 dollars and 23 cents: wl the amount of the bill? Paid two 20-dollar bills: much change should be received in return?

132. Sold 5 bushels of corn for 3 dollars 87 6 bushels of rye for 7 dollars 14 cents, 9 bushe potatoes for 3 dollars 17 cents, 11 bushels oa 5 dollars 5 cents, 37 bushels of wheat for 35 do what should be received on the above bill? much does it lack of 100 dollars?

ANALYSIS OF SECTION III.

Subtraction.

1 ILLUSTRATION.
2. DEFINITIONS.
3. SIGN.
4. AXIOMS.
5. ABSTRACT PROBLEMS.
    (a.) *Process.*
    (b.) *Analysis.*
    (c.) *Rule.*
6. CONCRETE PROBLEMS.
    (a.) *Formula.*
    (b.) *Analytical Steps.*
    (c.) *Analysis.*

TEST QUESTIONS FOR EXAMINATION.

1. What is a unit? Name a unit. Is a book a unit? A horse? A h

2. What is a number? Write a number on the blackboard. Show a of books. Can there be numbers of different kinds? Give an example.

3. What is a concrete number? Write a concrete number on the blac Name five concrete numbers of different kinds. Name five concrete nu the same kind.

4. When are concrete numbers called denominate? Is 5 ounces a den number? 3 horses? 5 dimes? 6 cows? 7 yards?

NOTE. — It is important that one day in the week should be spe review of the principles over which the class have passed during the ot *days. The* answers to these questions may be written on slate or paper in *ical form, to be read* as a class-exercise, subject to criticism by the *cher.*

5. What is an abstract number? Name an abstract number. What is the difference between a concrete and an abstract number? Name three abstract numbers. Can one abstract number be greater than another? Write an abstract number on the blackboard. Write a concrete number on the blackboard. What is the difference between a number and a unit? Is a unit always a number? Is a number always a unit? Give examples on the blackboard.

6. Of what does arithmetic treat?

7. What is notation? Illustrate the meaning of notation. Give an example of each method. Which is the most convenient? Why? Which is chiefly used in arithmetic? Why?

8. What is the Roman notation? Why called Roman? What characters are used in the Roman notation? How many? What is the value of each? Which has the greatest value? Which the least? What is the effect of repeating a letter? What is the effect of placing a letter at the left of one of greater value? What of placing a letter at the right of one of greater value? When a dash is placed over a letter, or a combination of letters, how is the value affected? Write forty-eight on the blackboard in Roman characters. Change it to forty-eight thousand. To forty thousand and eight. Ten thousand fifty-eight. Forty-five thousand and three. Forty-seven thousand and one.

9. What is the Arabic notation? Why so called? What characters are used in the Arabic notation? How many? By what name are nine of them sometimes called? How many single things does each represent? Which character has no value? How many names has it? Which is the most common? What do figures represent? Is a figure, or a combination of figures, a number? Why? What is the difference between a picture and a thing? When you look at figures, what do you think about?

10. What is the simple value of a figure?

11. What is the local value? Has every digit a simple value? When is the simple value the same as the local value? When are they different? In the number 395, how many values has the figure five? How many values has the figure three. Does its local value change its simple value? Does it increase it, or decrease it?

12. What is numeration? What is a period? How many figures in a period? What is the name of the first period? Of the second period? Of the third period? How many places in a period? Name the periods from right to left. From left to right. Write on the blackboard a number containing four periods. Read it. At which hand do you commence to write a number? Repeat the numeration-table by periods, from right to left. From left to right. What should be done with a vacant place in the period of a number? Write a number on the blackboard, each period of which has a vacant place. What should be done with vacant periods of a number? Write a number on the blackboard having a vacant period. What should be done with the vacant left-hand places of the left-hand period? Tell how a number is written. Tell how a number is read. How are periods usually separated in writing numbers?

13. A digit in units' place has how many times the value of the same digit in *tenths'* place? A digit in tens' place has how many times the value of the same digit in units' place?

14. What is a rule? Which is the more important, — a rule, or an analysis? Can a rule be made *from an* analysis? Why are rules used in arithmetic? [illegible]

the teacher explain to the pupil that rules are deduced from analyses, and th an analysis is far more important than a rule.]

15. Recite the rule for notation.

16. Recite the rule for numeration.

17. What is addition? What is the first process in arithmetic? *Notatio* What is the next? *Numeration.* What is the next? *Addition.*

18. What is the sum of two or more numbers? When two numbe are added together, what is the result called? What is the sum of 6 and What is the 14 called? Of what numbers is the 14 composed? Can 14 be sep rated into more than two parts? Which is the greater, — 14, or the sum of the parts of 14? What is putting-together two or more numbers into one su called?

19. What is the sign of addition? What is a sign? Illustrate the use of t sign of addition.

20. What is the sign of equality? Illustrate its use.

21. What is the dollar-sign? Illustrate its use.

22. What is an axiom? What is meant by "a self-evident truth"? Can axiom be proved? Can it be illustrated? Do axioms need to be proved? Illu trate at the blackboard the axiom used in addition.

23. What is a problem? What is meant by the solution of a problem?

24. What are simple problems? How many questions in a simple proble How many answers can be given? What is the meaning of "complex"?

25. What is a complex problem? Can a complex problem have more th one elementary question? Can it have more than one answer?

26. What is an analytical step? How many steps can there be in a sim problem? How many in a complex problem?

27. What is the order of solving problems?

28. Give a complete analysis of a problem in addition, following the giv order.

31. What is subtraction? What is the process of taking 7 from 14 calle Why?

32. What is the subtrahend? Which is the greater, — the subtrahend, or t minuend? How should the subtrahend be written? Why?

33. What is the minuend? Is the minuend the greater, or the small number?

34. What is the remainder? Which is the greater, — the remainder, or t minuend? Can the remainder be less than the subtrahend? Can it be greate What two numbers added together will give the minuend? What is the diffe ence of two numbers? How does the difference differ from the remainder? A they always alike? If the difference and minuend are given, how can the sub trahend be found? When the remainder and the subtrahend are given, how ca the minuend be found?

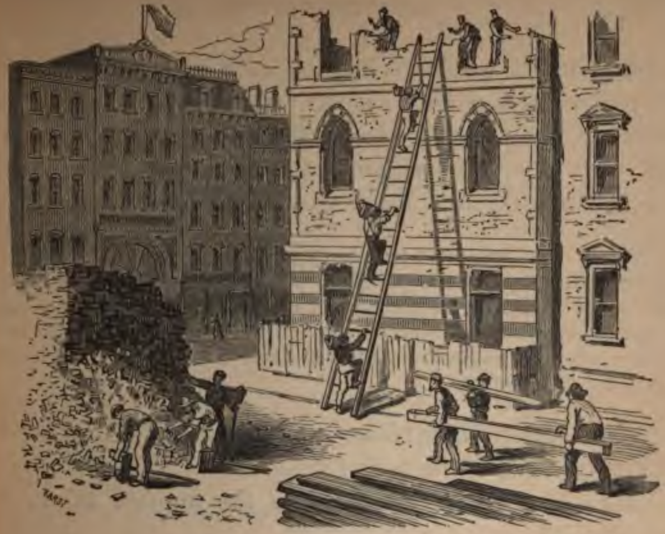
35. Illustrate the use of the sign in subtraction.

36. What are the axioms used in subtraction? Illustrate the first on th blackboard. The second

37. Give a complete analysis of an example in subtraction by first method.

38. Analyze an example by the second method.

39. *Illustrate the* use of the parenthesis in arithmetic.



SECTION IV.

MULTIPLICATION.

In the above picture some men are engaged in building a house. Three of the men are busy taking bricks from a pile, 3 others are carrying lumber, 3 are each carrying a hod of bricks up the ladder, while 3 bricklayers are placing the bricks in a wall.

How many men are at work?

How many groups of 3 men each are there?

How many are 3 men and 3 men and 3 men and 3 men? How many 3's of men are there?

Each of the 3 men climbing the ladder has 10 bricks in his hod: how many are 10 bricks + 10 bricks + 10 bricks? How many are 3 tens counted together?

In the building with a flagstaff are some windows. *If you count them* one way, you have 3 rows of windows, and 4 windows in each row, or 4 windows + windows + 4 windows; but, if you count them th other way, you have 4 rows of windows, and 3 window in each row, or 3 windows + 3 windows + 3 window + 3 windows: how many windows in the building?

When equal numbers are to be counted together, there is a shorte process than by addition. This process is called *multiplication*; thu 3 times 4 windows = 12 windows, or 4 times 3 windows = 12 window Any one of the equal numbers to be united is called the *multiplicand* the number of the equal numbers is called the *multiplier*; the resu is called the *product*.

40. *Multiplication* is the process of taking number a given number of times.

41. *The Multiplicand* is the number taken, c multiplied.

42. *The Multiplier* is the number by which w multiply, and shows how many times the multiplica is taken.

43. *The Product* is the result of the multiplic tion.

44. *The Factors* of the *product* are the multip cand and multiplier.

45. *The Sign of Multiplication* is an o lique cross, thus, ×; and, when placed between nu bers, it denotes that they are to be multiplied together as, $4 \times 8 = 32$.

46. *The Axiom* used in multiplication is,—

The product is the same whichever factor is taken fo the multiplier.

47. *Principles:*—

1. *The multiplicand may be abstract or concrete.*
2. *The multiplier is always abstract.*
3. *The product is like the multiplicand.*

MULTIPLICATION TABLE.

2 × 1 = 2	3 × 1 = 3	4 × 1 = 4	5 × 1 = 5
2 × 2 = 4	3 × 2 = 6	4 × 2 = 8	5 × 2 = 10
2 × 3 = 6	3 × 3 = 9	4 × 3 = 12	5 × 3 = 15
2 × 4 = 8	3 × 4 = 12	4 × 4 = 16	5 × 4 = 20
2 × 5 = 10	3 × 5 = 15	4 × 5 = 20	5 × 5 = 25
2 × 6 = 12	3 × 6 = 18	4 × 6 = 24	5 × 6 = 30
2 × 7 = 14	3 × 7 = 21	4 × 7 = 28	5 × 7 = 35
2 × 8 = 16	3 × 8 = 24	4 × 8 = 32	5 × 8 = 40
2 × 9 = 18	3 × 9 = 27	4 × 9 = 36	5 × 9 = 45
2 × 10 = 20	3 × 10 = 30	4 × 10 = 40	5 × 10 = 50
2 × 11 = 22	3 × 11 = 33	4 × 11 = 44	5 × 11 = 55
2 × 12 = 24	3 × 12 = 36	4 × 12 = 48	5 × 12 = 60
6 × 1 = 6	7 × 1 = 7	8 × 1 = 8	9 × 1 = 9
6 × 2 = 12	7 × 2 = 14	8 × 2 = 16	9 × 2 = 18
6 × 3 = 18	7 × 3 = 21	8 × 3 = 24	9 × 3 = 27
6 × 4 = 24	7 × 4 = 28	8 × 4 = 32	9 × 4 = 36
6 × 5 = 30	7 × 5 = 35	8 × 5 = 40	9 × 5 = 45
6 × 6 = 36	7 × 6 = 42	8 × 6 = 48	9 × 6 = 54
6 × 7 = 42	7 × 7 = 49	8 × 7 = 56	9 × 7 = 63
6 × 8 = 48	7 × 8 = 56	8 × 8 = 64	9 × 8 = 72
6 × 9 = 54	7 × 9 = 63	8 × 9 = 72	9 × 9 = 81
6 × 10 = 60	7 × 10 = 70	8 × 10 = 80	9 × 10 = 90
6 × 11 = 66	7 × 11 = 77	8 × 11 = 88	9 × 11 = 99
6 × 12 = 72	7 × 12 = 84	8 × 12 = 96	9 × 12 = 108
10 × 1 = 10	11 × 1 = 11	12 × 1 = 12	13 × 1 = 13
10 × 2 = 20	11 × 2 = 22	12 × 2 = 24	13 × 2 = 26
10 × 3 = 30	11 × 3 = 33	12 × 3 = 36	13 × 3 = 39
10 × 4 = 40	11 × 4 = 44	12 × 4 = 48	13 × 4 = 52
10 × 5 = 50	11 × 5 = 55	12 × 5 = 60	13 × 5 = 65
10 × 6 = 60	11 × 6 = 66	12 × 6 = 72	13 × 6 = 78
10 × 7 = 70	11 × 7 = 77	12 × 7 = 84	13 × 7 = 91
10 × 8 = 80	11 × 8 = 88	12 × 8 = 96	13 × 8 = 104
10 × 9 = 90	11 × 9 = 99	12 × 9 = 108	13 × 9 = 117
10 × 10 = 100	11 × 10 = 110	12 × 10 = 120	13 × 10 = 130
10 × 11 = 110	11 × 11 = 121	12 × 11 = 132	13 × 11 = 143
10 × 12 = 120	11 × 12 = 132	12 × 12 = 144	13 × 12 = 156

DRILL CARD EXERCISE.

N.B.—Assign to the class such portions only of the followi graded drill exercises as will make the pupils prompt and accurate their calculations *without answers.* The key is for the use of teach only.

Table I., *Thousands.*

Exercise I.—2 multiplier.
II.—3 "
III.—4 "
IV.—5 "
V.—6 "
VI.—7 "
VII.—8 "
VIII.—9 "

Table II., *Billions.*

Ex. IX.—2 multiplier.
X.—3 "
XI.—4 "
XII.—5 "
XIII.—6 "
XIV.—7 "
XV.—8 "
XVI.—9 "

Table II., *Billions.*

Ex. XVII.—11 multiplier.
XVIII.—12 "
XIX.—22 "
XX.—33 "

Table II., *Thousands.*

Ex. XXI.—100 to 119 multiplie
XXII.—220 " 239 "
XXIII.—340 " 359 "
XXIV.—460 " 479 "

Ex. XXV.—5091 multiplier.
XXVI.—90090 "
XXVII.—50902 "
XXVIII.—903004 "

UNITED STATES CURRENCY.—TABLE III.

Ex. I.—2 multiplier.
II.—3 "
III.—4 "
IV.—5 "

Ex. V.—6 multiplier.
VI.—7 "
VII.—8 "
VIII.—9 "

Ex. IX.— 20 multipli
X.— 37 "
XI.—105 "
XII.—900 "

DIRECTIONS.

Exercise I. 1st Example.—Copy (Table I., Thousands' Period and multiply A by 2; multiply B by 2; multiply C by 2, &c.; thus

A.	242022	B.	442202	C.	442200
	2		2		2
Ans.	484044		884404		884400

Ex. II.—*Multiply* A by 3; B by 3; C by 3, &c.
Ex. III.—*Multiply* A by 4; B by 4; C by 4, &c.

FELTER'S

ARITHMETICAL DRILL CARDS.

TABLE I.		TABLE II.		TABLE III.
Thou. Units.		B. Mill. Thou. Units.		Dollars. Cts. m.
579,438		9,381,908,191		5,319.43 4
242,022	A	8,448,646,446	A	3,402 cts.
442,202	B	6,688,286,466	B	1,811 m.
442,200	C	868,424,246	C	60 dol.
240,220	D	606,486,644	D	403 cts.
402,222	E	428,664,246	E	10 dol.
331,110	F	75,351,535	F	121 m.
303,211	G	73,735,353	G	8 cts.
311,121	H	53,533,353	H	195 dol.
342,221	I	53,131,533	I	2,000 m.
221,121	K	33,713,555	K	715 dol.
550,112	L	29,036,632	L	86 cts.
404,201	M	19,241,747	M	7 dol.
040,121	N	9,866,474	N	9 m.
443,121	O	9,673,342	O	8,189 cts.
552,232	P	9,477,773	P	908 m.
243,110	Q	8,347,544	Q	4,003 cts.
152,231	R	887,744	R	500 dol.
435,112	S	347,654	S	6,001 cts.
543,121	T	281,263	T	2,100 m.
402,301	U	187,602	U	5,000 cts.

N.B.—A KEY *containing* the answers to the problems on the *above card may be obtained* by *mail*, by addressing the publishers.

48. ANALYSIS OF MULTIPLICATION.

1. **PROBLEM.**—How many are 24 times 873?

PROCESS.

$$\left.\begin{array}{r l} 873 & \text{multiplicand,} \\ 24 & \text{multiplier,} \end{array}\right\} \text{factors.}$$

$$\begin{array}{r l} \overline{3492} & \text{partial product, 4 times multiplicand.} \\ 1746 & \text{partial product, 20 `` ``} \\ \hline 20952 & \text{whole product, 24 `` `` .} \end{array}$$

2. **ANALYSIS.**—(1.) For convenience, write the units' figure of the multiplier under the lowest order of the multiplicand.*

(2.) For convenience, multiply first by the units' figure.

(3.) 4 times 3 units are 12 units, equal to 1 ten and 2 units: write the units in the place of units, and reserve the tens to be added to the product of the tens.

(4.) 4 times 7 tens are 28 tens, plus the ten reserved, are 29 tens, equal to 2 hundreds and 9 tens: write the tens in the place of tens, and reserve the hundreds to be added to the product of hundreds.

Continue thus until each figure of the multiplicand has been multiplied by the units' figure of the multiplier.

(5.) Next multiply by the tens' figure. Because it is in tens' place, it has a value tenfold greater than if in units' place, or 20. 20 times 3 are 60 units = 6 tens, which must be written in the tens' place.

(6.) 20 times 7 tens are 140 tens, or 14 hundreds, equal to, &c.

(7.) Add the partial products: hence

3. **CONCLUSION.**—24 times 873 are 20952.

4. **RULE.**—I. *Write the multiplier under the multiplicand.*

II. *Beginning with the unit figure, multiply each figure of the multiplicand by each figure of the multiplier successively, observing to write the first figure of each*

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* It should be remembered that the product is not changed by changing the order of the factors (**46**): hence, if more convenient, the multiplicand may be written under the multiplier. This analysis may be omitted until a final review *of the work.*

partial product directly under its multiplier, writing down and carrying as in addition.

III. *Add the partial products; and their sum will be the product sought.*

5. PROOFS.—1. *Repeat the work.*
2. *Change the factors.*
3. *Divide the product by either factor.*

ORAL AND WRITTEN PROBLEMS CONTAINING BUT ONE ELEMENTARY QUESTION.

In the following problems the first question in each one is usually oral, and should be recited as such with the appropriate analysis. Those questions containing numbers too large to be thus performed constitute the written problems. The advantages of this arrangement must be obvious to all.

The letter *W* will be used to separate the oral from the written questions, and the answers of the written only will be given at the end of the book.

1. (*a.*) What will 80 pounds of rice cost at 5 cents a pound? (*b.*) What will 4,572 pounds cost?

ANALYSIS.—(*a.*) Since 1 pound of rice costs 5 cents, 80 pounds will cost 80 times 5 cents, which is 400 cents, or $4.

FORMULA.—(*b.*) 5 cts. × 4,572 = 22,860 cts. = $228.60.

2. What will 50 spelling-books cost at 7 cents apiece? *W.* What will 325 books cost?

3. At 12 cents a pound, what will 40 pounds of sugar cost? *W.* 580 pounds?

4. At 3 cents apiece, what will 90 oranges cost? *W.* 97 oranges?

5. At 11 cents apiece, what will 40 pine-apples cost? *W.* *270 pine-apples?*

6. What will 8 yards of cloth cost at 21 cents a yard? *W.* At 31 cents? At 91 cents?

7. If a quantity of provisions will last 2 men 8 days, how long will it last 1 man? *W.* If it will last 2 men 47 days, how long will it last one man?

8. If 2 men can do a piece of work in 31 days, how long will it take 1 man? *W.* If 9 men can do it in 149 days, how long will it take 1 man?

9. If 3 cows eat a ton of hay in 31 days, how long will it last 1 cow? *W.* If 12 cows eat it in 125 days, how long will it last 1 cow?

10. If 3 horses eat 20 bushels of oats in 20 days, how long will it take 1 horse to eat the same quantity? *W.* If 11 horses eat 70 bushels in 37 days, how long will the same quantity last 1 horse?

11. If 30 horses eat 4 bushels of oats in 8 days, how many horses will eat as many in one day? *W.* If 54 horses eat 37 bushels in 137 days, how many horses will eat as much in 1 day?

12. What will 3 sheep cost at 11 dollars each? *W.* At $13?

13. What will 11 pencils cost at 7 cents apiece? *W.* At 27 cents?

14. At 3 cents apiece, what will 4 nutmegs cost? *W.* 57 nutmegs?

15. At 4 cents apiece, what will 5 apples cost? *W.* 89 apples?

16. What will 5 arithmetics cost at 12 cents apiece? *W.* At 129 cents?

REVIEW.—What is a unit? (1.) Define a number. (2.) A concrete number. *(3.) A denominate* number. (4.) An abstract number. (5.) Of what does arith- *tic treat? (6.) How many* methods of notation are in common use? (7.) De- *be the Roman* method. (8.) The Arabic. (9.)

17. What will 7 pens cost at 3 cents apiece? *W.* At 127 cents?

18. At 8 cents apiece, what will 3 oranges cost? *W.* 125 oranges?

19. At 7 cents apiece, what will 6 lemons cost? *W.* 314 lemons?

20. How many are 8 times 7? 6? *W.* 952?

21. *W.* If 99 men eat 40 cheeses in 16 days, how many men will be required to eat the same quantity in 1 day?

22. *W.* If 70 cows can eat 9 tons of hay in 8 days, how many cows will be required to eat the same quantity in 1 day?

WRITTEN PROBLEMS CONTAINING NOT MORE THAN TWO ELEMENTARY QUESTIONS.

23. A man bought 27 cords of wood at $8 per cord, and gave in exchange produce worth $89: how much remained unpaid?

FORMULA.—1. $8 × 27 = $216.
2. $216 — $89 = $127. Ans.

ANALYSIS.

ANAL. STEPS.—1. *Find the cost of the wood.*

Since 1 cord costs $8, 27 cords will cost 27 times $8, which is $216.

2. *Find balance unpaid.*

If the wood costs $216, and the produce was worth $89, the balance unpaid will be the difference of these amounts, which is $127.

24. A man bought 27 sheep at $4 apiece, and gave

REVIEW.—What is the difference between the simple and local values of a figure? (10, 11.) Repeat the numeration table from right to left. (12.) From left to right. What is meant by numbers increasing in a tenfold ratio? (13.) Give rule for notation. (15.) Give rule for numeration. (16.)

in payment a note for $47, and the remainder in cash: how much cash did he pay?

25. A tailor had a piece of broadcloth containing 42 yards; he cut from it 17 yards: how much was the remainder worth at $4 a yard?

26. A man sold 17 sheep at $5 apiece; he received $37 in cash, and the rest in wood: how much did his wood cost him?

27. A man started to make a journey of 279 miles; he travelled 4 days at 38 miles a day: how far was he from the end of his journey?

28. A farmer had 108 bushels of potatoes; he sold 47 bushels: how much was the remainder worth at 7 shillings a bushel?

29. A tailor has a piece of cloth containing 57 yards; he cuts from it 13 suits containing 4 yards each: how many yards remain?

30. I have 337 yards of calico: how much remains if 37 dresses, each containing 9 yards, are cut off?

31. Charles bought 93 oranges at 4 cents apiece; he paid towards them 178 cents: how much did he still owe?

32. A man earned 537 cents in a week; he paid 39 cents a day for board and washing: how much had he left at the expiration of the 7 days?

33. A man owing $189 gave in part payment 23 sheep at $4 per head: how much did he still owe?

34. I bought 28 sheep at $6 per head, and a span of horses for $297: how much more did the horses cost than the sheep?

35. A man paid $189 toward 147 bushels of wheat at *$2 a bushel: how* much did he still owe?

36. I have 7 bags of grain; 6 of them contain 89 pounds each; they all contain 722 pounds: how many pounds does the seventh contain?

37. I have a cistern containing 189 gallons; the faucet will discharge 2 gallons every minute: if the faucet is left open 83 minutes, how much water will remain in the cistern?

38. At 2 cents a foot, how much will 12 boards cost, each board containing 13 feet?

39. What will 17 dozen of eggs cost at 2 cents apiece?

40. At 3 shillings a yard, how many shillings will 13 pieces of cloth cost, if each piece contains 37 yards?

41. What will 7 dozen arithmetics cost at 85 cents apiece?

42. How many pounds in 8 bags of wheat, each bag containing 2 bushels of 60 pounds each?

43. If I buy 6 barrels of pork, at $16 a barrel, and sell it for $105, how much do I gain?

44. If I buy 12 pounds of sugar at 16 cents per pound, and sell it for 205 cents, how much do I gain?

45. In an orchard are 26 peach-trees, and 6 times as many apple-trees as peach-trees: how many trees in the orchard?

46. In an orchard there are 8 rows of trees, and 125 trees in each row: how many trees in the orchard?

47. If 8 horses eat 12 bushels of oats in 6 days, how many bushels will they eat in 32 days?

REVIEW.—Define addition. (17.) The sum. (18.) What is the answer in addition called? Illustrate the sign of addition. (19.) What is it called? Illustrate the use of the sign of equality. (20.) The dollar-sign. (21.) What is an axiom? (22.) Define a problem. (23.) What is the difference between a simple *and a complex problem.* (24, 25.) What is meant by an analytical step? (26.)

49. To Multiply by 10, 100, 1,000, &c.

ILLUSTRATION.

$347 \times 10 = 3{,}470.$
$347 \times 100 = 34{,}700.$
$347 \times 1{,}000 = 347{,}000.$

It is shown, by the above illustration, that a number can be m tiplied by *ten* by annexing *one* cipher, also that, by annexing an ac tional cipher, it is multiplied by *ten* twice, or 100, &c.: hence,

1. *Annexing one cipher to any number multiplies it by* 10.
2. *Annexing two ciphers to any number multiplies it by* 100.
3. *Annexing three ciphers to any number multiplies it by* 1,000, &

50. Table United States Currency.

$$\left.\begin{array}{l}10 \text{ mills} = 1 \text{ cent,}\\ 10 \text{ cents} = 1 \text{ dime,}\\ 10 \text{ dimes} = 1 \text{ dollar,}\end{array}\right\} \text{ or } \left\{\begin{array}{r}1{,}000 \text{ mills} =\\ 100 \text{ cents} =\\ 10 \text{ dimes} =\end{array}\right\} 1 \text{ dollar.}$$

$1 = 100 cents.
$\frac{1}{2}$ = 50 cents.
$\frac{1}{4}$ = 25 cents.
$\frac{3}{4}$ = 75 cents.
$\frac{1}{2}$ cent = 5 mills.

(GOLD.)

(BRONZE.)

(SILVER.)

48. How many cents in 8 dollars?

Formula. — 100 cents × 8 = 800 cents.

Analysis. — Since $1 equals 100 cents, 8 dollars must times 100 cents, or 800 cents.

49. How many dimes in 12 dollars? In $3 In $420? In $35?

50. How many mills in 4 dollars? *W.* In 20 *In 75 dollars?*

51. How many mills in 3 cents? *W.* In 250 cents? In 183 cents?

MISCELLANEOUS WRITTEN PROBLEMS.

52. What will 2 barrels of flour cost at 8 dollars and 10 cents per barrel?

PROCESS.

$$\begin{array}{r} \$8.10 \\ 2 \\ \hline \$16.20 \end{array}$$

ANALYSIS.—If 1 barrel of flour costs $8.10, 2 barrels will cost twice $8.10, or $16.20.

53. What will 3 pairs of gloves cost at 87 cents a pair? What will 389 pairs cost?

54. Give the cost of each of the following items: 3 pairs of silk hose at $1.20 each, 5 pairs of boots at $3.50 each, 4 pairs of shoes at $1.50 each, 4 pairs of cotton gloves at $.38 each, 7 pairs kid gloves at $1.00 each.

55. Give the cost of the following: 4 First Readers at 30 cents, 5 Second Readers at 50 cents, 3 Third Readers at 70 cents, 9 Fourth Readers at 80 cents, and 8 Fifth Readers at $1.00.

56. Give the cost of the following: 5 pounds of sugar at 11 cents per lb., 3 pounds of tea at 90 cents per lb., 4 pounds of coffee at 22 cents per lb., 8 pounds of raisins at 31 cents per lb., 4 pounds of rice at 16 cents per lb.

57. Bought 3 yards of calico at 16 cents a yard; I gave the clerk a half-dollar: how much change should I receive?

REVIEW.—Give the order of solving problems. (27.) Add a column of 2's to 50. (29.) Add a column of 3's in the same way. Define subtraction. (31.) Give an example, and name and define the subtrahend (32); the minuend (33); the *remainder (34).*

58. Bought a book for $1.30, a slate for 16 cents gave the shopkeeper a 2-dollar bill: how much change should I receive? Gave him a 5-dollar bill? A 10-dollar bill?

59. A little girl went shopping with 1 dollar; she bought 2 spools of thread at 10 cents a spool, 3 spools of silk at 15 cents each: how much did the thread and silk cost? How much money had she left?

60. His father gave Charles $1.50; he spent 41 cents for a knife, 21 cents for a kite, and 11 cents for a top: how much money had he left?

61. What must I give for 3 books at $1.25 each, and 5 slates at 27 cents each?

62. Thomas attended public school 76 days, at a tuition of 3 cents per day: how much was his bill? How much at 2½ cents per day?

FORMULA.—2½ cents = $.025. $.025 × 76 = $1.900.

ANALYSIS.—1. 2½ cents = 25 mills.

2. Since the tuition for 1 day costs 25 mills, the tuition for 76 days will cost 76 times as much, or 1,900 mills, equal to $1.90.

ORAL AND WRITTEN PROBLEMS.

63. At 5 cents each, how much should a newsboy receive for 6 papers? *W.* For 876 papers?

64. If a man spend $3½ per day, to how much will his expenses amount in 3 days? *W.* In 1 month, or 30 days? In 1 year, or 365 days?

65. At $6¼ each, how much will 2 army-coats cost? *W.* 5 army-coats cost? 476 army-coats?

REVIEW.—Illustrate the use of the sign of subtraction. (35.) Repeat the axioms used in subtraction. (36.) Give the analysis of a problem in subtraction. (37.) Illustrate the use of the parenthesis. (39.)

66. *W.* How many pounds of cotton in 2 bales, if 1 bale weighs 478 pounds? In 5 bales? In 384 bales?

67. How much is 1 bale worth at 10 cents per pound? *W.* How much are 5 bales worth? 384 bales?

68. How many gallons are 8 times 3 gallons? 5 gallons? *W.* 47 gallons? 3,947 gallons?

69. How many pounds are 16 times 1 pound? 2 pounds? *W.* 46 pounds? 3,478 pounds?

70. At 3 cents each, what will 2 oranges cost? 5 oranges? *W.* 147 oranges? 1,973 oranges?

71. If 1 bag of apples is worth $2¾, what are 2 bags worth? *W.* 5 bags? 167 bags?

72. If 1 quart of peanuts cost 9 cents, what will 8 quarts, or 1 peck cost? *W.* 16 quarts, or 2 pecks? 56 quarts, or 7 pecks?

73. If 1 peck of apples is worth 60 cents, what are 3 pecks worth? 4 pecks, or 1 bushel? *W.* 8 pecks, or 2 bushels?

74. *W.* A man bought 6 pounds of veal at 11 cents per pound, and 7 pounds of beef at 16 cents per pound: how much did both cost? He gave the shopkeeper a 10-dollar bill: how much change should he receive?

75. At 6 cents per quart, what do 3 quarts of milk cost? 4 quarts, or 1 gallon? *W.* 5 gallons? 16 gallons?

76. *W.* If I have a 10-dollar bill, and pay out 37 cents, how much shall I have left? If I afterwards receive $1.87, how much shall I have? If I then loan Mr. Smith $3 and 4 cents, how much will remain?

77. *W.* 4 dollars and 20 cents, *plus* $1.50, *minus* 10

REVIEW.—Define multiplication. (40.) Give an example, point out, name, and define the multiplicand. *(41.)* Multiplier. (42.) Product. (43.) Factors. (44.) *Give sign of multiplication. (45.)* Illustrate its use.

cents, *plus* $4.13, *minus* $3½, = how much? $100 *plus* 100 cents, *minus* 100 mills, = how much?

78. *W.* 100 cents *plus* 100 dollars *minus* $3¾ = how much?

79. *W.* $300 + 20 cents, — $3.00, + $2.20, — 4½ dollars = ?

80. *W.* $4.20 × 3, + $1.20, — $8.00, + 40 cents = ?

81. *W.* $5 × 20, + 200 cents, — $87½, + 87½ cts. = ?

82. *W.* 40 dollars + 40 cents, + 30 dollars, — 128 cents = ?

83. *W.* $4.17 × 8, + $9¼, — 87½ cents, + $87½ = ?

84. *W.* $0.17 × 9, — 27 cents, + $27, — 27 mills =

85. *W.* 300 dollars — 300 cents, + 300 mills = ?

86. *W.* 92 cents × 8, — $6.924, + 17 mills = ?

87. *W.* $100 × 3, — 5½ cents, + 3 dollars = ?

88. *W.* $217 — 1½ cents, × 3, + 7½ cents = ?

ANALYSIS OF SECTION IV.

Multiplication.

1. ILLUSTRATION.
2. DEFINITIONS.
3. SIGN.
4. AXIOM.
5. PRINCIPLES.
6. ABSTRACT PROBLEMS.
    a. *Process.*
    b. *Analysis.*
    c. *Proof.*
    d. *Rule.*
7. CONCRETE PROBLEMS.
    a. *Formula.*
    b. *Analytical Steps.*
    c. *Analysis.*

REVIEW.—Name the axiom used in multiplication. (46.) Repeat the princ ples, and illustrate each. (47.) Give the analysis of multiplication. (48.) Illustrat the multiplication by 10, 100, 1,000, &c. (49.) Repeat the table of United Stat currency. (50.)

SECTION V.

DIVISION.

ARTHUR and his two sisters are dividing some fruit equally among themselves. They have six apples: into how many equal parts must they separate them?

How many will each have?

They also have 9 pears: into how many equal parts must they be separated?

How many will each have?

Arthur has just received a number of peaches. He commenced dividing them by giving each of his two sisters 4 peaches, and took 4 himself; he then had one peach left. How many times did he take away 4 peaches from the number?

How many were left?

How many peaches did he receive in all?

This process of finding how many times one number is containe in another, or of separating a number of things into equal parts, is called *division*. The number to be divided is called the *dividend*. The number of equal parts into which it is to be divided is called the *divisor*. The result of the division, or the number of things in each of the equal parts, is called the *quotient*. The number of things left after the equal parts are taken out is called the *remainder*.

51. *Division* is the process of finding how many times one number is contained in another, or of separating a number into equal parts.

52. *The Dividend* is the number divided.

53. *The Divisor* is the number by which the dividend is divided.

54. *The Quotient* is the result obtained by dividing.

55. *The Remainder* is the part of the dividend left undivided.

56. *The Sign of Division* is a short horizontal line between two dots; thus, ÷. It shows that the number before it is to be divided by the number after it; thus, $20 \div 4$ means that 20 is to be divided by 4.

57. *The Fractional Sign of Division* consists in writing the dividend above a line, and the divisor below it; thus, $\frac{20}{4}$ denotes that 20 is to be divided by 4.

58. In ***Short Division,*** the quotient only is written. The intermediate steps are omitted.

59. In ***Long Division,*** each step of the process is written.

60. The axiom used in division is,—

If a number be both multiplied and divided by the same number, its value will not be changed.

Review. — What is a unit? (1.) A number? (2.) A concrete number? (3.) A denominate number? (4 [illegible] ct numb

61. PRINCIPLES, —

1. *The dividend may be either abstract or concrete.*

2. *When the divisor is abstract, the quotient is like the dividend.*

3. *When the divisor is concrete, the quotient is abstract.*

62. ANALYSIS OF SHORT DIVISION.

1. PROBLEM. — Divide 70,301 by 4.

PROCESS.

Divisor. 4)70,301 Dividend.
17,575¼ Quotient.

Proof.
17,575¼ Quotient.
4 Divisor.
70,301 Dividend.

2. ANALYSIS.[*] — (1.) For convenience write the divisor at the left of the dividend, and commence by dividing the left-hand digit.

(2.) *One-fourth* of 7 tens of thousands is 1 ten of thousands with a remainder of 3 tens of thousands, equal to 30 thousand. Write the 1 in the place of tens of thousands, and add the 30 thousands to the thousands.

(3.) *One-fourth* of 30 thousands is 7 thousands with a remainder of 2 thousands, which is equal to 20 hundreds. Write the 7 in the place of thousands, and add the 20 hundreds to the hundreds: 20 hundreds and 3 hundreds are 23 hundreds.

(4.) *One-fourth* of 23 hundreds is 5 hundreds with a remainder of 3 hundreds, equal to 30 tens. Write the 5 in hundreds' place, and add the 30 tens to the tens.

(5.) *One-fourth* of 30 tens is 7 tens with a remainder of 2 tens, equal to 20 units. Write the 7 in the place of tens, and add the 20 units to the units: 20 units and 1 unit are 21 units.

(6.) *One-fourth* of 21 units is 5 units with a remainder of 1 unit,[†] which, divided by 4, gives one-fourth of a unit, written ¼. Write the 5 in the place of units, and annex the ¼ of a unit.

3. CONCLUSION. — Therefore *one-fourth* of 70,301 is 17,575¼.

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[*] In this analysis, we prefer the *fractional* form, as it is simpler, and more readily understood, although liable to some objection. The teacher will find another form in the analysis of long division (**64**), which he can use here, if he prefers.

[†] *It will be seen by this* that a remainder in division can be changed to a *fractional quotient by* writing the divisor under it, in the form of a *fractional sign of division.*

4. RULE FOR SHORT DIVISION. — I. *Write the divisor at the left of the dividend.*

II. *Beginning at the left, divide each figure of the dividend by the divisor, and set each quotient figure under its dividend.*

III. *If there is a remainder after any division, prefix it to the next figure of the dividend, and divide as before.*

IV. *If any partial dividend is less than the divisor, write a cipher for the quotient, prefix the partial dividend to the next figure of the dividend, and divide as before.*

V. *If there is a remainder after dividing the last figure of the dividend, it can be changed to a fractional quotient by writing the divisor under it, and annexing the result to the quotient.*

5. PROOF. — *Division may be proved by multiplying the quotient by the divisor, and adding the remainder, if there is any, to the product. The result will be equal to the dividend, if the work is correct.*

DRILL CARD EXERCISES.

TABLE I., *Thousands.*	TABLE II., *Billions.*
Exercise I. — 2 Divisor.	Exercise II. — 2 Divisor.
III. — 3 "	IV. — 3 "
V. — 4 "	VI. — 4 "
VII. — 5 "	VIII. — 5 "
IX. — 6 "	X. — 6 "
XI. — 7 "	XII. — 7 "
XIII. — 8 "	XIV. — 8 "
XV. — 9 "	XVI. — 9 "

REVIEW. — What is notation? (7.) What methods are most used? Which employs letters? Which figures? How many, and what, are the letters used? (8.) When are numbers written by the use of letters? Give the value of each letter. What is the effect of repeating a letter? What effect has a letter of less value placed *before* one of greater value? When placed after? How is the value *affected* when a dash is placed over a letter, or a combination of letters?

FELTER'S
ARITHMETICAL DRILL CARDS.

TABLE I.		TABLE II.		TABLE III.
Thou. Units.		B. Mill. Thou. Units.		Dollars. Cts. m.
579,438		9,381,908,191		5,319.43 4
242,022	A	8,448,646,446	A	3,402 cts.
442,202	B	6,688,286,466	B	1,811 m.
442,200	C	868,424,246	C	60 dol.
240,220	D	606,486,644	D	403 cts.
402,222	E	428,664,246	E	10 dol.
331,110	F	75,351,535	F	121 m.
303,211	G	73,735,353	G	8 cts.
311,121	H	53,533,353	H	195 dol.
342,221	I	53,131,533	I	2,000 m.
221,121	K	33,713,555	K	715 dol.
550,112	L	29,036,632	L	86 cts.
404,201	M	19,241,747	M	7 dol.
040,121	N	9,866,474	N	9 m.
443,121	O	9,673,342	O	8,189 cts.
552,232	P	9,477,773	P	908 m.
243,110	Q	8,347,544	Q	4,003 cts.
152,231	R	887,744	R	500 dol.
435,112	S	347,654	S	6,001 cts.
543,121	T	281,263	T	2,100 m.
402,301	U	187,602	U	5,000 cts.

N.B.—A Key containing the answers to the problems on the *above card may be obtained* by *mail*, by addressing the publishers.

MENTAL DIVISION.

ILLUSTRATION.

One.

2 halves.

3 thirds.

4 fourths.

5 fifths.

How many *halves* in one apple? How many *thirds* in one pear? How many *fourths* in one orange? How many *fifths* in one melon? How many *sixths* in one dollar? How many *sevenths* in one dime? How many *eighths* in one pound?

How many halves make one apple? How many thirds make one pear? How many fourths make one orange? How many fifths make one melon? How many tenths make one mile? How many twenty-fifths make one peach?

1 half.

1 third.

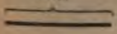
2 fourths.

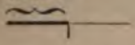
3 fifths.

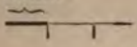
What is meant by one *half?* Ans. — *One of the two equal parts into which a unit, or number of units, is divided.*

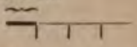
Review. — What characters are used in the Arabic notation? (9.) How is 10 *written? Why? How* is 12 written? Why? What is the simple value of a *gure? (10.)* The local value? (11.) What is numeration? (12.) Repeat th ble by periods from right to left. From left to right.

What is meant by one *third?* What by 2 fourths? What by 3 fifths? What by 4 sevenths? What by 7 twelfths? What by 4 fifths?

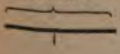

One.

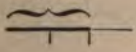

1 half.

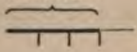

1 third.

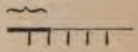

1 fourth.

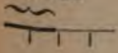
Which is the greater, — one, or one *half?* One *half*, or one *third?* One *third*, or one *fourth?* One *half*, or one *fourth?* *One*, or one *third?* *One*, or one *fourth?* One *fourth*, or one *fifth?* One *seventh*, or one *tenth?* One *sixteenth*, or one *seventeenth?*

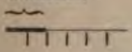

2 halves.

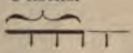

2 thirds.

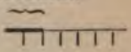

3 fourths.


2 sixths.

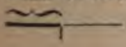

2 fourths.

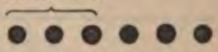

2 sixths.


3 fifths.


2 sevenths.

Which is the greater, — 2 halves, or 2 fourths? 2 thirds, or 2 sixths? 3 fourths, or 3 fifths? 2 sixths, or 2 sevenths? 6 sevenths, or 6 eighths? 5 sevenths, or 5 elevenths? 5 ninths, or 5 thirteenths?


1 half of one.


1 half of 6.

What do we mean by one-half of 1? ANS. — *One of the two equal parts into which one is divided.*

What do we mean by one-half of 2? ANS. — *One of the two equal parts into which two is divided.*

What do we mean by one-half of 6? What by one-half of 8? What by one-half of 12? What by one-half of 16? Of 18? Of 20? Of 10?

How many units in one-half of 1? ANS. — *One-half of one unit.*

How many dollars in 4,235 mills?

FORMULA. — 4,235 mills ÷ 1,000 mills = $4.235.

ANALYSIS. — Since in 1,000 mills there is 1 dollar, in 4,235 mills there are as many dollars as 1,000 mills are contained times in 4,235 mills, or 4 dollars with a remainder of 23 cents and 5 mills.

The teacher should dictate problems until the pupils can readily divide or multiply by 10, 100, 1,000, &c.

EXERCISES.

Change each of the following to dollars, cents, and mills, by dividing by 10, 100, or 1,000, as the case may be.

PROCESS.	(1.)	(2.)
434 cents = $4.34	58 mills.	1,304 mills.
434 mills = .434	20 cents.	2,050 mills.
404 mills = .404	938 cents.	250 cents.
20 cents = .20	715 cents.	452 cents.

N.B. — Additional problems should be dictated by the teacher.

ORAL PROBLEMS.

PROBLEM. — If 15 apples are divided equally among 3 boys, how many apples does each boy receive? If 3,474 apples are so divided?

PROCESS.

3)3,474 apples.

1,158 apples. Ans.

ANALYSIS. — If 3,474 apples are divided equally among 3 boys, 1 boy will have 1 third of 3,474 apples, or 1,158 apples.*

1. If 9 peaches cost 27 cents, what is the cost of 1 peach? If 7 cost 35 cents? If 12 cost 144 cents?

---

* *Let the pupil be* taught that one-*third* of a thing, or number, is the same as *the quotient of that* thing, or number, divided by 3.

63. To DIVIDE by 10, 100, 1,000, &c.

ILLUSTRATION.

4,000 ÷ 10 = 400
4,000 ÷ 100 = 40
4,000 ÷ 1,000 = 4

4,346 ÷ 10 = 434 and 6 Rem.
4,346 ÷ 100 = 43 and 46 Rem.
4,346 ÷ 1,000 = 4 and 346 Rem.

It is shown by the above illustrations that a number can be divided by 10 by cutting off one figure from the right, also, that, by cutting off two figures from the right of a number, it is divided by 10 twice, or 100, &c.; and the figures cut off are a remainder.

Hence,

1. *Removing a figure from the right of a number divides it by* 10.

2. *Removing two figures from the right of a number divides it by* 100.

3. *Removing three figures from the right of a number divides it by* 1,000; *&c.*

UNITED STATES CURRENCY.

To change cents and mills to dollars.

ILLUSTRATION.

3,454 mills ÷ 1,000 = $3.454.
3,454 cents ÷ 100 = $34.54.
424 mills ÷ 10 = $ .424.

From the above it is seen that, —

1. *Mills are changed to dollars by pointing off three figures from the right, or dividing by* 1,000.

2. *Cents are changed to dollars by pointing off two figures from the right, or dividing by* 100.

3. *Mills are changed to cents by pointing off one figure from the right, or by dividing by* 10.

13. 9 pounds of sugar cost 81 cents: how much is that a pound? How much for 2 pounds? *W.* For 380 pounds?

14. In 9 rows of trees there are 72 trees: how many trees in each row? In 4 rows? *W.* In 85 rows?

15. A man travelled 72 miles in 6 hours: how far did he travel in 1 hour? In 7 hours? *W.* In 24 hours?

16. 8 bins of equal size contain 72 bushels of wheat: how many bushels does each contain? How many would 3 contain? *W.* Would 50 contain?

17. If 81 cents be paid for 9 dozen eggs, what will be the cost of 1 dozen? Of 5 dozen? *W.* Of 280 dozen?

18. 11 tons of hay are worth $77: what is the value of 1 ton? Of 9 tons? *W.* Of 386 tons?

PROBLEM. — At 5 cents each, how many lemons can I buy for 20 cents? For $38.75?

PROCESS. $5\overline{)3,875}$

$\overline{775}$ lemons.

ANALYSIS. — 1. $38.75 = 3,875 cents.

2. At 5 cents each, as many lemons can be bought as 5 cents are contained times in 3,875 cents, or 775 lemons.

19. How many boxes of cheese at $5 a box can be bought for $30? *W.* For $6,755?

20. If I receive $12 interest in 1 year, in how many years shall I receive $132 interest? *W.* $864 interest?

21. How many pencils can I buy for 360 cents, if 1 pencil costs 3 cents? *W.* How many for 1,041 cents?

REVIEW. — What is subtraction? (31.) Define minuend. (33.) Subtrahend 32.) *Remainder. (34.)* Minus means what? (35.) Recite the axioms of sub-action. *(36.)*

22. At 7 cents per pound, how many pounds of lead may be bought for $8.40? *W.* For $6.23?

23. How many barrels of apples at $3 a barrel can be bought for $183? *W.* For 2,922 dollars?

24. If 300 men accomplish a certain piece of work in 1 day, how many men will be required to do it in 4 days? In 5 days? *W.* In 10 days?

25. If 168 men use 2 barrels of flour in 1 week, how many men will use the same in 6 weeks? *W.* In 12 weeks?

26. If 750 persons eat 80 loaves of bread in 1 week, how many persons will it require to use the same in 5 weeks? *W.* In 15 weeks?

27. If 3 yards of cloth are enough for a coat, how many coats can be made from 33 yards? *W.* From 384 yards?

28. A man travelled 7 miles in 1 hour: at that rate, how long would it take him to travel 63 miles? *W.* 861 miles?

29. For 49 cents, how many pounds of sugar can be bought at 7 cents a pound? *W.* For 343 cents?

30. For 240 cents, how many oranges can be bought at 3 cents apiece? How many for 9,248 cents?

31. For 360 cents, how many pounds of sugar can be bought at 9 cents a pound? *W.* At 12 cents?

32. If 12 men can build 240 rods of wall in 20 days, how many rods can 1 man build in the same time? *W.* 7 men?

REVIEW. — Give the first method of analysis of subtraction. (37.) The second. (38.) What is the use of the parenthesis? (39.) Define multiplicatio [illegible] Multiplicand. (41.) Multiplier. (42.) Product. (43.) Factors. (44.) [illegible] the sign of *multiplication.* (45.) Show its use. Give the axiom. (46.) [illegible] *principles.* (47.)

WRITTEN EXAMPLES.

33. If 12 men can build 240 rods of wall in 20 days, how many rods can they build in 1 day?

34. If 9 men can do a piece of work in 3 days, how long will it take 1 man to do it? How long will it take 3 men?

35. Two men own equal shares in a farm worth $3,417: what is the value of each man's share?

FORMULA.—$3,417 ÷ 2 = $1,708½.

PROCESS.

$$\begin{array}{r} 2)\$3,417 \\ \hline 1,708\tfrac{1}{2} \end{array}$$

ANALYSIS.—If 2 equal shares are worth $3,417, 1 of the shares is worth one-half as much, or $1,708½.

36. In 3 miles there are 190,080 inches: how many inches in 1 mile?

37. In 4 hogsheads of wine there are 8,064 gills: how many gills in 1 hogshead?

38. In 8 pounds there are 7,680 farthings: how many farthings in 1 pound?

39. How many families are there in a city of 18,000 inhabitants, if there are an average of 6 persons in each family?

40. Sound moves 8,960 feet in 8 seconds: how many feet does it move in 1 second?

41. In 7 equal regiments there are 8,050 men: what number in each?

42. If 1 man can perform a piece of work in 1,728 hours, how long will it take 9 men to do the same?

43. A man dying left an estate valued at $3,874, to be equally divided among his wife and four children: how much did each receive?

44. Divide the number 1,347 into 3 equal parts: how many units in each part?

45. The sum of the 4 equal parts of a number is 147,342: what is one of the parts?

46. 5 times a certain number is 41,324: what is the number?

47. How many oranges can be purchased for 3,416 cents at 8 cents each?

Formula. — 3,416 cents ÷ 8 cents = 427, number of oranges.

Analysis. — At 8 cents each, as many oranges can be bought as 8 cents are contained times in 3,416 cents, or 427 oranges.

48. How many pounds of beef can be purchased for 4,768 cents at 10 cents a pound?

64. ANALYSIS OF LONG DIVISION.*

1. PROBLEM. — Divide 26,306 by 45.

Steps. — 1. Find number of times.
2. Multiply.
3. Subtract.
4. Bring down.

DIVISION.

Divisor, Dividend, Quotient.
45) 26,306 ($584\frac{26}{45}$, or
225 (584.57+
380
360
206
180
Remainder, units. 26.0
22.5
3.50
3.15
Remainder, hundredths. .35

PROOF.

1st Quotient.
$584\frac{26}{45}$
45 Divisor.
2920
2336
26 Remainder.
26,306

PROOF.

2d Quotient.
584.57+
45 Divisor.
2922.85
23382.8
.35 Remainder.
26,306.00

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* *This analysis may be omitted until a final review of the work.*

2. ANALYSIS. — (1.) For convenience,* place the divisor at the left of the dividend, and the quotient, when found, at the right.

(2.) 45 is contained in 263 5 times: as the 263 is hundreds, the 5 is also hundreds, and must be written in the quotient in hundreds' place. 5 times 45 are 225, really 225 hundreds, and written to indicate hundreds under the dividend. 225 hundreds from 263 hundreds leave 38 hundreds, equal to 380 tens.

(3.) 45 is contained in 380 tens 8 tens times with a remainder of 20 tens, equal to 200 units. Write the 8 in the tens' place of the quotient, and to the 200 units add the 6 units of the dividend, making 206 units.

(4.) 45 is contained in 206 units 4 times with a remainder of 26 units. Write the 4 in the units' place of the quotient, and indicate the division of 26 by 45 (**57**) as $\frac{26}{45}$ of a unit, to be written at the right of the quotient.

(5.) If desired, the division may be carried still further: thus, the remainder, 26 units, equals 260 tenths: 45 is contained in 260 tenths 5 tenths times with a remainder of 35 tenths, equal to 350 hundredths. Write the 5 tenths in the tenths' place, &c.

3. RULE FOR LONG DIVISION. — I. *Write the divisor at the left of the dividend, as in short division.*

II. *Find how many times the divisor is contained in the fewest left-hand figures of the dividend that will contain it, and place the quotient at the right of the dividend.*

III. *Multiply the divisor by this quotient figure; subtract the product from the partial dividend; and to the remainder annex the next figure of the dividend.*

IV. *Divide as before, until all the figures of the dividend have been annexed and divided.*

V. *If any partial dividend does not contain the divisor, place a cipher in the quotient, annex the next figure of the dividend, and divide as before.*

REVIEW. — Give analysis of multiplication. (48.) Illustrate the method of multiplying by 10, 100, &c. (49.) Recite the table of United States currency. (50.) How many cents make a dollar? How many mills make a dollar?

* Those who prefer may select any other place, not in the way of subsequent work, for either divisor or quotient. The class should have a method, and adhere to it.

VI. *If there is a remainder after dividing all the figures of the dividend, it may be changed to a fractional quotient by writing the divisor underneath it.*

4. PROOF. — *The proof of long division is the same as that of short division.*

NOTES. — 1. The first step in long division is to find the quotient figure of the highest denomination.

2. Multiply the divisor by that figure, and write the product under the first partial dividend; then observe, that, if the product is greater than the partial dividend, the quotient figure is too large.

3. Subtract that product from the first partial dividend, and observe, that, if the remainder is equal to or greater than the divisor the quotient figure is too small.

4. Change the remainder to the next lower denomination, and add to it the corresponding denomination in the dividend.

WRITTEN EXAMPLES.

49. When 16 acres of land cost $2,304, what is the cost per acre?

50. If 21 bushels of wheat weigh 1,260 pounds, what is the weight of 1 bushel? Of 5 bushels?

51. How many bushels in 5,040 pounds of wheat, if 1 bushel weighs 60 pounds?

52. There are 25 barrels of cider, each barrel contains 40 gallons: how many gallons in all?

53. I have 1,000 gallons of wine: how many barrels must I have to contain it, if each barrel contains 40 gallons?

54. I have 1,000 gallons of vinegar which I wish to put into barrels, each holding 25 gallons: how many barrels shall I require?

REVIEW. — Define division. (51.) Dividend. (52.) Divisor. (53.) Quotient. (54.) Remainder. (55.) Describe the sign. (56.) Show its use. Describe the fractional sign. (57.) What *is meant by* short division? (58.) By long division. (59.) What *is the axiom of division?* (60.) Give the principles. (61.)

55. In every mile there are 320 rods: how many rods in 2 miles? In 42 miles?

56. Since every 320 rods make a mile, how many miles in 640 rods? In 7,234 rods?

57. $4 \div 2 =$ what? $14 \div 7 =$ what? $473 \div 9 =$ what? $9{,}134 \div 46 =$ what?

58. $55 \times 2 \times 4 - 26 + 47 \div 46 =$ what?

59. The dividend is 25, the divisor is 5: what is the quotient?

Point out and name the terms, and give the answers, in the following:—

60. $4 + 3 + 4 + 25 =$ what?
61. $115 - 63 =$ what?
62. $137 \times 14 =$ what?
63. $571 \div 92 =$ what?
64. $15 \times 13 =$ what?
65. $374 + 4 =$ what?
66. $913 \div 21 =$ what?
67. $517 - 118 =$ what?

What is the answer to each of the following?

68. $\overline{29 + 7} + \overline{56 - 23}$?
69. $\overline{29 + 7} - \overline{56 - 23}$?
70. $27 + \overline{84 \div 4} - \overline{0 + 8}$?
71. $27 + \overline{84 \times 4} - \overline{0 \times 8}$?
72. $6 + 12 - 11 \times \overline{15 \div 3}$?
73. $12 \times 3, + 4, - 6, \times 1, \div 4$?
74. $\overline{12 \times 3} + 4, - 6, \times 1, \div 4$?
75. $12 \times \overline{3 + 4} - 6, \times 1, \div 4$?

Review.—Give the **analysis of an example in short division. (62.) Illustrate** *the method of dividing* **by 10, 100, &c. (63). Give the analysis of an example in** *division. (64.)*

76. $12 \times 3, + (14 - 6) \times 1, \div 4$?
77. $12 \times 3, + 14 - (6 \times 1) \div 4$?
78. $12 \times 3, + 14, - 6 \times \overline{12 \div 4}$?
79. $25 \times 0, + (86 \div 2) - \overline{7 - 2}$?

Indicate in the proper manner, by signs, the following, and give the results: —

80. Multiply the sum of 10 and 8 by 3, and divide the product by 9.

81. From the product of 8 and 7 subtract 25, and divide the remainder by 11.

82. Multiply by 35 the quotient of 60 less 8 divided by 4.

83. How much is 3 less than the quotient of 408 divided by 12?

Write the formula for each of the following problems: —

84. A boy spent 20 cents on Monday, $1.30 on Tuesday, 87½ cents on Wednesday, and $2.30 on Thursday: how much did he spend on the three days?

85. A boy earned 80 cents one time, and 40 cents at another; he spent a dollar: how much had he left?

86. What will 8 pounds of rice cost at 22 cents per pound?

87. What is the value of 5 tons of hay, at the rate of $72 for 12 tons?

88. A man bought 18 yards of cloth for $54, and 12 yards of cheaper cloth for $25: what was the difference of the price per yard?

DRILL CARD EXERCISES.

LONG DIVISION.

Table I., *Thousands.*		**Table II.**, *Thousa*
Exer. I. —	Divide A to K inclusive, by 10. " L to U " " 11.	Exer.
III. —	Divide by 11, 21, 31, 41, &c., to 101 inclusive. " " 12, 22, 32, 42, &c., to 102 "	
V. —	Divide by 13, 23, 33, 43, &c., to 103 " " " 14, 24, 34, 44, &c., to 104 "	
VII. —	Divide by 20, 30, 40, 50, &c., to 110 " " " 19, 29, 39, 49, &c., to 109 "	V
IX. —	Divide by 18, 28, 38, 48, &c., to 108 " " " 17, 27, 37, 47, &c., to 107 "	
XI. —	Divide by 16, 26, 36, 46, &c., to 106 " " " 15, 25, 35, 45, &c., to 105 "	
XIII. —	Divide by 100, 101, 102, &c., to 109 " " " 210, 211, 212, &c., to 219 "	X
XV. —	Divide by 760, 761, 762, &c., to 769 " " " 870, 871, 872, &c., to 879 "	X
XVII. —	Divide by 980, 981, 982, &c., to 989 " " " 990, 991, 992, &c., to 999 "	XV

Table II., *Billions.*

Exercise XIX. — Divide by 5009, 5109, 5209, &c., to 5909 inclusi
" " 6090, 6190, 6290, &c., to 6990 "

XX. — Divide Table II. by the corresponding numbe
Table I., thus: — A (Table II.) ÷ A (Table
B ÷ B, C ÷ C, &c., to R.

UNITED STATES CURRENCY. — TABLE III.

Exer. I. —	2 divisor.	Exer. V. —	6 divisor.	Exer. IX. —	10 divi
II. —	3 "	VI. —	7 "	X. —	25 "
III. —	4 "	VII. —	8 "	XI. —	84 "
IV. —	5 "	VIII. —	9 "	XII. —	59 "

Directions. — Exer. III. Divide A (Table I.) by 11, B by C by 31, &c., and K by 101. Then divide L by 12, M by 22, N b *&c., and U by 102.*

Exer. IV. Divide A (Table II.) by 11, B by 21, C by 31, D b *E by 51, &c., as in* Exercise III. (See Key.)

FELTER'S

ARITHMETICAL DRILL CARDS.

TABLE I.		TABLE II.		TABLE III.
Thou. Units.		B. Mill. Thou. Units.		Dollars. Cts. m.
579,438		9,381,908,191		5,319.43 4
242,022	A	8,448,646,446	A	3,402 cts.
442,202	B	6,688,286,466	B	1,811 m.
442,200	C	868,424,246	C	60 dol.
240,220	D	606,486,644	D	403 cts.
402,222	E	428,664,246	E	10 dol.
331,110	F	75,351,535	F	121 m.
303,211	G	73,735,353	G	8 cts.
311,121	H	53,533,353	H	195 dol.
342,221	I	53,131,533	I	2,000 m.
221,121	K	33,713,555	K	715 dol.
550,112	L	29,036,632	L	86 cts.
404,201	M	19,241,747	M	7 dol.
040,121	N	9,866,474	N	9 m.
443,121	O	9,673,342	O	8,189 cts.
552,232	P	9,477,773	P	908 m.
243,110	Q	8,347,544	Q	4,003 cts.
152,231	R	887,744	R	500 dol.
435,112	S	347,654	S	6,001 cts.
543,121	T	281,263	T	2,100 m.
402,301	U	187,602	U	5,000 cts.

N.B. — *A Key containing* the answers to the problems on the *above card may be obtained* by *mail*, by addressing the publishers.

MISCELLANEOUS WRITTEN PROBLEMS WHICH CONTAIN NOT MORE THAN THREE ELEMENTARY QUESTIONS.

89. A farmer bought a horse for $65, a saddle for 16 dollars, and a bridle for 8 dollars: how much did he pay for all?

90. A lady paid $200 for a piano, $37 for a sofa, and $20 for an easy-chair: how much did she pay for all? How much had she left of a five-hundred-dollar bill?

91. Julia bought 2 pieces of tape for 20 cents each, 2 papers of pins for 26 cents, and a pair of scissors for 50 cents: how much did she pay for all? How much change should she receive for a ten-dollar bill?

92. A farmer has 115 sheep in one lot, 75 in another, and 40 in his barnyard: how many sheep has he? How many had he after selling 37 sheep?

93. James has 39 nuts in one pocket, and 14 less in another: how many has he in both?

ANAL. STEPS. — 1. *Find how many nuts in the second pocket.*

If James has 14 nuts less in the second pocket than in the first, he must have in the second pocket the difference of these numbers, or 25 nuts.

2. *Find how many nuts in both.*

If James has 39 nuts in one pocket, and 25 nuts in another, he must have in both the sum of these numbers, or 64 nuts.

94. A man bought 40 sheep of three farmers. He bought 9 of the first, and 13 of the second: how many did he buy of the third? How many more must he buy to have 250 sheep?

95. A grocer had a tub of butter containing 48 pounds. He sold 13 pounds to Mr. Jacob Wilson, and

15 pounds to John Huntsman: how many pounds were left?

96. Mr. Clark paid $25.50 for a cow, and for a horse $56 more than he paid for the cow: how much did he pay for both?

97. What is the difference between the cost of 6 pieces of tape at four cents apiece, and of 4 yards of ribbon at 6 cents a yard?

ANAL. STEPS. — 1. *Find cost of tape.*
2. *Find cost of ribbon.*
3. *Find difference of cost.*

98. How many cents must a boy have to buy 18 apples at 3 cents apiece, and 14 cakes at 2 cents apiece? How much change should he receive for a 5-dollar bill?

99. I bought 6 barrels of pork at 11 dollars a barrel, and sold it for $89.47: how much did I gain?

100. What is the cost of 13 sticks of candy at 2 cents apiece, and 12 pounds of raisins at 15 cents a pound? I paid for these from a $5 bill, and spent the remainder for rice at 12 cents a pound: how many pounds did I buy?

101. If 3 dresses of 16 yards each are cut from 86 yards of calico, how many yards will be left?

102. A man started from home, and travelled 5 miles an hour for 13 hours, and then set out to return, at the rate of 6 miles per hour: how far was he from home, after having travelled 9 hours?

103. Henry and John started together, and travelled in the same direction. Henry walked 3 miles an hour, and John 4 miles an hour: how far apart were they at the end of 9 hours? At the end of 48 hours?

104. Robert and Joseph started together from home, and travelled in the same direction. Robert travelled 2 miles an hour, and Joseph 4 miles an hour: how far were they apart at the end of 36 hours? At the end of 5 days, travelling 8 hours each day?

105. The next week, Robert and Joseph started together from home, and travelled in opposite directions. Robert travelled 2 miles an hour, and Joseph 3 miles an hour: how far were they apart at the end of 19 hours? At the end of 49 hours?

106. If 3 yards of cloth cost $27, what will 967 yards cost?

ANAL. STEPS. —1. *Find the cost of* 1 *yard.*
2. *Find the cost of* 967 *yards.*

If 3 yards cost $27, 1 yard will cost 1 third of $27, which is $9, and 967 yards will cost 967 times $9, or $8,703.

107. If 7 oranges cost 63 cents, how much will 127 oranges cost? 149 oranges?

108. If 9 yards of tape cost 27 cents, how much will 120 yards cost? 346 yards?

109. If 7 sheep cost $56, how much will 321 sheep cost? 7,487 sheep?

110. Susan can solve 80 problems in 8 days: how many can she solve in 76 days? In 200 days, or a school-year?

111. I pay $84 for 12 sheep: how much must I pay for 196 sheep? For 9,327 sheep?

112. A man rode 75 miles in 25 hours: how far, at that rate, can he ride in 324 hours? In 93 days of 24 hours each?

113. A man travels 63 miles in 7 hours: how far, at that rate, can he ride in 72 hours? In 297 days of 24 'ours each?

114. If 84 dollars are paid for 21 barrels of flour, how much must be paid for 9,386 barrels?

115. How much will 8,437 hats cost, if 7 cost $21?

116. How much can a person earn in 114 days, if he can earn $48 in 24 days? How much in 30 days?

117. What must I pay for 2,746 apples, if 5 cost me 25 cents? If 9 cost 18 cents?

118. A farmer sold 114 calves at the rate of 2 calves for $24: what did he receive for them?

119. If 36 men can dig a cellar in 18 days, how many men will be required to dig it in 3 days?

ANALYSIS. — 18 days are 6 times 3 days: therefore to do the work in 3 days will take 6 times 36 men, or 216 men.

120. If 4 men can dig a ditch in 40 days, how many men will be required to dig it in 5 days?

121. How long will it take 7 men to reap a field that 28 men can reap in 2 days? That 42 men can reap in 5 days?

122. Two ships 168 miles apart sail directly toward each other, — one at the rate of 8 miles an hour, and the other at the rate of 4 miles an hour: how soon will they meet? If they are 8,904 miles apart?

123. A man bought a number of lemons at the rate of 11 for 25 cents: how many could he buy for 250 cents?

ANAL. STEPS. — 1. *Find how many times 11 lemons can be bought for 250 cents.*
2. *Find the number.*

124. When potatoes are sold at the rate of 4 barrels for $8, how *many* barrels must be given for 4 cwt. of
r at $14 a hundred?

151051

ANAL. STEPS. — 1. *Find value of* 1 *barrel of potatoes.*
2. *Find entire value of the sugar.*
3. *Find number of barrels required.*

125. How many dozen of eggs at 18 cents a dozen will pay for 9 gallons of molasses at 36 cents a gallon? How many will pay for 36 gallons?

126. When eggs are sold at 24 cents for 8, how many will it take to pay for 13 pounds of sugar at 6 cents a pound? For 926 pounds at 12 cents?

127. How many gallons of molasses at 35 cents a gallon will pay for 70 pounds of butter at 30 cents a pound? 67,280 pounds of butter?

128. A man received 32 pounds of sugar in exchange for 920 pounds of butter at 16 cents a pound: what did the sugar cost him per pound?

129. A man bought 80 turkeys at the rate of 4 for 3 dollars; he sold them for $80.30: did he make, or lose? and how much?

ANAL. STEPS. — 1. *Find cost* 1 *turkey.*
2. *Find cost of* 80 *turkeys.*
3. *Find gain or loss.*

130. A farmer bought 24 sheep for 60 dollars. At how much per head must he sell them in order to gain 12 dollars? To gain $36?

131. 5 men bought a threshing-machine for $150. They rented it 4 weeks at $10 a week, and then sold it for $100: did they make, or lose? and what was each man's share?

132. 5 barrels of flour are worth $60: how many hundred weight of pork at $18 a hundred will 3 barrels buy? How many will 6 barrels buy?

133. 4 men agree to build a barn for $160; but, when

it is half built, 2 of the men quit work, and the remainder complete the job: how much should each man receive?

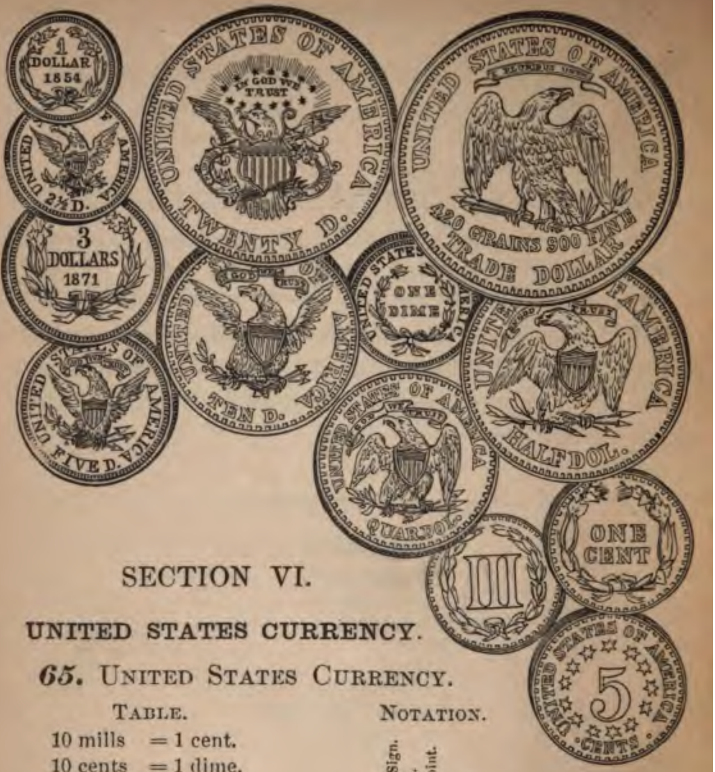
ANAL. STEPS. — 1. *Find what each man should receive for building the first half.*
2. *Find what each man should receive for building the second half.*
3. *Find the entire amount each man should receive.*

134. 4 men agree to chop 25 cords of wood for $48; but, when it is half done, 3 of the men quit, leaving the remaining man to finish the work. How shall the money be divided?

ANALYSIS SECTION V.

Division.

1. ILLUSTRATION.
2. DEFINITIONS
3. SIGNS.
4. KINDS.
5. AXIOMS.
6. PRINCIPLES.
7. ABSTRACT PROBLEMS.
   - *a. Process.*
   - *b. Analysis.*
   - *c. Rule.*
   - *d. Proof.*
8. CONCRETE PROBLEMS.
   - *a. Formula.*
   - *b. Analytical Steps.*
   - *c. Analysis.*


1 DOLLAR 1854
UNITED STATES OF AMERICA 2½ D.
3 DOLLARS 1871
UNITED STATES OF AMERICA FIVE D.
UNITED STATES OF AMERICA TWENTY D.
IN GOD WE TRUST
UNITED STATES OF AMERICA TEN D.
UNITED STATES OF AMERICA 420 GRAINS 900 FINE TRADE DOLLAR
UNITED STATES OF AMERICA ONE DIME
UNITED STATES OF AMERICA HALF DOL.
UNITED STATES OF AMERICA QUAR. DOL.
III
ONE CENT
UNITED STATES OF AMERICA 5 CENTS

SECTION VI.

UNITED STATES CURRENCY.

65. UNITED STATES CURRENCY.

TABLE.

10 mills = 1 cent.
10 cents = 1 dime.
10 dimes = 1 dollar.
10 dollars = 1 eagle.

NOTATION.

Dollar Sign.	Dollars.	Dec. Point.	Cents.	Mills.
$	11	.	11	1

66. TABLE OF ALIQUOT PARTS.

½ dollar = 50 cents.
¼ dollar = 25 cents.
⅕ dollar = 20 cents.
⅛ dollar = 12½ cents.
$\frac{1}{10}$ dollar = 10 cents.
¾ dollar = 75 cents.
½ cent = 5 mills.
¼ cent = 2½ mills.
⅓ dollar = 33 cts. 3 mills.

67. The ***coins**** of the United States are made of gold, silver, nickel, and bronze (or copper), as follows: —

---

* *The silver five and three cent pieces, the bronze two cent, and the nickel one cent, are no longer coined.*

Gold, the double-eagle, eagle, half-eagle, quarter-eagle, three-dollar and one-dollar pieces.

Silver, the trade-dollar, half-dollar, quarter-dollar, twenty-cent piece and dime.

Nickel, the five-cent and three-cent pieces.

Bronze, one cent.

68. Paper money consists of notes issued by the United States Treasury and banks as substitutes for coin.

WRITTEN PROBLEMS. — REVIEW OF UNITED STATES CURRENCY.

PROBLEM. — (*a.*) How many times are 12½ cents contained in $4?

(*b.*) In $3¾?

```
   (a.) PROCESS.                    (b.) PROCESS.
$0.125)$4,000(32 Ans.          $0.125)$3,750(30 Ans.
        375                            375
        ———                            ———
         250                             0
         250
```

ANALYSIS. — (*a.*) 12½ cents = 125 mills. $4 = 4,000 mills. 125 mills are contained in 4,000 mills 32 times.

(*b.*) 12½ cents = 125 mills. $3¾ = 3,750 mills. 125 mills are contained in 3,750 mills 30 times: hence the following

RULE. — *Change the divisor and dividend to the lowest denomination found in either, and divide as in simple division.*

1. How many times are 3 cents contained in $3? In $3.33?

2. How many times are 5½ cents contained in $5½? In $203½?

3. How many times are 20 cents contained in $20? In $2? In $5.20?

4. How many times are $1.12½ contained in $10.12½? In $12.37½?

5. $4.40 are how many times 4 cents? 40 cents? 5½ cents?

6. $6¾ are how many times 25 cents? 5 cents? 9 cents?

BUSINESS PROBLEMS.

7. A boy bought a book for $1, a slate for 50 cents, and a sponge for 12½ cents: how much did he give for all?

8. A lady bought some calico for 37½ cents, and some lace for $1¾: what did she give for both?

9. A boy had a three-dollar bill; he paid out $1.37½: how much had he left?

Note for the Teacher.—The analyses to be used in the solution of these problems are the same as those used in simple numbers.

10. Lucy bought a book for 30 cents, an inkstand for 62½ cents, and some paper for 12½ cents: how much did she spend?

11. John had $11.37½; he gave 2½ dollars to William: how much had he left?

12. A boy earned $5¾ in a week; he gave his father $4½: how much had he left?

13. A boy found 30 cents; he then earned 2½ dollars how much had he in all?

14. What will 120 apples come to at 2 cents apiece At 3½ cents? At 2½ cents?

Review.—Recite the table of United States currency. (65.) Give the tabl *of aliquot parts.* (66.) Name the gold coins of the United States. (67.) The silve *coins. The nickel.* The bronze.

15. If the interest on 1 dollar for one year is 6 cents, what is the interest on $347? On $276?

16. If 1 eighth of a house is worth $371.26, what is 8 eighths, or the whole house, worth?

17. A man gave $387.27½ for 1 eighteenth of a ship: what is 15 eighteenths of the ship worth?

18. 1 sixth of a church cost $1397.87½: what did 5 sixths of the church cost? 6 sixths?

19. 1 ninth of a factory is worth $876.43: what are 5 ninths worth? 9 ninths, or the whole?

20. Seven boys had $12½: how much is that apiece?

ANALYSIS. — $12½ are equal to 12,500 mills. 1 boy must have had 1 seventh of 12,500 mills, which is 1,785+ mills, or $1.785.

21. If 9 boys earned $4¼, how much did 1 boy earn? If they earned 5¾? $10.87½?

22. If I give $1.37½ for a book, how much will 2 copies cost? 27 copies? 1,002 copies?

23. If 1 pound of beef cost 16½ cents, how much will 3 pounds cost? 307 pounds? 4,003 pounds?

24. 7 pounds of sugar cost $1.17: how much is that per pound? If it cost $1.20? If it cost $1¼?

25. 8 pounds of cheese cost ¾ of a dollar: how much was that a pound? If it cost $1¼?

26. I have a dollar-bill: how many oranges can I buy with it at 5 cents apiece? At 2½ cents?

27. How many pounds of beef can be bought for $91.62½ at 12½ cents per pound?

28. How many oranges can be bought for $14.80 at 5 cents apiece? At 2½ cents?

29. How many pounds of rice can be bought for $4.32 at 4 cents per pound? At 8 cents per pound?

30. At 50 cents a yard, how many yards of cloth can be bought for $12? For $816? For $9,218½?

31. How much will 75 baskets of peaches cost at $1.62½ per basket? At $1.87½? At 90½ cents?

32. What will 117 pounds of beef cost at 16½ cents per pound? At 12½ cents? At 16 cents?

33. What must I give for 1 yard of lace, if 7 yards cost $7.50? $13¾? $9.87½?

34. What must I give for 93 bushels of potatoes, if 13 bushels cost $8.97? $11.882? $13?

35. What will 17 pounds of sugar cost, if 190 pounds cost $38? $36.10? $28½? $33.44?

36. What must I pay per gallon for vinegar, if 63 gallons cost $50? $35?

37. A boy had $4.00; he gave away 12½ cents: how much had he left?

38. If the interest on 1 dollar for a year is 5½ cents, how much will it be on $20? On $75? On $375?

39. If the interest on 1 dollar for a year is 7½ cents, how much will it be for 3 years? For 30 years?

40. If the interest on 1 dollar for a year is 7½ cents, how much will it be for $3? For $2,003?

41. What cost 15 pounds of tea at 37 cents per pound, 19 pounds of coffee at 13½ cents per pound, 46 gallons of molasses at $0.37½ per gallon, and 37 gallons of vinegar at 11 cents per gallon?

42. What cost 37 chests of tea at $37.46 each; 37 bags of coffee, 97 pounds each, at $0.12½ per pound; 49 casks of rice at $42 each; and 60 barrels of oil, 38 gallons each, at $1.08 per gallon?

43. What cost 49,748 feet of boards at $0.03 per foot, 18,764 shingles at ½ cent each, 987,864 square feet of

boards at 3 cents per foot, and 49,187 lath at 3½ cents each?

44. What cost 46 bushels of turnips at $0.36 per bushel, 97 bushels of carrots at $0.25 per bushel, 47 bushels of beets at 63 cents per bushel, and 138 bushels of potatoes at $0.87½ per bushel?

BILLS.

69. ***A Bill or Invoice of Goods*** is a written statement of the goods purchased, given by the seller to the buyer.

The Debtor is the party who owes.

The Creditor is the party who is owed.

An Account is a bill which contains both debit and credit items.

The teacher should dictate simple bills like the following, requiring, in the solution, the business form to be strictly observed.

NEW YORK, July 1, 1875.

J. W. SCHERMERHORN & Co.,
To SCRIBNER, ARMSTRONG, & Co., DR.

1875.						
April 10	To 25 Sheldon's Ob. Less. at $1.75	$43	75			
" 16.	" 200 First Lessons at 25c	50				
May 10.	" 500 Primary Arith. at 30c	150				
		243	75			
	Less 20 per cent	48	75	195		
	CR.					
May 23.	By School apparatus	137				
	Less 10 per cent	13	70	123	30	
	Balance due S. A. & Co.			$71	70	

Received Payment,
SCRIBNER, ARMSTRONG, & Co.,
per VALENTINE.*

---

* When a clerk or agent receipts a bill, he writes the signature of the firm, and attaches his own as factor or agent.

45.

TOPEKA, KAN., Sept. 3, 1875.

MR. T. H. CHURCH,

Bo't of L. T. GAGE,

36 pounds of sugar, at $0.09½ per pound,
3 hhd. molasses, 63 gallons each, at $0.27 per gal.,
5 casks of rice, 285 pounds each, at $0.05 per pound,
2 chests of tea, 86 pounds each, at $0.96 per pound,
4 barrels of oil, 31 gallons each, at $0.87½ per gal.

$

46.

MANCHESTER, N.H., June 17, 1874.

Mr. Z. F. RILEY,

Bo't of WILLIAM PRATT,

69 pieces of carpeting, 47 yards each, at $1.13 per yd.
87 " of Brussels do. 35 " at $1.87½ "
147 " of sheeting, 47 " at $0.05½ "
378 " of calico, 39 " at $0.08 "

$

47.

LEAVENWORTH, April 27, 1875.

Mr. THOS. SCHOLES,

To T. W. SCUDDER, DR.

1875.
Jan. 17. To 72 tons coal at $8.62½ per ton.
April 8. " 15 cords oak wood at $7.25 per cord.
" 25. " 18 cords pine wood at $6.33⅓ per cord.

CR.

Feb. 17. By merchandise, as by his bill $550.50
" 24. " cash, 200.00

What is the balance due T, W. Scudder?

48.

SOUTHAMPTON, MASS., March 3, 1875.

HENRY ROSE & Co.,

Bo't of MOSES ATWOOD & Co.

86 shovels, at $0.50.
96 spades, at $0.86.
18 ploughs, at $11.00.
23 handsaws, at $3.50.
14 hammers, at $0.62.

$

Received payment,

MOSES ATWOOD & Co.

49.

NEWARK, N.J., Sept. 29, 1875.

Mr. J. F. HUNTSMAN,

Bo't of A. WARD & SON,

37 chests of green tea, at $23.75.
42 chests of black do., at $17.50.
43 casks of wine, at $99.00.
12 crates of Liverpool ware, at $75.00.
19 barrels Genesee flour, at $7.00.
23 bushels rye, at $1.52.

$

50.

BUTTERNUTS, N.Y., May 3, 1875.

Mr. LYMAN A. BURR,

Bo't of HENRY FISK,

69 chests of tea, at $55.65.
126 bags of coffee, 100 pounds each, at $0.12½.
167 boxes of raisins, at $2.75.
800 bags of almonds, at $18.50.
9,004 barrels of shad, at $7.50.

$

Received payment,

HENRY FISK.

ANALYSIS, SECTION VI.

United States Currency.

1. ILLUSTRATION.
2. TABLE.
3. COINS.
4. PAPER CURRENCY.
5. INVOICE, OR BILL.
6. FUNDAMENTAL OPERATIONS.
    - *a. Process.*
    - *b. Analysis.*
7. APPLICATIONS.
    - *a. Formula.*
    - *b. Analytical Steps.*
    - *c. Analysis.*

TEST QUESTIONS FOR EXAMINATION.

TO THE TEACHER. — It is very important that each pupil should be subjected to a searching examination before leaving the Fundamental Rules and United States currency. A standard of about *eighty-five* per cent should be required to entitle him to proceed. The examination should test the pupils upon the following points: —

1. *Rapidity and accuracy in abstract operations.*
2. *Familiarity with the terms and definitions, and knowledge of their meaning.*

3. *Ability to analyze problems of not more than three elementary questions.*

The pupil must not be expected to understand the principles of notation, or to explain multiplication and division. The task is too severe. Children easily commit to memory, but do not readily understand principles. Better leave these explanations until the mind is more matured.

40. What is multiplication? Multiplication is a short method of what? Can all problems in addition be done by multiplication? Can all problems in multiplication be done by addition? What is the difference between a problem that can be done only by addition, and one that can be more easily done by multiplication? Ans. — *In one, the numbers to be added are equal: in the other, they are unequal.* What is the answer called in addition? What is it called in multiplication?

41. What is the multiplicand? Has the multiplicand any corresponding number in addition? Add 184 to itself. What is the 184 called in addition? Can the problem be done by multiplication? What is the 184 called in multiplication?

42. What is the multiplier? What does the multiplier show? What do we call the number that shows how many times the 184 is taken?

43. What is the product? To what in addition does the product correspond? [The teacher should show at the blackboard the real difference between addition and multiplication.]

44. Which are the factors? What is the meaning of "factor"? How many terms in multiplication? Which two do we multiply together to make the third? Which two terms must be given in order to find the third? If the multiplicand and multiplier are given, how can the product be found? If the product and multiplier are given, how can the multiplicand be found? If the product and multiplicand are given, how can the multiplier be found? Illustrate each.

45. What is the sign of multiplication? How is it used? Should the multiplier be placed before, or after, the sign? Where should the product be placed? What sign should be placed between the factors and the product?

46. What is the axiom of multiplication? Illustrate it. What is the axiom of addition? Can unlike things be added together? Can things be subtracted from unlike things? Give an illustration.

47. Give an example in which the multiplicand is abstract. In which it is concrete. What is always abstract? What is like the multiplicand?

48. How should the numbers be written in multiplication? Must the multiplier be written under the multiplicand? Is the less number always the multiplier? Does it make any difference in the result, which number is used as the multiplier? Where should the products of the figures be placed? Why? What is a rule? What is the rule for multiplication? How may you prove an answer in multiplication?

49. How may numbers be multiplied by 10? by 100? by 1,000, &c.?

50. Give the table of United States currency. Show how dollars can be changed to mills; to cents.

51 What is division? Is there any other way of finding how many times one number contains another? Ans. — *By subtraction.* Illustrate.

Review. — *What can be said of paper money?* (68) What is an invoice, or bill? (69.) *What is meant by debtor?* By creditor? An account?

52. What is the dividend?

53. What is the divisor?

54. What is the quotient? To what term in division does the *minuend* correspond? To what does the *subtrahend* correspond? How many times is 6 contained in 18? How many times can 6 be subtracted from 18? Has the quotient 3 any corresponding number in subtraction? In subtraction, how many times is the subtrahend taken from the minuend? In division, how many times is the divisor contained in the dividend? To what does the remainder in subtraction correspond in division?

55. What is the remainder in division? What is the remainder in subtraction?

56. What is the sign of division? Illustrate the use of the sign of division. Is the sign placed before, or after, the divisor? What sign is placed between the divisor and quotient, when it is to be shown that 18 divided by 6 is 3?

57. In what does the fractional sign of division consist? How are the numbers written? Which is the dividend? Which is the divisor? Illustrate the use of both signs, and show their difference.

58 & 59. What is the difference between short division and long division?

60. Give and illustrate the axiom of division.

61. Give an example in which the dividend is abstract; one in which it is concrete When the divisor is abstract, what is the quotient like? When the divisor is concrete, what is the quotient?

62. Give the analysis of a problem in short division. Repeat the rule. State the method of proof. What is the remainder in division? How can a remainder be changed to a fractional quotient? What is the answer called in division? What is to be done with the fractional quotient?

63. How may a number be divided by 10, 100, 1,000, &c. By this means change mills to dollars, cents to dollars, mills to cents.

64. What is the method of long division? Repeat the rule for long division. What is the first step in long division? The second? The third? The fourth? When the next figure is brought down to the remainder, and it will not then contain the divisor, what is to be done? If the product of the divisor and the quotient figure is more than the partial dividend, what does it show? If any remainder equals or exceeds the divisor, what does it show? What is the opposite of division? How can division be proved? How can multiplication be proved? How does multiplying the divisor affect the quotient? How does multiplying the dividend affect the quotient? How does dividing the divisor affect the quotient?

65. What is the currency of the United States? When was it adopted? Ans.—1786. What are the denominations? In business, how are eagles read? How are dimes read? Which denomination of United States money is the unit? How many cents make a dollar? How many mills make a cent? A dime? A dollar?

66. How many cents in a half-dollar? In a quarter-dollar? In three quarter-dollars? How many cents in 1 eighth of a dollar? In 1 tenth? In 1 fifth? In 2 fifths? A mill is what part of a cent? A cent is what part of a dollar? Into how many parts is a cent divided? What are the parts called?

67. What are coins? Of what are coins made? What is the place called where they are made? What metals are called precious? Which precious metal is worth the most? What are the gold coins of the United States? The silver coins? The nickel? The bronze?

68. What is meant by paper money?

69. What is a bill or invoice of goods? By whom are bills given? What is their use? What is a debtor? What is a creditor? When is a bill called an account? Write a bill on the blackboard. Write an account. What is it to receipt a bill? Is the bill receipted by the debtor, or by the creditor? How is a bill receipted by a clerk or agent?

GENERAL QUESTIONS IN UNITED STATES CURRENCY.

How are dollars reduced to cents? Which of the fundamental rules is used to reduce dollars to cents? Cents to mills? Mills to cents? Mills to dollars? Cents to dollars? Cents to mills? Give the formula for reducing $2 to cents. 500 cents to dollars. 4 cents to mills. 80 mills to cents. 5,000 mills to dollars. Recite the rule for reducing dollars to cents. Dollars to mills. Mills to cents. Mills to dollars. Cents to dollars. Cents to mills.

How are numbers in Federal money added? Can dollars be added to cents? Cents to mills? Mills to dollars? How are the numbers written? Why? What is the answer called? How may the answer be proved? How many places do mills take? Cents? Where is the decimal point placed in the answer? What sign is placed before dollars?

What is subtraction of United States money? How are the numbers written? Which is the subtrahend? Which the minuend? Which the remainder? Can cents be taken from dollars? Mills from cents. Cents from cents? Show at the blackboard how 1,000 mills can be taken from a thousand cents. A thousand mills from a thousand dollars. A thousand cents from a thousand dollars. How may subtraction be proved? What is the opposite of subtraction?

What is multiplication? What do we call the number to be multiplied? Show at the blackboard how 1 dollar 1 cent and 1 mill can be multiplied by 100? Which is the multiplicand? Which the multiplier? Which the product? How should the product be written? The cost of 2 apples is how many times as much as the cost of 1 apple? 20 cows are how many times as many as 1 cow? The cost of 20 cows is how many times as much as the cost of 1 cow? How may multiplication be proved?

What is division? What do we call the number to be divided? What do we call the number by which we divide? What is the answer called? When the dividend is divided into equal parts, of what denomination is the quotient? Is it like, or unlike, the dividend? When the dividend is cents, what is the quotient? When the dividend is mills what is the quotient? When we want to find how many times one sum of money is contained in another, what must first be done? ANS. — *If they are unlike, reduce them to the same denomination.* If the divisor is mills, and the dividend is cents, what must be done? What if the divisor is dollars and cents, and the dividend is dollars? If the divisor is cents and mills, and the dividend is dollars and cents? If the divisor is mills, and the dividend is dollars? If the dividend is cents, and the divisor is dollars? How many times are 6 cents contained in 6 dollars? 20 mills in 20 dollars? 5 dimes in 5 eagles? 1 mill in 1 eagle? 1 dime in 1 dollar? Divide 20 dollars into 5 equal parts? Into 4 equal parts? Into 3 equal parts? ANS. — $6.666+. Why reduce the dollars to mills before dividing? ANS. — *That the remainder may be mills, and therefore of small value.* What does the sign *plus* after the answer show? What is 1 *third* of 30 *cents? 1 third* of 10 mills? What is it customary in business to do with the remainder of mills? ANS. — *When it equals or exceeds 5, add* ONE *to the mills; if less, reject it as of no value.* In the final answer, what is done with the mills? ANS. — *When the mills equal or exceed 5, add* ONE *to the cents; if less, reject them.*

SECTION VII.

THE GREATEST COMMON DIVISOR.

NOTE TO THE TEACHER. — There are many who prefer to teach compound numbers, and their application to the transactions of the shop, store, and market, *before* common and decimal fractions. This book is so arranged, that no difficulty will be experienced by the teacher or pupil in omitting the latter subjects (Sections VII., VIII., IX.), and in taking them up after denominate numbers.

THE practical value of the greatest common divisor and the least common multiple is very slight indeed, as they are seldom used, except in school-books. Cancellation is of more utility; but in business transactions its importance has been overrated.

When 2 and 3 are multiplied together, the product, 6, is said to be a composite number; and the numbers multiplied together are said to be factors of 6. A number that cannot be so produced by the multiplication of factors is said to be a *prime number*, and can have no *divisor;* as, 2, 3, 7, 11, &c. Since 2 and 3, the factors of 6, are prime numbers, they are said to be the *prime factors* of 6.

70. *A Composite Number* is one produced by the multiplication of two or more factors.*

71. *A Prime Number* is one that cannot be produced by the multiplication of factors. Numbers are said to be prime to each other when they have no common factor or divisor.

72. *A Factor, or Divisor,* of a number is one of the numbers which multiplied together will produce it. When the factor is a prime number, it is said to be a *prime factor*.

---

* The words *factor* and *divisor* refer to factors and divisors that are integral only.

73. TO RESOLVE A NUMBER INTO ITS PRIME FACTORS.

PROBLEM. — What are the prime factors of 60?

PROCESS.

$$\begin{array}{r|l} 2 & 60 \\ \cline{2-2} 2 & 30 \\ \cline{2-2} 3 & 15 \\ \cline{2-2} & 5 \end{array}$$

$60 = 2 \times 2 \times 3 \times 5$

ANALYSIS. — (1.) By trial, 60 is found to be the product of the prime factor 2, and the composite factor 30.

(2.) 30 is the product of the prime factor 2, and the composite factor 15.

(3.) 15 is the product of the prime factors 3 and 5.

CONCLUSION. — Therefore by trial the prime factors of 60 are found to be $2 \times 2 \times 3 \times 5$.

RULE. — *Divide by the least prime factor, and each quotient by its least prime factor, until a quotient is found which is a prime factor. The several divisors and the last quotient are the factors required.*

What are the prime factors of the following numbers?

1. 24.
2. 36.
3. 48.
4. 144.
5. 120.
6. 136.
7. 145.
8. 225.
9. 796.
10. 576.
11. 256.
12. 344.
13. 864.
14. 945.
15. 320.
16. 500.
17. 965.
18. 7007.
19. 8148.
20. 719.
21. 913.
22. 24024.
23. 645.
24. 840.

The numbers 8 and 12 can both be divided by 2 and 4: hence 2 and 4 are said to be *common factors*, or *common divisors*, of 8 and 12. Since 4 is the greatest number that will divide both 8 and 12, 4 is said to be the *greatest common divisor*, or factor, of 8 and 12: hence,

74. *The Greatest Common Divisor,* or factor, of two or more numbers is the greatest number that will divide each of them without a remainder.

REVIEW. — *Define* a unit. (1.) A number. (2.) A concrete number. (3.) A *denominate number.* (4.) An abstract number. (5.)

TO FIND THE GREATEST COMMON DIVISOR OF TWO OR MORE NUMBERS.

PROBLEM. — Find the greatest common divisor of 6, 12, and 18.

PROCESS.

$6 = 3 \times 2$

$12 = 3 \times 2 \times 2$

$18 = 3 \times 2 \times 3$

ANALYSIS. — By inspection we find that 3 and 2 are the only common factors, or divisors, of 6, 12, and 18: hence the product of 3 and 2, or 6, is the greatest common divisor of 6, 12, and 18.

RULE. — *Resolve the numbers into their prime factors. The product of the factors which are common to all the numbers will be the greatest common divisor.*

Find the greatest common divisors

25. Of 6, 12. Of 8, 12, 16. Of 24, 30, 36.
26. Of 9, 15, 21. Of 16, 20, 28. Of 36, 42, 88.
27. Of 36, 81, 135. Of 39, 65, 104. Of 72, 168.
28. Of 78, 234, 468. Of 16, 60, 72, 96.

When numbers cannot be easily factored, the pupil may use the following

RULE. — I. *Divide the greater number by the less: if a remainder appears, divide the preceding divisor by it; and so continue to divide the last divisor by the last remainder, until nothing remains. The last divisor used is the greatest common divisor.*

II. *When there are more than two numbers, find the greatest common divisor of two of them, then of that divisor and one of the other numbers, &c. The last divisor will be the greatest common divisor.*

REVIEW. — Define notation. (7.) For what is the Roman notation used? (8.) *What letters are used?* What is the value of each? How does repeating a letter *affect its value? How is 9 represented?* How 11?

EXAMPLES.

29. Find the greatest common divisor of 2219 and 4501.

30. Find the greatest common divisor of 16082 and 25762.

31. Find the greatest common divisor of 1496, 2312, and 10761.

32. Find the greatest common divisor of 3298 and 10761.

TO FIND THE LEAST COMMON MULTIPLE OF TWO OR MORE NUMBERS.

When a number, as 20, can be divided by an integral number, as 5, without a fractional quotient, it is said to be a *multiple* of that number. When a number, as 20, can be so divided by two or more numbers, as by 4 and 5, it is said to be a *common multiple* of those numbers. Since 20 is the least number that can be so divided by 4 and 5, 20 is said to be the *least common multiple* of 4 and 5.

75. *A Common Multiple* of two or more numbers is a number which can be divided by each of them without a remainder.

76. *A Least Common Multiple* of two or more numbers is the least number which can be divided by each of them without a remainder.

PROBLEM. — What is the least common multiple of 6, 12, and 18?

PROCESS.

$6 = 3 \times 2$

$12 = 3 \times 2 \times 2$

$18 = 3 \times 3 \times 2$

$3 \times 3 \times 2 \times 2 = 36$

ANALYSIS. — Resolve the numbers 6, 12, and 18 into their prime factors. A multiple to be divided by 18 must contain all the factors of 18, or $3 \times 3 \times 2$: to be divided by 12, it must contain also the additional factor 2, not found in 18. All the factors of 6 are found in 12 and 18: hence the factors of the multiple are $3 \times 3 \times 2 \times 2 = 36$. Since 36 is

the least number divisible by 6, 12, and 18, it is also the least common multiple of these numbers.

The following *process* and *rule* have the advantage of brevity.

PROBLEM. — What is the least common multiple of 24, 16, 15, 20, and 8?

PROCESS.

$$\begin{array}{r|rrrrr} 2 & 24 & 16 & 15 & 20 & 8^* \\ \hline 2 & 12 & 8 & 15 & 10 & \\ \hline 2 & 6 & 4 & 15 & \cancel{5}^* & \\ \hline {}^* & \cancel{3}^* & 2 & 15 & & \end{array}$$

$2 \times 2 \times 2 \times 2 \times 15 = 240$ least common multiple.

RULE. — I. *Write the numbers in a horizontal line, and cancel those which are factors of any of the rest.*

II. *Divide by the smallest number that will divide them without a remainder, and write the quotients and undivided numbers below.*

III. *Cancel and divide as before, until no number will divide two of them without a remainder. The product of the divisors and undivided numbers is the least common multiple.*

Find the least common multiple of

33. Of 10, 6, 18, 15.

34. Of 24, 16, 15, 20.

35. Of 25, 60, 72, 35.

36. Of 63, 12, 84, 72.

37. Of 54, 81, 14, 63.

38. Of 12, 72, 36, 144.

39. Of 1, 2, 3, 4, 5, 6, 7, 8, 9.

40. Of 8, 9, 55, 49.

41. Of 720, 336, 1,736.

42. Of 8, 12, 16, 24, 36, 48, 72, 144.

43. Of 3, 7, 5, 9, 186.

44. Of 5, 3, 7, 9, 91.

45. Of 4, 3, 6, 7, 8, 16, 19.

46. Of 4, 8, 12, 24, 48, 84.

---

* *See process.* Cancel the 8 because it is a factor of 24, also 5 and 3 because *they are factors of 15.*

CANCELLATION.

77. Cancellation. When one number is to be divided by another, the process may often be shortened by striking out factors common to both divisor and dividend. This process is called *cancellation.*

PROBLEM. — Divide 28 by 14.

PROCESS.

Dividend. Divisor. Quotient.

$28 \div 14 = 2$

$(\cancel{7} \times \cancel{2} \times 2) \div (\cancel{7} \times \cancel{2}) = 2$

ANALYSIS. — Since the dividend, 28, is composed of the factors 7 × 2 × 2, and the divisor, 14, is composed of the factors, 7 × 2, if the factors 7 × 2 in the dividend are cancelled, and the equal factors, 7 × 2, in the divisor, are cancelled, the quotient of the remaining factor will still be 2: hence,

78. PRINCIPLES. — I. *Cancelling a factor of a number divides the number by that factor.*

II. *Cancelling equal factors in both numbers will not change the quotient.*

PROBLEM. — Divide 11 × 3 × 4 by 11 × 6.

PROCESS.

$$\begin{array}{l} \text{Dividend} \\ \text{Divisor} \end{array} \frac{\cancel{11} \times \cancel{3} \times \overset{2}{\cancel{4}}}{\cancel{11} \times \underset{2}{\cancel{6}}} = 2 \text{ Quotient.}$$

RULE. — ***Write the dividend over the divisor, and cancel all common factors.***

47. Divide 48 × 3 × 4 by 8 × 7 × 5.
48. Divide 40 × 10 × 3 by 5 × 6 × 2.
49. Divide 25 × 7 × 19 by 5 × 48 × 2.
50. Divide 21 × 21 × 25 by 7 × 5 × 5.
51. Divide 4 × 13 × 8 by 2 × 6.
52. *Divide* 81 × 8 × 3 by 2 × 27.
53. (6 × 3 × 4 × 9) ÷ (4 × 3 × 8) = **what?**

54. $(7 \times 4 \times 8 \times 2) \div (3 \times 4 \times 2) =$ what?

55. $144 \div (3 \times 4 \times 2) =$ what?

56. $(5 \times 8 \times 9) \div 120 =$ what?

57. $\frac{6 \times 8 \times 4 \times 9 \times 3}{324} =$ what?

58. A farmer sells 45 bushels of apples worth 40 cents a bushel, and receives in payment 20 pounds of tea. What is the tea worth per pound?

59. How many tons of hay, at $16 per ton, must be given for 12 barrels of flour, at $8 per barrel?

60. A farmer exchanged 9 tubs of butter, each containing 56 pounds, worth 25 cents per pound, for 4 chests of tea containing 42 pounds each. What was the price per pound of the tea?

NOTE. — Additional examples should be dictated by the teacher, or placed on the blackboard to be copied, and solved, at seats.

ANALYSIS OF SECTION VII.

I. Composite Numbers.

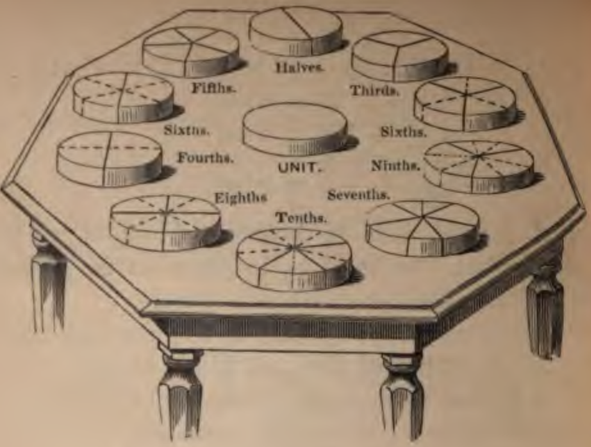
1. ILLUSTRATION.
2. PRIME NUMBERS.
3. PRIME FACTORS.
4. COMPOSITE NUMBERS.
5. GREATEST COMMON DIVISOR.
   - (*a.*) *Definition.*
   - (*b.*) *Problems.*
   - (*c.*) *Analysis.*
   - (*d.*) *Rule.*

II. Least Common Multiple.

1. ILLUSTRATION.
2. DEFINITIONS.
3. PROBLEMS.
   - (*a.*) *Analysis.*
   - (*b.*) *Rule.*

III. Cancellation.

1. ILLUSTRATION.
2. PRINCIPLES.
3. PROBLEMS.
   - (*a.*) *Analysis.*
   - (*b.*) *Rule.*



SECTION VIII.

FRACTIONS.

1. WHAT is meant by halves? By fifths? By thirds? By eighths? By ninths? By sevenths?

2. How are halves obtained? How thirds? How fifths? How sevenths?

3. In how many ways are sixths obtained?

ANS. — *By dividing a unit into six equal parts, or dividing halves each into* 3 *equal parts, or thirds each into* 2 *equal parts.*

How can quarters be obtained? eighths? tenths?

4. Point out the unit in the picture, halves, thirds, fifths, eighths, tenths, ninths, fourths, sevenths.

5. Into how many equal parts must a unit be divided to make halves? thirds? fifths? tenths? eighths, &c.?

6. How do halves compare in size with thirds? Halves with fourths? Thirds with sixths? Fifths with tenths? Ninths with sevenths? Fourths with eighths?

7. How is the size of the parts affected by cutting the unit into a larger number of equal parts? By cutting into a less number of equal parts?

One.

2 halves ($\frac{2}{2}$).

3 thirds ($\frac{3}{3}$).

4 fourths ($\frac{4}{4}$).

5 fifths ($\frac{5}{5}$).

8. When an apple is divided into two equal parts, what are the parts called? What is one of the parts called? What is each of the parts called?

9. When a pear is divided into three equal parts, what are the parts called? What is any *one* of the parts called?

10. When an orange is divided into four equal parts, what are the parts called? What is each of the parts called?

11. Three thirds of a pear are equal to how many whole pears? Five fifths of a melon are equal to what?

When any thing, or number, is divided into equal parts, one or more of these equal parts is called a *fraction:* one of these equal parts is sometimes called a *fractional unit.*

That which shows the number of equal parts into which a thing, or number, is divided is called the *denominator*, and shows the *size* of the parts.

1 half ($\frac{1}{2}$).

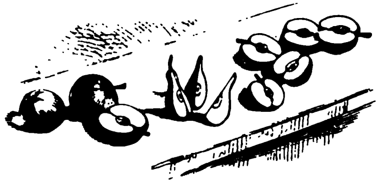
1 third ($\frac{1}{3}$).

2 fourths ($\frac{2}{4}$).

3 fifths ($\frac{3}{5}$).

One of the equal parts of a thing, or number, is called *one-half*, or *one-third*, or *one-fourth*, &c. Two or more of the equal parts are called *two-thirds*, *two-fourths*, *three-fifths*, &c.

That which shows *how many* of the equal parts are expressed is called the *numerator*.



When all the parts represented are *less* than a single thing, or unit, the expression is said to be a *proper fraction;* as, 1 third, 2 fourths, 3 fifths. When all the parts represented equal, or exceed, a single thing, the expression is said to be an improper fraction; as, 5 halves, 3 thirds. When one or more single things, and one or more parts of a single thing, are used together, the expression is called a *mixed number;* as, $2\frac{1}{2}$ apples.

79. A Fraction is one or more equal parts of a unit, as, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{8}$.

80. The Denominator shows into how many equal parts the thing, or number, is divided; as, in $\frac{3}{4}$, the denominator 4 shows that the unit is divided into four equal parts.

EXERCISE. — Write twenty fractions, point out the denominator of each, and tell its use.

81. The Numerator shows how many of the equal parts are included in the fraction; as, in $\frac{3}{5}$, the numerator 3 denotes that three of the five equal parts are expressed by the fraction.

EXERCISE. — Write twenty fractions, point out the numerator of each, and tell its use.

82. A Proper Fraction is one whose value is less than a unit; as, $\frac{2}{3}$, $\frac{5}{8}$, $\frac{3}{9}$.

EXERCISE. — Write twenty proper fractions, and tell why so called

83. *An Improper Fraction* is one whose value is equal to, or greater than, a unit; as, $\frac{4}{3}$, $\frac{17}{13}$, $\frac{5}{5}$.

EXERCISE. — Write twenty improper fractions, and tell why so called.

84. *A Mixed Number* consists of an integer and a fraction; as, $2\frac{1}{2}$, $5\frac{7}{8}$. The sign + is understood between the integer and the fraction.

EXERCISE. — Write twenty mixed numbers, and tell why so called.

85. PRINCIPLES. — I. *The smaller the number of equal parts into which the unit is divided, the greater is the size of each part.*

II. *The greater the number of equal parts into which a unit is divided, the smaller is the size of each part.*

Fractions may be written in words; as, four-tenths, five-sevenths; or by figures, thus, $\frac{3}{7}$, $\frac{4}{11}$, $\frac{5}{7}\begin{matrix}\text{Numerator.}\\ \text{Denominator.}\end{matrix}$

ORAL EXERCISES.

1. When an apple is divided into 3 equal parts, what are the parts called? When divided into 4 equal parts? When into 7?

2. When an orange is divided into 8 equal parts, what are the parts called? When divided into 4 equal parts? 11 parts? 9 parts? 6 parts?

3. When a peach is divided into 2 equal parts, what is each one of the parts called? When divided into 5 equal parts? Into 3 equal parts? Into 7 equal parts? Into 11 equal parts?

4. When a melon is divided into 5 equal parts, what is 1 of the parts called? What are 2 parts called? *3 parts? 5 parts?*

5. When a pie is cut into 8 equal parts, what are 2 parts called? 6 parts? 3 parts? 1 part? 7 parts? 4 parts?

6. What is meant by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{6}$, &c.?

ANS. — *By one-half is meant* ONE *of the* TWO *equal parts into which a thing, or number, is divided; one-third is* ONE *of the* THREE *equal parts; one-fifth is* ONE *of the* FIVE *equal parts; &c.*

7. What is meant by $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{9}$, &c.?

ANS. — *By two-thirds is meant* TWO *of the* THREE *equal parts into which a unit, or number of units, is divided. The term three-fifths means* THREE *of the* FIVE *equal parts into which a unit, or number of units, is divided, &c.*

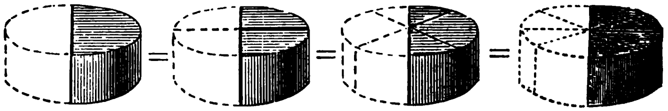
NOTE. — When no definite number of units is expressed, *one unit is always understood.*

8. When a melon is divided into six equal parts, what is one of the parts called?

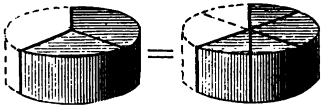
ANS. — One-sixth of a melon.

REDUCTION OF FRACTIONS TO HIGHER TERMS.

ILLUSTRATIONS.



$\frac{1\times 2}{2\times 2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$



$\frac{2\times 2}{3\times 2} = \frac{4}{6}$

From the above illustrations is deduced the following: —

86. PRINCIPLE. — *Multiplying both terms of a fraction by the same number* does not change its value.

ORAL, SLATE, AND BLACKBOARD EXERCISES.

Write a series of ten equivalent fractions from each of the following, to be read as a class-exercise.

PROCESS.

$\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}=\frac{5}{10}=\frac{6}{12}=\frac{7}{14}=\frac{8}{16}=\frac{9}{18}$, &c.

$\frac{1}{3}=\frac{2}{6}=\frac{3}{9}=\frac{4}{12}=\frac{5}{15}=\frac{6}{18}=\frac{7}{21}=\frac{8}{24}$, &c.

$\frac{2}{3}=\frac{4}{6}=\frac{6}{9}=\frac{8}{12}=\frac{10}{15}=\frac{12}{18}=\frac{14}{21}=\frac{16}{24}$, &c.

$\frac{3}{5}=\frac{6}{10}=\frac{9}{15}=\frac{12}{20}=\frac{15}{25}=\frac{18}{30}$, &c.

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)	(9.)	(10.)	(11.)	(12.)
$\frac{1}{2}$	$\frac{1}{7}$	$\frac{1}{12}$	$\frac{2}{11}$	$\frac{3}{10}$	$\frac{4}{11}$	$\frac{5}{11}$	$\frac{7}{9}$	$\frac{8}{11}$	$\frac{1}{13}$	$\frac{6}{13}$	$\frac{11}{13}$
$\frac{1}{3}$	$\frac{1}{8}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{3}{11}$	$\frac{5}{6}$	$\frac{5}{12}$	$\frac{7}{10}$	$\frac{9}{10}$	$\frac{2}{13}$	$\frac{7}{13}$	$\frac{12}{13}$
$\frac{1}{4}$	$\frac{1}{9}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{5}{7}$	$\frac{6}{7}$	$\frac{7}{11}$	$\frac{9}{11}$	$\frac{3}{13}$	$\frac{8}{13}$	$\frac{1}{14}$
$\frac{1}{5}$	$\frac{1}{10}$	$\frac{2}{7}$	$\frac{3}{7}$	$\frac{4}{7}$	$\frac{5}{8}$	$\frac{6}{11}$	$\frac{7}{12}$	$\frac{10}{11}$	$\frac{4}{13}$	$\frac{9}{13}$	$\frac{3}{14}$
$\frac{1}{6}$	$\frac{1}{11}$	$\frac{2}{9}$	$\frac{3}{8}$	$\frac{4}{9}$	$\frac{5}{9}$	$\frac{7}{8}$	$\frac{8}{9}$	$\frac{11}{12}$	$\frac{5}{13}$	$\frac{10}{13}$	$\frac{5}{14}$

Problem.—Change $\frac{5}{6}$ to an equivalent fraction of the denomination of 30ths.

PROCESS.

$\frac{5 \times 5}{6 \times 5} = \frac{25}{30}$

Analysis.—Since $\frac{1}{6}$ of one unit is equal to $\frac{5}{30}$, $\frac{5}{6}$ of a unit will equal five times as many thirtieths, or $\frac{25}{30}$.

In the same manner, reduce and analyze each of the following:—

1. $\frac{4}{7}$ to 21sts.
2. $\frac{5}{8}$ to 40ths.
3. $\frac{5}{7}$ to 35ths.
4. $\frac{2}{11}$ to 22ds.
5. $\frac{4}{13}$ to 26ths.
6. $\frac{3}{4}$ to 36ths.
7. $\frac{2}{5}$ to 60ths.
8. $\frac{4}{7}$ to 63ds.
9. $\frac{5}{9}$ to 72ds.

Review.—For what is the Arabic notation used? (9.) How many figures are there? *What is the value of each?* What is the use of the cipher? Which are *called digits? What is the simple* value of a figure? (10.) What the local value? (11.)

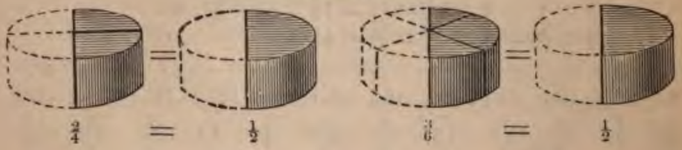
TO REDUCE A FRACTION TO ITS LOWEST TERMS.

87. A *fraction* is at its *lowest terms* when its numerator and denominator are prime to each other.

ORAL AND WRITTEN EXERCISES.

How many halves in $\frac{2}{4}$? In $\frac{3}{6}$?

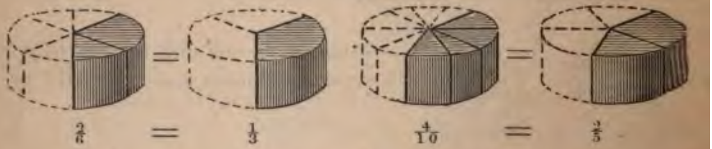
ILLUSTRATION.



1. How many halves in $\frac{3}{6}$? In $\frac{4}{8}$? In $\frac{8}{16}$? In $\frac{25}{50}$? In $\frac{6}{12}$? In $\frac{3}{6}$? In $\frac{12}{24}$? In $\frac{5}{10}$? In $\frac{7}{14}$? *W.* In $\frac{17}{34}$? In $\frac{11}{22}$? In $\frac{71}{142}$? In $\frac{95}{190}$? In $\frac{62}{124}$?

2. Reduce $\frac{2}{6}$ and $\frac{4}{10}$ to their lowest terms.

ILLUSTRATION.



ANALYSIS.—Since every 2 *tenths* equals 1 *fifth*, in 4 *tenths* there are as many *fifths* as 2 *tenths* are contained times in 4 *tenths*, or $\frac{2}{5}$.

3. Reduce $\frac{5}{15}$ to its lowest terms. $\frac{3}{9}$. $\frac{6}{18}$. $\frac{4}{12}$. $\frac{7}{21}$. $\frac{11}{33}$. $\frac{14}{21}$. $\frac{16}{32}$. $\frac{4}{12}$. *W.* $\frac{21}{63}$. $\frac{15}{45}$. $\frac{31}{93}$. $\frac{48}{144}$. $\frac{16}{48}$. $\frac{24}{72}$.

REVIEW.—What is numeration? (12.) Repeat the table from right to left *From left to right* by periods. How do numbers increase? (13.) What is mean *by tenfold ratio?* What is a rule? (14.) Give the rule for notation. (15.) Giv *the rule for* numeration. (16.)

PROBLEM. — (*a.*) How many fifths in $\frac{4}{10}$? (*b.*) In $\frac{24}{60}$?

FORMULA. — *b.* $\frac{24}{60} \div \frac{12}{12} = \frac{2}{5}$; or,

$12)\frac{24}{60} = \frac{2}{5}$.

(*b.*) **ANALYSIS.** — Since every 12 sixtieths is equal to 1 fifth, in 24 sixtieths, there are as many *fifths* as 12 sixtieths are contained times in 24 sixtieths, or 2 *fifths*, ($\frac{2}{5}$).

From the above is deduced the following: —

88. PRINCIPLE. — *Dividing both terms of a fraction by the same number does not change its value.*

To reduce a fraction to its lowest terms, use the following: —

RULE. — *Divide the numerator and the denominator of the fraction by any number that will divide them both without a remainder. Continue dividing in this way until no number greater than* ONE *will divide them.*

4. How many fourths in $\frac{6}{8}$? In $\frac{4}{16}$? *W.* In $\frac{54}{135}$?

PROCESS. — $3)\frac{54}{135}$ $3)\frac{18}{45}$ $3)\frac{6}{15}$ $\frac{2}{5}$.

5. Change $\frac{8}{12}$ to thirds. $\frac{4}{14}$ to sevenths. *W.* $\frac{18}{45}$ to fifths. $\frac{24}{36}$ to thirds.

6. Reduce $\frac{7}{63}$ to ninths. $\frac{4}{10}$ to fifths. *W.* $\frac{13}{91}$ to sevenths. $\frac{24}{84}$ to sevenths.

ORAL, SLATE, AND BLACKBOARD EXERCISES.*

Reduce each of the following fractions to its lowest terms, to be read as a class-exercise.

MODEL.	(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
$\frac{2}{4} = \frac{1}{2}$	$\frac{5}{10}$	$\frac{4}{6}$	$\frac{14}{77}$	$\frac{21}{28}$	$\frac{20}{25}$	$\frac{10}{12}$	$\frac{18}{21}$
$\frac{2}{8} = \frac{1}{4}$	$\frac{8}{16}$	$\frac{6}{9}$	$\frac{16}{88}$	$\frac{21}{49}$	$\frac{24}{30}$	$\frac{15}{18}$	$\frac{28}{32}$
$\frac{4}{6} = \frac{2}{3}$	$\frac{7}{14}$	$\frac{8}{12}$	$\frac{4}{26}$	$\frac{15}{50}$	$\frac{36}{45}$	$\frac{20}{28}$	$\frac{14}{84}$
$\frac{9}{12} = \frac{3}{4}$	$\frac{11}{33}$	$\frac{6}{21}$	$\frac{6}{8}$	$\frac{27}{99}$	$\frac{16}{36}$	$\frac{15}{27}$	$\frac{42}{66}$
$\frac{12}{16} = \frac{3}{4}$	$\frac{14}{42}$	$\frac{16}{56}$	$\frac{15}{20}$	$\frac{24}{104}$	$\frac{20}{45}$	$\frac{20}{44}$	$\frac{56}{96}$
$\frac{10}{15} = \frac{2}{3}$	$\frac{9}{45}$	$\frac{18}{81}$	$\frac{24}{32}$	$\frac{6}{26}$	$\frac{24}{42}$	$\frac{10}{26}$	$\frac{14}{26}$

---

* Since the fractions used in business transactions are few, and the terms of most are small, it is of great importance that the pupils should be able to recognize the lowest terms of each at a glance: hence the large number of these exercises

SUGGESTION. — The greatest common divisor can be readily discovered by finding what part of the numerator is contained in the denominator; as, the whole, a half, a third, a fourth, &c.

The following suggestions are valuable in discovering the divisor of a number:—

1. Every even number is divisible by 2; as, 384, 672.
2. Every number ending with 5 or a zero is divisible by 5; as, 305, 240.
3. If the sum of the digits of any number is divisible by 3, the number itself is divisible by 3; as, 612, 822.

7. What are the lowest terms of $\frac{4}{8}$? Of $\frac{5}{10}$? Of $\frac{6}{14}$? Of $\frac{7}{21}$? Of $\frac{8}{10}$? Of $\frac{9}{12}$? *W.* Of $\frac{64}{72}$? Of $\frac{27}{36}$? Of $\frac{29}{232}$? Of $\frac{31}{279}$?

8. What are the lowest terms of $\frac{9}{12}$? Of $\frac{8}{16}$? Of $\frac{21}{28}$? Of $\frac{16}{48}$? Of $\frac{25}{30}$? Of $\frac{22}{27}$? Of $\frac{88}{99}$? *W.* Of $\frac{63}{105}$? Of $\frac{121}{132}$? Of $\frac{100}{120}$?

9. What are the lowest terms of $\frac{56}{24}$? Of $\frac{81}{27}$? Of $\frac{120}{180}$? Of $\frac{48}{96}$? Of $\frac{36}{84}$? Of $\frac{45}{90}$? *W.* Of $\frac{84}{132}$? Of $\frac{121}{220}$? Of $\frac{750}{890}$?

10. What are the lowest terms of $\frac{7}{14}$? $\frac{25}{100}$? $\frac{20}{100}$? $\frac{40}{100}$? $\frac{45}{100}$? $\frac{75}{100}$? $\frac{15}{100}$? *W.* $\frac{30}{100}$? $\frac{35}{100}$? $\frac{132}{240}$? $\frac{220}{1230}$?

11. What are the lowest terms of $\frac{38}{100}$? $\frac{4}{100}$? $\frac{5}{100}$? $\frac{6}{100}$? $\frac{8}{100}$? $\frac{12}{100}$? $\frac{14}{100}$? $\frac{16}{100}$? *W.* $\frac{18}{100}$? $\frac{88}{100}$? $\frac{55}{200}$? $\frac{128}{448}$? $\frac{81}{108}$? $\frac{678}{321}$?

12. What are the lowest terms of $\frac{46}{100}$? $\frac{54}{100}$? $\frac{64}{100}$? $\frac{22}{100}$? $\frac{24}{100}$? $\frac{26}{100}$? *W.* $\frac{75}{100}$? $\frac{88}{100}$? $\frac{95}{100}$? $\frac{75}{100}$? $\frac{402}{807}$? $\frac{837}{534}$? $\frac{3003}{5005}$?

ALWAYS CHANGE FRACTIONS TO THE LOWEST TERMS PERMITTED BY THE CONDITIONS OF THE WORK.

REVIEW. — *What is addition?* (17.) What is the sum of two or more numbers? *(18.)* Describe the sign of addition. (19.) Show its use. Describe the sign of equality. (20.) Show its use. Describe the dollar-sign. (21.)

89. TO CHANGE INTEGRAL OR MIXED NUMBERS TO IMPROPER FRACTIONS.

PROBLEM.—How many quarters in $2\frac{3}{4}$ apples? *W.* In $37\frac{1}{4}$ oranges?

WRITTEN PROCESS.

$$\begin{array}{r} 37\frac{1}{4} \\ 4 \\ \hline 149 \\ \hline 4 \end{array}$$

ANALYSIS.—Since in 1 apple there are 4 quarters, in 2 apples there are 2 times 4 quarters, or 8 quarters, and 8 quarters and 3 quarters are 11 quarters: hence $2\frac{3}{4} = \frac{11}{4}$.

RULE.—*Multiply the integral number by the denominator of the fraction, and to the product add the numerator, and write the result over the denominator.*

ORAL, SLATE, AND BLACKBOARD EXERCISES.

Write the corresponding improper fraction for each of the following mixed numbers.

MODEL.	(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
$3\frac{1}{4} = \frac{13}{4}$	$4\frac{3}{7}$	$4\frac{3}{4}$	$6\frac{3}{4}$	$14\frac{3}{2}$	$5\frac{3}{7}$	$7\frac{2}{3}$
$4\frac{1}{7} = \frac{29}{7}$	$6\frac{7}{8}$	$8\frac{6}{7}$	$4\frac{5}{9}$	$11\frac{4}{7}$	$2\frac{5}{4}$	$8\frac{6}{7}$
$3\frac{1}{8} = \frac{25}{8}$	$5\frac{5}{7}$	$5\frac{8}{9}$	$7\frac{5}{8}$	$16\frac{3}{7}$	$6\frac{3}{17}$	$9\frac{5}{11}$
$5\frac{1}{9} = \frac{46}{9}$	$3\frac{3}{8}$	$4\frac{3}{7}$	$8\frac{6}{7}$	$14\frac{1}{6}$	$8\frac{6}{13}$	$8\frac{17}{14}$
$5\frac{4}{7} = \frac{39}{7}$	$7\frac{1}{8}$	$6\frac{6}{7}$	$13\frac{1}{4}$	$13\frac{5}{8}$	$4\frac{7}{11}$	$6\frac{13}{17}$
$4\frac{5}{8} = \frac{37}{8}$	$6\frac{8}{9}$	$5\frac{3}{7}$	$17\frac{2}{7}$	$6\frac{7}{8}$	$8\frac{3}{16}$	$5\frac{2}{7}$

WRITTEN EXAMPLES.

Change each of the following numbers to fractions.

13. $42\frac{3}{4}$.
14. $28\frac{5}{7}$.
15. $91\frac{2}{3}$.
16. $67\frac{4}{9}$.
17. $59\frac{7}{9}$.
18. $46\frac{3}{4}$.
19. $876\frac{4}{5}$.
20. $86\frac{3}{9}$.
21. $413\frac{1}{2}$.
22. $682\frac{5}{8}$.
23. $83\frac{5}{9}$.
24. $413\frac{4}{13}$.
25. $917\frac{5}{11}$.
26. $763\frac{9}{17}$.
27. $412\frac{19}{79}$.

NOTE.—*For additional* examples, see drill-card exercises.

90. TO CHANGE IMPROPER FRACTIONS TO INTEGRAL OR MIXED NUMBERS.

PROBLEM. — How many melons in 15 halves?

FORMULA. $\frac{15}{2} = 7\frac{1}{2}$.

ANALYSIS. — Since in 2 halves there is 1 melon, in 15 halves there are as many melons as 2 halves are contained times in 15 halves; or 7 with 1 half remaining: hence $\frac{15}{2} = 7\frac{1}{2}$.

RULE. — *Divide the numerator by the denominator.*

ORAL, SLATE, AND BLACKBOARD EXERCISES.

Write corresponding integral or mixed numbers for each of the following improper fractions.

MODEL.	(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
$\frac{43}{7} = 6\frac{1}{7}$	$\frac{46}{7}$	$\frac{47}{8}$	$\frac{56}{3}$	$\frac{49}{21}$	$\frac{46}{37}$	$\frac{59}{72}$	$\frac{43}{6}$
$\frac{27}{3} = 9$	$\frac{37}{2}$	$\frac{38}{6}$	$\frac{72}{3}$	$\frac{73}{16}$	$\frac{97}{81}$	$\frac{41}{36}$	$\frac{94}{6}$
$\frac{63}{4} = 15\frac{3}{4}$	$\frac{46}{9}$	$\frac{97}{6}$	$\frac{86}{7}$	$\frac{42}{13}$	$\frac{143}{67}$	$\frac{138}{71}$	$\frac{83}{21}$
$\frac{47}{4} = 11\frac{3}{4}$	$\frac{86}{8}$	$\frac{97}{9}$	$\frac{96}{11}$	$\frac{72}{16}$	$\frac{48}{31}$	$\frac{68}{3}$	$\frac{47}{3}$
$\frac{68}{5} = 13\frac{3}{5}$	$\frac{47}{3}$	$\frac{57}{3}$	$\frac{57}{9}$	$\frac{83}{18}$	$\frac{67}{32}$	$\frac{48}{21}$	$\frac{168}{82}$
$\frac{47}{13} = 3\frac{8}{13}$	$\frac{67}{9}$	$\frac{67}{2}$	$\frac{68}{3}$	$\frac{89}{17}$	$\frac{47}{36}$	$\frac{67}{28}$.	$\frac{46}{3}$

WRITTEN EXAMPLES.

Change the following fractions to integral or mixed numbers.

28. $\frac{683}{4}$, $\frac{213}{4}$.
29. $\frac{678}{4}$, $\frac{897}{4}$.
30. $\frac{896}{5}$, $\frac{371}{5}$.
31. $\frac{680}{5}$, $\frac{413}{5}$.
32. $\frac{6713}{6}$, $\frac{312}{4}$.
33. $\frac{4913}{7}$, $\frac{317}{5}$.
34. $\frac{9836}{8}$.
35. $\frac{4163}{8}$.
36. $\frac{7888}{11}$.
37. $\frac{83764}{10}$.
38. $\frac{4873}{16}$.
39. $\frac{2839}{22}$.
40. $\frac{61381}{37}$.
41. $\frac{2134}{67}$.
42. $\frac{4134}{382}$.
43. $\frac{6713}{42}$.
44. $\frac{789}{631}$.
45. $\frac{567}{567}$.
46. $\frac{697}{397}$.
47. $\frac{413}{413}$.
48. $\frac{8643}{241}$.
49. $\frac{6783}{874}$.
50. $\frac{637}{83}$.
51. $\frac{6913}{92}$.

For additional examples, see drill-card exercises.

ORAL REVIEW.

1. How many thirds in 1? In 2? In 4?

Analysis.—Since there are 3 thirds in 1, in 4 there are 4 times 3 thirds, or $\frac{12}{3}$.

2. How many quarters in 2 melons? How many fourths? How many thirds? Sixths? Elevenths?

3. How many quarters in 2 yards? In $2\frac{1}{4}$ yards? In $2\frac{3}{4}$ yards? In $6\frac{3}{4}$ yards?

4. How many units in $\frac{3}{3}$? In $\frac{5}{5}$? In $\frac{7}{7}$? In $\frac{6}{3}$? In $\frac{12}{3}$? In $\frac{18}{3}$? In $\frac{10}{5}$? In $\frac{21}{7}$?

5. How many apples in $\frac{8}{4}$? In $\frac{16}{4}$? In $\frac{5}{4}$? In $\frac{4}{4}$? In $\frac{7}{4}$?

6. How many ones in $\frac{3}{3}$? In $\frac{7}{7}$? In $\frac{14}{7}$? In $\frac{18}{9}$?

7. How many times 1 are $\frac{13}{3}$?

Analysis.—Since 3 thirds equal 1, 13 thirds will equal as many times 1 as 3 thirds are contained times in 13 thirds, or $4\frac{1}{3}$.

8. How many yards in $\frac{14}{7}$ of a yard? In $\frac{21}{3}$? In $\frac{16}{7}$?

9. How many fifths in 3 apples? In $6\frac{1}{5}$ apples? In $3\frac{4}{5}$ oranges? In $8\frac{1}{5}$?

10. How many halves in 1 apple? How many quarters in $\frac{1}{2}$ of an apple? How many eighths in $\frac{1}{4}$ of an apple?

11. In $\frac{1}{2}$ of an apple, how many eighths? How many fourths? How many sixteenths? Eighteenths?

12. In $3\frac{3}{4}$ how many quarters? Eighths? Twelfths? Sixteenths? Twentieths? Twenty-fourths?

Review.—What is subtraction? (31.) Name the terms of subtraction. (32, 33, 34.) Describe the sign of subtraction. (35.) Show its use. Give the axioms used in subtraction. (36.) Analyze a short example in subtraction. (37.) By a *different method.* (38.) What is the use of a parenthesis? (39.)

DRILL CARD EXERCISES. — TABLE IV.

Note. — The teacher should select such portions of these drill exercises as are adapted to the wants of the class.

REDUCTION OF FRACTIONS.

Exercise I. — Write a series of ten fractions, each equivalent to the fractions in column **V** (see card); thus, A. $\frac{1}{2}=\frac{2}{4}=\frac{3}{6}$, &c. B. $\frac{1}{5}=\frac{2}{10}=\frac{3}{15}=\frac{4}{20}$, &c. C. $\frac{1}{8}=\frac{2}{16}=\frac{3}{24}$, &c.

Ex. II. — In the same way write ten fractions, each equivalent to the fractions in column **W**.

Ex. III. — Write ten fractions, each equivalent to the fractions in column **X**.

Ex. IV. — Prefix to each fraction the integral number opposite expressed in column **1**, and change the resulting mixed number to an improper fraction; thus,

Column **V**.	Column **W**.	Column **X**.
A. $1\frac{1}{2}$ = what?	A. $1\frac{1}{3}$ = what?	A. $1\frac{1}{4}$ = what?
B. $6\frac{1}{5}$ = what?	B. $6\frac{1}{6}$ = what?	B. $6\frac{1}{7}$ = what?
C. $8\frac{1}{8}$ = what?	C. $8\frac{1}{9}$ = what?	C. $8\frac{1}{10}$ = what?
D. $9\frac{1}{11}$ = what?	D. $9\frac{1}{12}$ = what?	D. $9\frac{1}{13}$ = what?
&c.	&c.	&c.

Ex. V. — Prefix to each fraction the integral numbers opposite expressed in column **2**, and change the resulting mixed numbers to improper fractions; thus,

Column **V**.	Column **W**.	Column **X**.
A. $38\frac{1}{2}$ = what?	A. $38\frac{1}{3}$ = what?	A. $38\frac{1}{4}$ = what?
B. $59\frac{1}{5}$ = what?	B. $59\frac{1}{6}$ = what?	B. $59\frac{1}{7}$ = what?

Ex. VI. — Prefix the integral number expressed in column **3** to each of the fractions as before, and change to improper fractions.

Ex. VII. — In the place of each numerator in each of the columns, substitute one five times as large, and change to integral or mixed numbers when possible; thus,

Column **V**.	Column **W**.	Column **X**.
A. $\frac{5}{2}$ = what?	A. $\frac{5}{3}$ = what?	A. $\frac{5}{4}$ = what?
B. $\frac{5}{5}$ = what?	B. $\frac{5}{6}$ = what?	B. $\frac{5}{7}$ = what?
C. $\frac{5}{8}$ = what?	C. $\frac{5}{9}$ = what?	C. $\frac{5}{10}$ = what?

Note. — The fractions should always be given in the lowest terms.

Ex. VIII. — In the place of each numerator in each of the columns, *substitute one ten times* as large, and reduce as before.

Ex. IX. — *In the* place of each numerator, in each of the columns, *substitute one* twenty-five times as large, and reduce as before.

FELTER'S
ARITHMETICAL DRILL CARDS.

	TABLE IV. FRACTIONS.						TABLE V.	
	3	2	1	V	W	X	UNITS. Dec. Point. 10ths. 100ths. 1000ths. 10000ths. 100000ths. 1000000ths. 5.834709	
A	472	38	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{3}{1}$	A
B	604	59	6	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{50}{1}$	B
C	103	16	8	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{9}{10}$	C
D	504	20	9	$\frac{1}{11}$	$\frac{1}{12}$	$\frac{1}{13}$	$\frac{20}{10}$	D
E	300	47	4	$\frac{2}{3}$	$\frac{2}{5}$	$\frac{2}{7}$	$\frac{63}{10}$	E
F	412	32	1	$\frac{2}{9}$	$\frac{2}{11}$	$\frac{2}{13}$	$\frac{25}{100}$	F
G	755	93	3	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{3}{7}$	$\frac{300}{100}$	G
H	411	51	7	$\frac{3}{8}$	$\frac{3}{10}$	$\frac{3}{11}$	$\frac{208}{100}$	H
I	640	80	6	$\frac{4}{5}$	$\frac{4}{7}$	$\frac{4}{9}$	$\frac{732}{1000}$	I
K	832	21	8	$\frac{4}{11}$	$\frac{4}{13}$	$\frac{4}{15}$	$\frac{8000}{1000}$	K
L	253	73	5	$\frac{5}{6}$	$\frac{5}{7}$	$\frac{5}{8}$	$\frac{8021}{1000}$	L
M	414	86	2	$\frac{5}{9}$	$\frac{5}{11}$	$\frac{5}{12}$	$\frac{2143}{10000}$	M
N	674	13	1	$\frac{6}{7}$	$\frac{6}{11}$	$\frac{6}{13}$	$\frac{105}{10}$	N
O	305	14	8	$\frac{7}{8}$	$\frac{7}{9}$	$\frac{7}{10}$	$\frac{6134}{100}$	O
P	501	73	3	$\frac{7}{11}$	$\frac{7}{12}$	$\frac{7}{13}$	$\frac{43}{1000}$	P
Q	244	89	7	$\frac{8}{9}$	$\frac{8}{11}$	$\frac{8}{13}$	$\frac{1009}{100}$	Q
R	106	67	1	$\frac{9}{10}$	$\frac{9}{11}$	$\frac{9}{13}$	$\frac{5}{100}$	R
S	243	45	6	$\frac{10}{11}$	$\frac{10}{13}$	$\frac{10}{17}$	$\frac{2}{100000}$	S
T	182	38	2	$\frac{11}{12}$	$\frac{11}{13}$	$\frac{11}{14}$	$\frac{70}{1000000}$	T
U	100	72	3	$\frac{12}{13}$	$\frac{12}{17}$	$\frac{12}{19}$	$\frac{82000}{10}$	U

N.B.—*For answers to the problems on the above drill card, see Key.*

MISCELLANEOUS EXAMPLES.

ORAL AND WRITTEN.

52. *W.* Change $121\frac{3}{5}$ to fifths.

53. *W.* Change $27\frac{1}{3}$ to thirds.

54. *W.* Reduce $43\frac{2}{7}$ to sevenths.

55. Change $\frac{9}{4}$ to an equivalent mixed number. *W.* Change $\frac{112}{4}$ to an integer.

56. How many units in $\frac{9}{3}$? *W.* In $\frac{637}{4}$? In $\frac{943}{8}$?

57. To how many whole apples are $\frac{12}{4}$ equal? Are $\frac{36}{3}$ equal? *W.* Are $\frac{413}{5}$ equal?

58. How many oranges will be required to give 10 boys $\frac{1}{2}$ of an orange each? To give 12 boys $\frac{1}{3}$ of an orange each? *W.* To give 112 boys $\frac{1}{18}$ of an orange each?

59. Arthur can pick $\frac{1}{3}$ of a peck of berries in a day: how many thirds of a peck can he pick in 6 days? *W.* In 36 days? How many pecks?

60. If a man spend $\frac{1}{4}$ of a dollar in one day, how many fourths of a dollar will he spend in 13 days? *W.* In 68 days? How many dollars?

61. Harry had \$$6\frac{1}{2}$ to give some poor boys: to how many could he give a half-dollar each? A quarter of a dollar each? *W.* A tenth of a dollar each? A twenty-fourth of a dollar each?

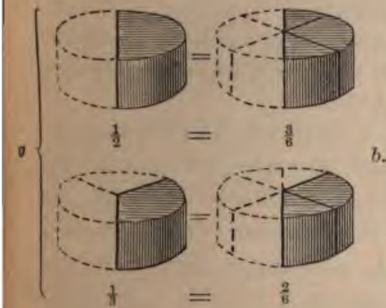
62. A boy has $5\frac{3}{5}$ apples: to how many schoolmates can he give a fifth of an apple each? A tenth of an apple? *W.* A seventy-fifth of an apple?

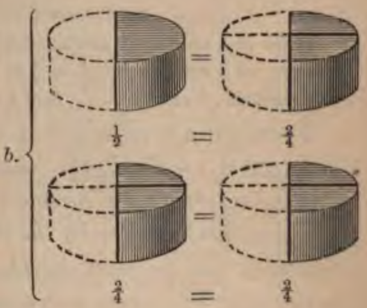
63. A man had $3\frac{4}{7}$ barrels of flour. He sold each seventh of a barrel for 1 dollar: how much did he receive for the flour? *W.* If he received 87 cents for *each seventh?* If [illegible] for each seventh?

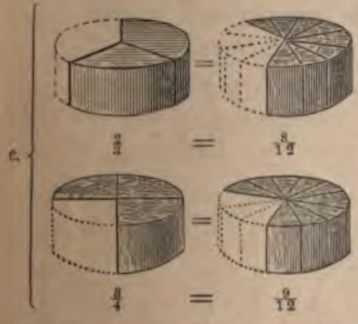
91. To CHANGE FRACTIONS TO EQUIVALENT FRACTIONS HAVING A COMMON DENOMINATOR.

92. ***Common Denominator.*** — When fractions are of the same denomination, they are said to have a common denominator.

ILLUSTRATION.







It will be seen by the illustration (*a*), that, if $\frac{1}{2}$ of a block of wood is divided into 3 equal parts, the pieces will be *sixths* of the block, and that, if $\frac{1}{3}$ of a block is divided into 2 equal parts, the pieces will also be sixths: hence, the pieces being of the same size, the fractions representing them will have a *common denominator.*

In the same way (*b*) $\frac{1}{2}$ and $\frac{2}{4}$ of a block may be reduced to pieces of the same *size*, or *fourths*. (*c*) $\frac{2}{3}$ and $\frac{3}{4}$ may each be changed to *twelfths* by dividing the quarters each into 3 equal parts, and the *thirds* each into 4 equal parts.

PROBLEM. — Change $\frac{2}{3}$ and $\frac{3}{4}$ to a common denominator.

PROCESS.

$\frac{2}{3} = \frac{8}{12}$

$\frac{3}{4} = \frac{9}{12}$

ANALYSIS.* — Since the least common multiple of the denominators of the fractions (c) $\frac{2}{3}$ and $\frac{3}{4}$ is 12, then *twelfths* will be the least number of equal parts into which $\frac{2}{3}$ and $\frac{3}{4}$ can be divided, and have all the pieces of equal size.

ANAL. STEPS. — 1. *Change $\frac{2}{3}$ to twelfths.*

Since in one unit there are 12 *twelfths,* in $\frac{1}{3}$ of a unit there are 1 third of 12 *twelfths,* or 4 *twelfths;* and in $\frac{2}{3}$ of a unit there are 2 times 4 *twelfths,* or 8 *twelfths.*

2. *Change $\frac{3}{4}$ to twelfths.*

Since in 1 unit there are 12 *twelfths,* in $\frac{1}{4}$ of a unit there are $\frac{1}{4}$ of 12 *twelfths,* or 3 *twelfths;* and in $\frac{3}{4}$ of a unit there are 3 times 3 *twelfths,* or 9 *twelfths:* hence $\frac{2}{3} = \frac{8}{12}$, and $\frac{3}{4} = \frac{9}{12}$.

From the above analyses is deduced the following: —

RULE I. — *Find the least common multiple of the denominators for a common denominator. Divide the common denominator by the denominator of each fraction, multiply the quotient by the numerator,* and *write the result over the common denominator,* or,

RULE II. — *Multiply the terms of each fraction by any number that will make the denominators common.*

ORAL, SLATE, AND BLACKBOARD EXERCISES.

Change the fractions in each series of the following numbers to equivalent fractions having a common denominator.

MODEL.	(1.)	(2.)	(3.)	(4.)	(5.)
$\frac{3}{4}, \frac{2}{3} = \frac{9}{12}, \frac{8}{12}.$	$\frac{6}{7}, \frac{4}{8}$	$\frac{2}{3}, \frac{4}{7}$	$\frac{4}{7}, \frac{2}{3}$	$\frac{4}{5}, \frac{3}{7}$	$\frac{4}{13}, \frac{1}{2}$
$\frac{4}{7}, \frac{2}{3} = \frac{12}{21}, \frac{14}{21}.$	$\frac{2}{7}, \frac{13}{14}$	$\frac{3}{6}, \frac{2}{7}$	$\frac{4}{11}, \frac{13}{22}$	$\frac{3}{8}, \frac{4}{7}$	$\frac{5}{7}, \frac{3}{14}$
$\frac{5}{7}, \frac{3}{4} = \frac{20}{28}, \frac{21}{28}.$	$\frac{4}{7}, \frac{20}{21}$	$\frac{4}{5}, \frac{3}{4}$	$\frac{5}{11}, \frac{6}{7}$	$\frac{2}{6}, \frac{4}{5}$	$\frac{4}{5}, \frac{1}{15}$
$\frac{3}{4}, \frac{1}{8} = \frac{6}{8}, \frac{1}{8}.$	$\frac{2}{7}, \frac{25}{21}$	$\frac{2}{5}, \frac{3}{7}$	$\frac{4}{17}, \frac{11}{34}$	$\frac{1}{4}, \frac{3}{8}$	$\frac{2}{8}, \frac{3}{4}$
$\frac{5}{6}, \frac{1}{3} = \frac{5}{6}, \frac{2}{6}.$	$\frac{4}{7}, \frac{3}{4}$	$\frac{4}{5}, \frac{2}{7}$	$\frac{1}{6}, \frac{6}{11}$	$\frac{5}{7}, \frac{5}{14}$	$\frac{5}{7}, \frac{6}{7}$
$\frac{4}{7}, \frac{4}{5} = \frac{20}{35}, \frac{28}{35}.$	$\frac{5}{9}, \frac{13}{18}$	$\frac{6}{7}, \frac{3}{8}$	$\frac{4}{5}, \frac{3}{7}$	$\frac{5}{8}, \frac{3}{24}$	$\frac{2}{3}, \frac{3}{4}$

NOTE. — When one denominator can be divided exactly by the other, it is necessary to multiply only one of the fractions; thus, $\frac{2}{3}, \frac{1}{6} = \frac{4}{6}, \frac{1}{6}.$

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* *The teacher should refer to the illustration at every step of the analysis.*

WRITTEN EXERCISES.

Problem.—Change $3\frac{5}{8}$, $2\frac{1}{6}$, $\frac{3}{4}$, $4\frac{5}{12}$, to equivalent fractions having a common denominator.

PROCESS.

$3\frac{5}{8}=3\frac{15}{24}$
$2\frac{1}{6}=2\frac{4}{24}$
$\frac{3}{4}=\frac{18}{24}$
$4\frac{5}{12}=4\frac{10}{24}$

Analysis.—Require the pupil to write an analysis of this problem in full, and present it to the teacher as a class-exercise.

64. $\frac{3}{7}$, $\frac{5}{8}$, $\frac{6}{12}$, $\frac{5}{14}$, $\frac{3}{4}$.
65. $\frac{6}{7}$, $\frac{3}{14}$, $\frac{7}{21}$, $\frac{11}{12}$, $\frac{11}{16}$.
66. $\frac{8}{9}$, $\frac{3}{27}$, $\frac{6}{7}$, $\frac{5}{28}$, $\frac{3}{21}$.
67. $\frac{2}{11}$, $\frac{6}{121}$, $\frac{11}{14}$, $\frac{3}{28}$.
68. $4\frac{3}{7}$, $\frac{6}{8}$, $\frac{9}{14}$, $11\frac{6}{7}$.
69. $3\frac{5}{8}$, $3\frac{4}{7}$, $5\frac{3}{8}$, $9\frac{4}{14}$.
70. $4\frac{3}{7}$, $5\frac{4}{28}$, $\frac{9}{12}$, $\frac{4}{15}$.
71. $4\frac{3}{4}$, $3\frac{11}{12}$, $\frac{6}{84}$, $\frac{3}{9}$.
72. $9\frac{4}{5}$, $3\frac{14}{28}$, $4\frac{8}{40}$, $\frac{3}{8}$.
73. $4\frac{3}{10}$, $8\frac{5}{14}$, $\frac{6}{7}$, $9\frac{2}{5}$.
74. $3\frac{6}{7}$, $5\frac{8}{9}$, $\frac{3}{27}$, $\frac{6}{28}$.
75. $3\frac{4}{7}$, $5\frac{9}{12}$, $\frac{1}{24}$, $3\frac{1}{28}$.
76. $\frac{6}{7}$, $5\frac{3}{21}$, $6\frac{6}{11}$, $3\frac{4}{9}$.
77. $\frac{8}{9}$, $6\frac{4}{7}$, $5\frac{3}{7}$, $6\frac{8}{14}$.

ADDITION OF FRACTIONS.

93. Since things can be added only to like things, it is evident, that, before fractions can be added or subtracted, they must be reduced to equivalent fractions having a common denominator.

Problem.—What is the sum of $5\frac{3}{4}$, $\frac{5}{6}$, and $4\frac{3}{8}$?

Formula. $5\frac{3}{4}+\frac{5}{6}+4\frac{3}{8}=5\frac{18}{24}+\frac{20}{24}+4\frac{9}{24}=10\frac{23}{24}$.

PROCESS.

$5\frac{3}{4}=5\frac{18}{24}$
$\frac{5}{6}=\frac{20}{24}$
$4\frac{3}{8}=4\frac{9}{24}$
$10\frac{23}{24}$

Analysis.—(1.) The least common denominator of $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{3}{8}$, is *twenty-fourths*.

(2.) $5\frac{3}{4}=5\frac{18}{24}$; $\frac{5}{6}=\frac{20}{24}$; and $4\frac{3}{8}=4\frac{9}{24}$.

(3.) The sum of 18 *twenty-fourths*, 20 *twenty-fourths*, and 9 *twenty-fourths*, is 47 *twenty-fourths*, equal to 1 unit and $\frac{23}{24}$; which, added to the sum of 4 and 5, equals $10\frac{23}{24}$.

Rule.—*Change the fractions to equal fractions having a common denominator, and add the numerators.*

Review.—What is multiplication? (40.) Define the terms. (41, 42, 43, 44.) Which are factors? (44.) Describe the sign of multiplication. (45.) Show its use. (45.) Repeat the axiom. (46.) Give the principles. (47.) Analyze a short example in multiplication. (48.)

NOTE. — 1. When adding mixed numbers, find the sum of th fractions, and add it to the sum of the integers.

2. Answers should always be given in the lowest terms, or reduce to integral or mixed numbers when possible.

ORAL, SLATE, AND BLACKBOARD EXERCISES.

Prepare the following like the model, and present o: slates as a class-exercise.

MODEL.	(1.)	(2.)	(3.)	(4.)
$\frac{1}{4}+\frac{1}{8}=\frac{3}{8}$	$\frac{1}{4}+\frac{1}{2}$	$\frac{3}{4}+\frac{1}{8}$	$4\frac{1}{7}+\frac{5}{28}$	$8\frac{1}{3}+\frac{2}{1}$
$\frac{1}{6}+\frac{7}{6}=1\frac{1}{3}$	$\frac{1}{3}+\frac{1}{6}$	$\frac{7}{8}+\frac{1}{2}$	$6\frac{1}{5}+4\frac{3}{5}$	$4\frac{9}{11}+\frac{9}{2}$
$\frac{4}{5}+\frac{1}{10}=\frac{9}{10}$	$\frac{3}{4}+\frac{7}{8}$	$\frac{4}{7}+\frac{1}{14}$	$2\frac{1}{4}+1\frac{1}{8}$	$6\frac{4}{5}+\frac{2}{3}$
$3\frac{3}{8}+1\frac{1}{8}=4\frac{1}{2}$	$\frac{1}{11}+\frac{2}{22}$	$\frac{3}{5}+\frac{2}{15}$	$7\frac{5}{7}+4\frac{3}{14}$	$9\frac{6}{7}+\frac{4}{4}$
$2\frac{5}{6}+\frac{1}{2}=3\frac{1}{3}$	$\frac{5}{10}+\frac{1}{5}$	$\frac{4}{7}+\frac{3}{14}$	$1\frac{2}{3}+2\frac{1}{15}$	$9\frac{4}{11}+10\frac{1}{2}$

DRILL CARD EXERCISES. — TABLE IV.

ADDITION OF FRACTIONS.

N.B. — For brief business methods of adding fractions, see Key t Drill Cards.

EXERCISE X. — Find the sum of **V** and **W**; as, A. $\frac{1}{2}+\frac{1}{3}$, &c.

Ex. XI. — Find the sum of **W** and **X**; as, A. $\frac{1}{3}+\frac{1}{4}$; B. $\frac{1}{6}+\frac{1}{7}$, &c

Ex. XII. — Find the sum of **V** and **X**; as, A. $\frac{1}{2}+\frac{1}{4}$; B. $\frac{1}{5}+\frac{1}{7}$, &c

Ex. XIII. — Find the sum of **V**, **W**, and **X**; as, A. $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$, &c

Ex. XIV. — Prefix 1 to **V**, and find the sum of two numbers; thus A + B, $1\frac{1}{2}+6\frac{1}{5}$; B + C, $6\frac{1}{5}+8\frac{1}{8}$; C + D, $8\frac{1}{8}+9\frac{1}{11}$; D + E, $9\frac{1}{11}+4\frac{2}{3}$ &c.

Ex. XV. — Prefix 1 to **W**, and find the sum of two numbers; thus A + B, $1\frac{1}{3}+6\frac{1}{6}$; B + C, $6\frac{1}{6}+8\frac{1}{9}$, &c.

Ex. XVI. — Prefix 1 to **X**, and find the sum of two numbers a: before.

Ex. XVII. — Prefix 2 to **V**, and find the sum of three numbers thus, A + B + C, $38\frac{1}{2}+59\frac{1}{5}+16\frac{1}{8}$; B + C + D, $59\frac{1}{5}+16\frac{1}{8}+20\frac{1}{11}$, &c.

Ex. XVIII. — Prefix 2 to **W**, and find the sum of three number as before.

Ex. XIX. — Prefix 3 to **V**, 2 to **W**, and 1 to **X**, then find the sun *of the resulting* mixed numbers; thus, A. $472\frac{1}{2}+38\frac{1}{3}+1\frac{1}{4}$; B. 604 $+59\frac{1}{6}+6\frac{1}{7}$, &c.

ORAL EXERCISES.

1. If a boy earns $\frac{2}{5}$ of a dollar at one time, $\frac{3}{5}$ of a dollar at another time, and \$$\frac{4}{7}$ at another, how many dollars does he earn in all?

FORMULA. $\$\frac{2}{5}+\$\frac{3}{5}+\$\frac{4}{7}=\$1\frac{4}{7}$.

ANALYSIS. \$$\frac{2}{5}$ and \$$\frac{3}{5}$ equal \$$\frac{5}{5}$ or, 1 dollar; and 1 dollar and \$$\frac{4}{7}$ equal \$$1\frac{4}{7}$.

SUGGESTION.—In adding more than two fractions, it is often more convenient to add *two* of them first, reduce, and add each of the others to the sum of the preceding addition.

2. If a boy spends $\frac{1}{6}$ of a dollar at one time, \$$1\frac{5}{6}$ at another time, and \$$\frac{1}{3}$ at another, how much does he spend in all?

3. There are 7 quarters of an apple in one pile, and 3 halves in another pile: how many apples in both?

4. A girl has 2 apples in one hand, and 3 quarter-apples in the other; her brother has 1 apple in one hand, and 1 half of an apple in the other: how many apples have both of the children?

5. Josie gave her mother $2\frac{1}{4}$ melons, and her father $2\frac{1}{3}$ melons: to which did she give the most? How much did she give to both?

6. Lillie gave to Laura $2\frac{1}{2}$ oranges, to Charlie $2\frac{3}{4}$ oranges, and to Delia $2\frac{3}{8}$ oranges: to which did she give the least? To which the most? How much did she give away in all?

7. A boy bought a pair of skates for \$$2\frac{3}{4}$, a knife for \$$\frac{7}{12}$, and a book for \$$1\frac{2}{3}$: how much did he give for all?

REVIEW.—Give method of multiplying by 10, 100, &c. (49.) Give the table of United States currency. (50.) What is understood by division? (51.) How many terms in division? (52, 53, 54, 55.) Define each. Give an example illustrating each. Describe the sign of division. (56.)

WRITTEN EXERCISES.

78. A man bought 4 pieces of calico. The first contained $37\frac{5}{9}$ yards; the second, $41\frac{5}{7}$ yards; the third, $27\frac{3}{8}$ yards; and the fourth, $38\frac{1}{12}$: how many yards did he buy?

79. If a pound of sugar costs $12\frac{5}{9}$ cents, a pound of tea \$$1.21\frac{1}{3}$, and a pound of coffee $22\frac{4}{9}$ cents, what will all cost?

80. A rubber cost $12\frac{1}{2}$ cents, a slate $15\frac{3}{8}$ cents, a pencil $\frac{11}{12}$ of a cent, and a sponge 5 cents: how much did they all cost?

81. A farmer raised $326\frac{7}{8}$ bushels of potatoes, $126\frac{3}{4}$ bushels of turnips, and $128\frac{1}{3}$ bushels of beets: how many bushels did he raise in all?

82. Henry has $24\frac{2}{3}$ dollars, Edward has $13\frac{1}{4}$ more than Henry: how many has Edward? How many have both?

83. One-third of a mill is valued at \$$416\frac{3}{4}$: what is the value of the whole mill?

84. A farm was divided into 5 parts; four of the parts contained $237\frac{4}{7}$ acres, one part contained $75\frac{2}{3}$ acres: how many acres in the farm?

85. \$$347\frac{2}{3}$ is \$$127\frac{3}{4}$ less than the value of my horses and carriage: what are they worth?

86. I sold a house for \$$1341\frac{3}{5}$, which was \$$137\frac{1}{4}$ less than it cost me: did I make, or lose? and how much?

87. A boy sold a sled for \$$1.86\frac{3}{7}$, which was $59\frac{5}{14}$ cents less than it cost him: did he make, or lose? and how much?

Review.—Describe the fractional sign. (57.) How many signs are there in *division?* What is their difference? What is the difference between *short division and long division?* (58, 59.)

88. I bought a mowing-machine for \$$280\frac{1}{3}$, and sold it so as to gain \$$23\frac{3}{5}$: for how much did I sell it?

89. A man bought $\frac{5}{8}$ of a share in an oil-well at one time, $\frac{3}{5}$ of a share at another time, and $3\frac{2}{5}$ shares at another time: how many shares did he buy?

90. How many times a number is $\frac{1}{2}$ of it, $\frac{1}{3}$ of it, $\frac{1}{4}$ of it, and $\frac{1}{5}$ of it?

91. How many units in 4, $3\frac{3}{7}$, $2\frac{4}{5}$, and $\frac{1}{2}$?

92. A boy had $6\frac{1}{2}$ apples; he gave his father $2\frac{1}{2}$ apples, his mother $\frac{2}{3}$ of one apple, and to his sister $\frac{7}{9}$ of another: how many apples did he give away?

93. Five times $16\frac{1}{3} + \frac{2}{3}$ of $16\frac{1}{3} + \frac{5}{9}$ of $16\frac{1}{3}$ are how many times $16\frac{1}{3}$?

94. $\frac{3}{8}$ of a pear $+ \frac{46}{3}$ of a pear $+ 3$ pears $+ 3\frac{9}{7}$ pears, make how many pears?

95. What is the sum of $\frac{5}{9}$, $\frac{41}{35}$, $7\frac{2}{3}$, $\frac{5}{14}$?

96. Three times a number plus $\frac{2}{3}$ of it, plus $4\frac{6}{7}$ times the number, plus $\frac{1}{9}$ of it, are how many times the number?

97. 34 times 28, plus $\frac{2}{3}$ of 28, plus $2\frac{5}{11}$ times 28, plus $\frac{7}{12}$ of 28, are how many times 28?

SUBTRACTION OF FRACTIONS.

94. TO SUBTRACT A FRACTIONAL NUMBER FROM AN INTEGER OR FROM ANOTHER FRACTIONAL NUMBER.

PROBLEM.—(*a.*) From 16 take $5\frac{2}{3}$. (*b.*) From $6\frac{3}{6}$ take $4\frac{5}{6}$. (*c.*) From $\frac{4}{5}$ take $\frac{2}{3}$.

PROCESSES.

(*a.*)	(*b.*)	(*c.*)
From 16	$6\frac{3}{6}$	$\frac{4}{5}=\frac{12}{15}$
Take $5\frac{2}{3}$	$4\frac{5}{6}$	$\frac{2}{3}=\frac{10}{15}$
Rem. $10\frac{1}{3}$	$1\frac{2}{3}$	$\frac{2}{15}$

c.) FORMULA. $\frac{12}{15} - \frac{10}{15} = \frac{2}{15}$.

Analysis. — (*a.*) Since there are no thirds in the minuend, change one unit to thirds; $\frac{3}{3}$ less $\frac{2}{3}$ equals $\frac{1}{3}$; 15 units less 5 units equals 10 units.

(*b.*) Since $\frac{5}{6}$ cannot be taken from $\frac{3}{6}$, change one of the 6 units to sixths, which, with $\frac{3}{6}$, makes $\frac{9}{6}$; $\frac{9}{6} - \frac{5}{6} = \frac{4}{6} = \frac{2}{3}$; and 5 less 4 = 1.

(*c.*) Change $\frac{4}{5}$ and $\frac{2}{3}$ to a common denominator; as, $\frac{12}{15}$ and $\frac{10}{15}$. $\frac{12}{15} - \frac{10}{15} = \frac{2}{15}$.

Rule. — *Change the fractions to equal fractions having a common denominator, and find the difference of the numerators.*

ORAL, SLATE, AND BLACKBOARD EXERCISES.

MODEL.	(1.)	(2.)	(3.)	(4.)	(5.)
$\frac{2}{3} - \frac{1}{2} = \frac{1}{6}$	$\frac{8}{9} - \frac{7}{12}$	$\frac{2}{3} - \frac{1}{2}$	$3\frac{1}{3} - 1\frac{2}{7}$	$\frac{3}{5} + \frac{2}{3}$	$4\frac{2}{11} - \frac{2}{22}$
$\frac{3}{4} - \frac{2}{3} = \frac{1}{12}$	$\frac{9}{10} - \frac{4}{5}$	$\frac{3}{4} - \frac{2}{3}$	$1\frac{4}{5} - \frac{3}{7}$	$\frac{7}{9} - \frac{5}{7}$	$2\frac{4}{7} + 1\frac{1}{3}$
$\frac{4}{5} - \frac{3}{4} = \frac{1}{20}$	$\frac{10}{11} - \frac{1}{22}$	$\frac{5}{6} - \frac{7}{12}$	$4\frac{3}{7} - 1\frac{1}{6}$	$\frac{3}{5} + \frac{8}{9}$	$5\frac{7}{9} - 2\frac{7}{10}$
$4\frac{5}{6} - \frac{4}{5} = 4\frac{1}{30}$	$\frac{11}{12} - \frac{5}{9}$	$\frac{6}{7} - \frac{7}{14}$	$8\frac{5}{7} - 5\frac{2}{9}$	$\frac{3}{7} - \frac{3}{8}$	$5\frac{7}{9} + 2\frac{7}{10}$
$3\frac{7}{8} - 2\frac{4}{7} = 1\frac{17}{56}$	$\frac{12}{13} - \frac{2}{3}$	$\frac{7}{8} - \frac{4}{5}$	$1\frac{2}{7} - 1\frac{2}{9}$	$\frac{3}{7} + \frac{3}{8}$	$5\frac{3}{7} + \frac{2}{3}$

DRILL CARD EXERCISES. — TABLE IV.

SUBTRACTION OF FRACTIONS.

N.B. — For brief business methods of subtracting fractions, see Key to Drill Cards.

Exercise I. — From **V** subtract **W**; thus, A. $\frac{1}{2} - \frac{1}{3}$; B. $\frac{1}{5} - \frac{1}{6}$; C. $\frac{1}{8} - \frac{1}{9}$, &c.

Ex. II. — From **W** subtract **X**; thus, A. $\frac{1}{3} - \frac{1}{4}$; B. $\frac{1}{6} - \frac{1}{7}$, &c.

Ex. III. — From **V** subtract **X**; thus, A. $\frac{1}{2} - \frac{1}{4}$; B. $\frac{1}{5} - \frac{1}{7}$, &c.

Ex. IV. — From **1** subtract **V**; thus, A. $1 - \frac{1}{2}$; B. $6 - \frac{1}{5}$, &c.

Ex. V. — From **2** subtract **W**; thus, A. $38 - \frac{1}{3}$; B. $59 - \frac{1}{6}$, &c.

Ex. VI. — From **3** subtract **X**; thus, A. $472 - \frac{1}{4}$; B. $604 - \frac{1}{7}$, &c.

Ex. VII. — Prefix column **1** to **V**, and subtract each mixed number from $23\frac{5}{11}$; thus, A. $23\frac{5}{11} - 1\frac{1}{2}$, &c.

Ex. VIII. — Prefix column **2** to **W**, and subtract each mixed number from $140\frac{1}{13}$.

Review. — Give the axiom used in division. (60.) Name the princples. (61.) Give an example illustrating each. Analyze a short example in division. (62.) Illustrate the method of dividing by 10, 100, 1,000, &c. (63.) Analyze a short example in long division. (64.)

Ex. IX. — Prefix column **3** to **X,** and subtract each mixed number from $920\frac{3}{5}$.

Ex. X. — Find the difference (column **V**) between A and B, $\frac{1}{2} - \frac{1}{5}$; B and C, $\frac{1}{5} - \frac{1}{8}$, &c.

Ex. XI. — Find the difference (column **W**) between A and B, B and C, &c.

Ex. XII. — Find the difference (column **X**) between A and B, B and C, &c.

Ex. XIII. — **2** — **V** + **W** + **1** — **X** = what? as, A. $38 - \frac{1}{2} + \frac{1}{3} + 1 - \frac{1}{4}$ = what? B. $59 - \frac{1}{5} + \frac{1}{6} + 6 - \frac{1}{7}$ = what?

ORAL EXERCISES.

1. What is meant by sixths? What by sevenths? What by ninths? By elevenths? By fifteenths?

2. What is the difference between halves and quarters? Thirds and fourths? Sixths and sevenths? Elevenths and twentieths?

3. Which is the greater, — a half, or a quarter? A third, or a fourth? A fifth, or a sixth? A ninth, or an eleventh?

4. Which is of the most value, $\frac{2}{3}$, or $\frac{2}{4}$? $\frac{2}{2}$, or $\frac{1}{2}$? $\frac{5}{8}$, or $\frac{5}{13}$? $\frac{2}{7}$, or $\frac{2}{9}$? $\frac{5}{8}$, or $\frac{5}{11}$? $\frac{3}{4}$, or $\frac{1}{4}$? $\frac{6}{7}$, or $\frac{3}{7}$? $\frac{2}{7}$, or $\frac{2}{9}$?

5. Jennie, having $\frac{3}{4}$ of an apple, gave $\frac{1}{4}$ to her sister: how much had she left?

6. What is the difference between $4\frac{3}{7}$ and $2\frac{2}{7}$?

7. If I pay $\$\frac{3}{4}$ from $\$3\frac{1}{4}$, how much remains?

8. If I sell $\frac{3}{4}$ of an acre of land from a lot which contains $\frac{11}{12}$ of an acre, what remains?

9. Ten acres less $\frac{1}{2}$ of an acre equals how much? Less $\frac{1}{3}$ of an acre? Less $\frac{3}{4}$ of an acre? Less $4\frac{2}{3}$ acres? Less $4\frac{4}{5}$ acres?

10. From $\frac{7}{8}$ take $\frac{3}{16}$; take $\frac{3}{4}$; take $\frac{8}{9}$; take $\frac{5}{7}$.

11. How much is $\frac{1}{3} + \frac{5}{6} - \frac{1}{4}$? $\frac{6}{7} + \frac{1}{2} - \frac{3}{14}$? $\frac{3}{5} - \frac{1}{3} + \frac{7}{15}$?

12. How much is $\frac{1}{2}$ plus $\frac{1}{3} - \frac{1}{4} + \frac{1}{8}$? $\frac{1}{2} + \frac{1}{10} - \frac{1}{5} + \frac{1}{15}$?

13. What is the difference between $4\frac{3}{7}$ and $2\frac{2}{9}$?

14. What must be added to $4\frac{3}{7}$ to make $8\frac{7}{9}$?

15. A boy bought a sled for $\$2\frac{1}{7}$; he then sold it for $\$\frac{1}{9}$ less than he gave for it: how much did he receive for it?

16. Kate had 25 cents; she bought apples for $3\frac{1}{3}$ cents, nuts for $5\frac{5}{6}$ cents, and an orange for $2\frac{5}{6}$ cents: how much money had she left?

17. Anna bought 5 oranges for $5\frac{1}{2}$ cents, 3 apples for $4\frac{1}{2}$ cents, $1\frac{3}{4}$ pounds of almonds for $16\frac{1}{2}$ cents: how much did she give for all?

WRITTEN EXERCISES.

98. From $\frac{11}{16}$ take $\frac{3}{16}$.
99. From $\frac{21}{22}$ take $\frac{5}{11}$.
100. From $\frac{4}{9}$ take $\frac{2}{11}$.
101. $\frac{4}{7} - \frac{5}{11} =$ what?
102. $\frac{4}{9} - \frac{5}{15} =$ what?
103. $\frac{61}{17}$ less $\frac{17}{19} =$ what?
104. $\frac{5}{11}$ minus $\frac{5}{12} =$ what?
105. $3\frac{1}{4} - \frac{4}{13} =$ what?
106. $27 - \frac{61}{12}$ plus $\frac{3}{8} =$ what?
107. $24\frac{1}{2} + \frac{1}{2}$ less $\frac{3}{92} =$ what?
108. $18\frac{3}{8}$ minus $1\frac{2}{7} + \frac{1}{2} =$ what?
109. $4\frac{3}{7} - \frac{2}{11} + 4\frac{3}{7} =$ what?
110. $5 + \frac{41}{7} - \frac{21}{7} + 8 =$ what?
111. $4 + \frac{8}{8} - \frac{9}{9} \overline{+ 3\frac{1}{2}} =$ what?
112. $(8\frac{1}{2} - \frac{3}{4}) + \frac{2}{3} + \frac{1}{2} =$ what?
113. $(11\frac{1}{2} + \frac{2}{3}) - (\frac{2}{3} + \frac{4}{5}) =$ what?

MISCELLANEOUS PROBLEMS FOR REVIEW.

114. A man bought a cow for $\$39\frac{3}{13}$, and sold it for *\$40: what* was the profit?

FORMULA. $\$40 - \$39\frac{3}{13} =$ what?

NOTE. — The pupil should be required to write a formula showing the relation of the numbers in each problem.

115. A grocer bought $371\frac{4}{7}$ pounds of butter; he then sold $147\frac{5}{8}$ pounds: how many pounds had he left?

116. A man bought at one time $34\frac{3}{4}$ barrels flour, at another time $47\frac{3}{15}$, and at another $44\frac{5}{12}$ barrels: how many barrels did he buy?

117. From a ten-dollar bill I spent at one time $\$1.25\frac{3}{5}$, at another $\$1.64\frac{2}{5}$, and at another $\$3.37\frac{1}{2}$: how much had I left?

118. $3\frac{4}{7}$ pounds of coffee cost $2\frac{2}{3}$ dollars, $4\frac{3}{8}$ pounds of tea cost $5\frac{5}{7}$ dollars: how much did both cost?

119. I had $347\frac{2}{3}$ pounds of tea, $325\frac{5}{8}$ pounds of coffee, and $123\frac{3}{7}$ pounds of rice; I sold $121\frac{5}{8}$ pounds of each: how many pounds of each had I remaining?

120. The minuend is $314\frac{2}{7}$, the subtrahend is $116\frac{4}{35}$: what is the difference?

121. The difference is $57\frac{3}{7}$, the minuend is $411\frac{8}{35}$: what is the subtrahend?

122. The subtrahend is $63\frac{5}{8}$, the difference is $51\frac{1}{2}$: what is the minuend?

123. Henry had $47\frac{3}{4}$ dollars, and Robert as many, lacking $\$8\frac{4}{15}$: how many dollars had Robert?

124. George bought a sled for $\$3\frac{3}{5}$, and sold it so as to gain $\$1\frac{7}{11}$: for how much did he sell it?

125. Mary bought a portfolio for $\$2.34\frac{5}{7}$; but she gave it to her cousin for $25\frac{2}{3}$ cents less than it cost her: for how much did she sell it?

126. The price paid for a book is $\$5.32\frac{1}{3}$, the gain was $\$1.12\frac{1}{2}$: what was the selling-price?

127. *The price paid* for a book is $\$3.24\frac{4}{7}$; the loss *was $87\frac{1}{2}$ cents:* what was the selling-price?

128. The selling-price of a house was $\$3474\frac{2}{3}$; the cost was $\$5234\frac{5}{8}$: what was the gain, or loss?

129. The selling-price of a horse was $\$47\frac{5}{7}$; the gain was $\$25\frac{3}{5}$: what was the cost?

130. The selling-price of a mowing-machine was $\$135\frac{4}{7}$; the loss was $\$25\frac{3}{8}$: what was the cost?

131. The selling-price of a barrel of flour was $\$5.38\frac{1}{3}$; the cost was $\$5.08\frac{3}{4}$: what was the gain, or loss?

MULTIPLICATION OF FRACTIONS.

95. TO MULTIPLY A FRACTION OR MIXED NUMBER BY AN INTEGER.

PROBLEM.—If I should give $\frac{2}{6}$ of a cake of maple-sugar to each of 7 boys, how many sixths of the cake would all the boys have?

FORMULA. $\frac{2}{6} \times 7 = \frac{14}{6} = 2\frac{1}{3}$.

ANALYSIS.—If 1 boy receives 2 sixths of a cake, 7 boys will receive 7 times 2 sixths, or 14 sixths: 14 sixths equals $2\frac{2}{6}$, or $2\frac{1}{3}$ cakes.

PRINCIPLE.—*Multiplying the numerator of a fraction multiplies the value of the fraction.*

The number of parts is increased, while their size remains the same.

ORAL EXERCISES.

1. If there are 3 halves of an apple in one pile, how *many* halves in 4 such piles? How many whole *apples?*

2. How many dollars in 5 times $\frac{3}{4}$ of a dollar? In 4 times $\frac{1}{2}$ of a dollar? In 5 times $\frac{2}{3}$ of a dollar?

3. At \$$\frac{2}{3}$ a pound, what will 4 pounds of coffee cost?

ANALYSIS. — Since 1 pound cost \$$\frac{2}{3}$, 4 pounds will cost 4 times \$$\frac{2}{3}$, or \$$\frac{8}{3}$, equal to \$$2\frac{2}{3}$.

4. At \$$\frac{3}{4}$ a bushel, what will 3 bushels of apples cost? 5 bushels? 6 bushels? 3 bushels? 10 bushels?

5. How many times 1 are 4 times $\frac{5}{7}$? 6 times $\frac{3}{4}$? 9 times $\frac{5}{9}$? 7 times $\frac{3}{4}$?

6. What will 5 lemons cost at $4\frac{1}{2}$ cents each?

ANALYSIS. — Since 1 lemon costs $4\frac{1}{2}$ cents, 5 lemons will cost 5 times $4\frac{1}{2}$ cents. 5 times $\frac{1}{2}$ cent $= \frac{5}{2}$, or $2\frac{1}{2}$ cents; 5 times 4 cents $= 20$ cents; 20 cents $+ 2\frac{1}{2}$ cents $= 22\frac{1}{2}$ cents.

7. What will 8 pounds of sugar cost at $12\frac{1}{2}$ cents per pound? At $8\frac{2}{3}$ cents? At $9\frac{3}{5}$ cents? At $10\frac{1}{2}$ cents?

ORAL, SLATE, AND BLACKBOARD EXERCISES.

MODEL.	(1.)	(2.)	(3.)	(4.)
$\frac{3}{4} \times 2 = \frac{6}{4}$* $= 1\frac{1}{2}$	$\frac{2}{3} \times 4$	$\frac{4}{7} \times 6$	$1\frac{2}{3} \times 3$	$4\frac{1}{4} \times 3$
$\frac{5}{8} \times 4 = \frac{20}{8} = 2\frac{1}{2}$	$\frac{5}{8} \times 7$	$\frac{8}{11} \times 4$	$4\frac{1}{2} \times 7$	$7\frac{4}{7} \times 4$
$\frac{3}{4} \times 5 = \frac{15}{4} = 3\frac{3}{4}$	$\frac{5}{9} \times 3$	$\frac{7}{9} \times 3$	$3\frac{1}{4} \times 6$	$2\frac{1}{4} \times 6$
$3\frac{2}{3} \times 3 = 9\frac{6}{3} = 11$	$\frac{7}{5} \times 5$	$\frac{4}{11} \times 9$	$7\frac{1}{3} \times 8$	$5\frac{3}{4} \times 7$
$4\frac{3}{7} \times 2 = 8\frac{6}{7} = 8\frac{6}{7}$	$\frac{3}{8} \times 9$	$\frac{7}{8} \times 3$	$5\frac{1}{2} \times 3$	$5\frac{5}{7} \times 3$

ORAL AND WRITTEN EXERCISES.

PROBLEM. — Multiply \$$251\frac{2}{5}$ by 3. By 5. By 7.

PROCESS.

$$\begin{array}{r} \$251\frac{2}{5} \\ 3 \\ \hline \$754\frac{1}{5} \end{array}$$

ANALYSIS. — (1.) For convenience multiply the integer and fraction separately.

(2.) 3 times \$$\frac{2}{5} = \$\frac{6}{5} = \$1\frac{1}{5}$.

(3.) 3 times \$251 = \$753. \$753 + \$$1\frac{1}{5}$ equals \$$754\frac{1}{5}$.

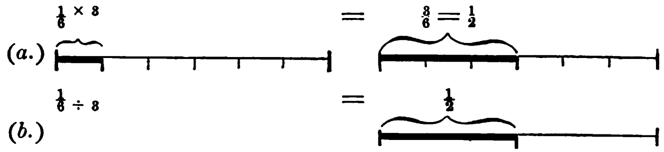
---

* *This intermediate step* should be omitted as soon as the pupil is able to per*form the work readily* in the mind.

1. *W.* Multiply $341\frac{2}{3}$ by 6. By 8. By 9. By 11.
2. *W.* Multiply $302\frac{1}{4}$ by 5. By 11. By 4. By 8.
3. *W.* Multiply $404\frac{3}{5}$ by 4. By 12. By 8. By 10.

Problem. — What part of a line is 3 times $\frac{1}{6}$ of a line?

ILLUSTRATION.



Analysis. — 3 times $\frac{1}{6}$ of a line equals $\frac{3}{6}$, or $\frac{1}{2}$, of a line.

Principle. — *Dividing the denominator of a fraction multiplies the value of the fraction.*

The number of parts remains the same, while their size is increased.

Rule. — *To multiply a fraction by an integer, multiply the numerator, or divide the denominator, by the integer.*

Note. — When the multiplicand is a mixed number, multiply the integer and fractions separately, and add the products.

4. *W.* What is the product of $453\frac{2}{3}$ multiplied by 3? By 5? By 8? By 13? By 11? By 29?

5. If 1 pound of tea costs $\$\frac{7}{8}$, what will 7 pounds cost? 4 pounds? 9 pounds? *W.* 341 pounds? 246 pounds?

6. If coffee is worth $37\frac{1}{2}$ cents per pound, what are 2 pounds worth? *W.* 7 pounds? 25 pounds? 87 pounds?

Review. — Recite the table of United States money. (65.) Give the table of *aliquot parts.* (66.) Of what are coins made? (67.) What is paper money? (68.) What is an invoice, or bill? (69.) Who is the debtor? Who the creditor? What is an account?

7. If a barrel of flour is worth \$9$\frac{3}{7}$, how much are 2 barrels worth? 3 barrels? *W.* 11 barrels? 223 barrels?

8. 2$\frac{2}{3}$ loaves of bread will last 1 man 2 days: at the same rate how much will 3 men require? 5 men? *W.* 68 men? 125 men?

96. TO MULTIPLY AN INTEGER BY A FRACTION, OR TO TAKE THE FRACTIONAL PART OF A NUMBER.

ORAL EXERCISES.

1. What is $\frac{1}{2}$ of an apple? ANS. — *One of the two equal parts into which an apple is divided.*

2. What is $\frac{1}{4}$ of an orange? $\frac{1}{7}$ of a pound? $\frac{1}{5}$ of an inch?

3. What is $\frac{2}{3}$ of a lemon? ANS. — *Two of the three equal parts into which a lemon is divided.* What is $\frac{3}{4}$ of a gallon? $\frac{5}{7}$ of an ounce?

4. Multiply \$2 by $\frac{3}{4}$, or, what is $\frac{3}{4}$ of \$2?

NOTE. — The word "of" is used instead of "times" when the multiplier is a fraction. We say "$\frac{3}{4}$ of 2," instead of "$\frac{3}{4}$ times 2."

FORMULA. $\$2 \times \frac{3}{4} = \$2 \times 3 \div 4$.

ANALYSIS. — (1.) $\frac{3}{4}$ of 1 dollar $= \frac{1}{4}$ of 3 times 1 dollar; or, $\$1 \times 3 \div 4$.

(2.) $\frac{3}{4}$ of 2 dollars $= \frac{1}{4}$ of 3 times 2 dollars; or, $\$2 \times 3 \div 4 = \$1\frac{1}{2}$. Hence the

RULE. — *To multiply by a fraction, multiply by the numerator, and divide by the denominator of the fraction.*

NOTE. — Whenever it is possible, the equal factors of the dividend and divisor should be canceled.

5. Multiply \$7 by $\frac{5}{6}$, by $\frac{2}{3}$, by $\frac{5}{7}$.

6. Multiply 8 by $\frac{4}{5}$, by $\frac{2}{3}$, by $\frac{7}{9}$.

7. Multiply 9 by $\frac{5}{6}$, by $\frac{2}{3}$, by $\frac{6}{7}$.

REVIEW. — Define a composite number. (70.) What is a prime number. (71 What is a factor, or divisor? (72.) What is a greatest common divisor? (74 *What is a common multiple?* (75.)

5. Multiply 8 by $\frac{3}{4}$. 7 by $\frac{5}{14}$. 9 by $\frac{7}{18}$. 12 by $\frac{5}{8}$.

By Cancellation. FORMULA. $8\times\frac{3}{4}=8\times3\div4=\overset{2}{\cancel{8}}\times3\div\cancel{4}=6$.

6. What is $\frac{6}{7}$ of 14? $\frac{5}{8}$ of 16? $\frac{2}{3}$ of 21? $\frac{5}{11}$ of 22?

ORAL, SLATE, AND BLACKBOARD EXERCISES.

MODEL.	(1.)	(2.)	(3.)	(4.)	(5.)
$4\times\frac{3}{4}=3$	$6\times\frac{5}{6}$	$5\times\frac{4}{7}$	$9\times\frac{2}{7}$	$4\times\frac{3}{4}$	$12\times\frac{4}{5}$
$3\times\frac{2}{5}=1\frac{1}{5}$	$5\times\frac{4}{7}$	$11\times\frac{3}{7}$	$1\times\frac{3}{4}$	$1\times\frac{2}{7}$	$14\times\frac{2}{7}$
$4\times\frac{4}{5}=3\frac{1}{5}$	$4\times\frac{8}{9}$	$2\times\frac{5}{8}$	$3\times\frac{6}{7}$	$3\times\frac{5}{6}$	$16\times\frac{1}{4}$
$5\times\frac{2}{7}=1\frac{3}{7}$	$3\times\frac{3}{7}$	$8\times\frac{3}{6}$	$4\times\frac{5}{9}$	$7\times\frac{4}{7}$	$15\times\frac{3}{5}$
$6\times\frac{3}{7}=2\frac{4}{7}$	$8\times\frac{6}{7}$	$9\times\frac{5}{9}$	$7\times\frac{2}{7}$	$1\times\frac{5}{9}$	$21\times\frac{3}{7}$

ORAL AND WRITTEN EXERCISES.

9. If 1 bushel of potatoes is worth $1.25, what are 2 bushels worth? *W.* $\frac{1}{2}$ of a bushel? $\frac{5}{8}$ of a bushel? $2\frac{5}{6}$ bushels?

10. If a bushel of wheat costs $2.10, what must be given for 2 bushels? *W.* For $\frac{2}{5}$ of a bushel? For $5\frac{3}{7}$ bushels?

11. *W.* An acre of land is valued at $314.27: what are 2 acres of such land worth? $\frac{3}{4}$ of an acre? $5\frac{7}{8}$ acres?

12. *W.* If I pay $4.37 for a cord of wood, what must I pay for 3 cords of such wood? For $4\frac{3}{5}$ cords? For $7\frac{5}{9}$ cords? For $\frac{6}{11}$ of a cord?

13. What will $\frac{5}{9}$ of a pound of indigo cost, at $4 per pound? *W.* At $3.40? At $8.96?

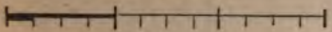
14. *W.* What will $1\frac{2}{3}$ bushels clover-seed cost, at 8 *dollars* per bushel? At $9.70? At $11.23?

REVIEW. — When is a fraction said to be proper? (82.) Illustrate by an object. Describe an improper fraction. (83.) Illustrate by an object. What is a mixed number? (84.)

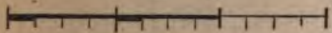
97. TO MULTIPLY A FRACTION BY A FRACTION OR MIXED NUMBER.

PROBLEM. — Multiply $\frac{2}{3}$ by $\frac{3}{4}$; or, what is $\frac{3}{4}$ of $\frac{2}{3}$?

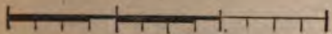
ILLUSTRATIONS.



$\frac{1}{4}$ of $\frac{1}{3}=\frac{1}{12}$.



$\frac{1}{4}$ of $\frac{2}{3}=\frac{2}{12}$.



$\frac{3}{4}$ of $\frac{2}{3}=\frac{6}{12}=\frac{1}{2}$.

ANALYSIS. — 1. If $\frac{1}{3}$ is divided into 4 equal parts, each part is $\frac{1}{4}$ of $\frac{1}{3}$: as there are 3 thirds in a unit, each of which may be divided into 4 such parts, the unit contains 12 such parts, and each part is $\frac{1}{12}$.

2. Since $\frac{1}{4}$ of $\frac{1}{3}$ is $\frac{1}{12}$, $\frac{1}{4}$ of $\frac{2}{3}=\frac{2}{12}$.

3. Since $\frac{1}{4}$ of $\frac{2}{3}$ is $\frac{2}{12}$, $\frac{3}{4}$ of $\frac{2}{3}=\frac{6}{12}$.

RULE. — *Multiply the numerators together for a new numerator, and the denominators for a new denominator.*

NOTE. — Mixed numbers may be changed to improper fractions when more convenient, or when the operation can be shortened by cancellation. Fractions with the word "of" between them are sometimes called "*compound fractions.*" The word "of" is equivalent to the word "times," or the sign of multiplication; thus, $\frac{3}{4}$ of $\frac{2}{3}=\frac{3}{4}$ times $\frac{2}{3}$; or, $\frac{3}{4}\times\frac{2}{3}$.

ORAL EXERCISES.

1. Multiply $\frac{3}{8}$ by $\frac{7}{12}$; or, what is $\frac{7}{12}$ of $\frac{3}{8}$?

FORMULA. $\frac{\cancel{3}}{8}\times\frac{7}{\underset{4}{\cancel{12}}}=\frac{7}{32}$.

2. What is $\frac{5}{7}$ of $\frac{14}{15}$? $\frac{5}{8}$ of $\frac{10}{11}$? $\frac{8}{9}$ of $\frac{18}{19}$? $\frac{6}{7}$ of $\frac{14}{15}$?

3. Multiply $5\frac{1}{2}$ by $\frac{4}{5}$; or, what is $\frac{4}{5}$ of $5\frac{1}{2}$?

FORMULA. $5\frac{1}{2}=\frac{11}{\cancel{2}}\times\frac{\overset{2}{\cancel{4}}}{5}=\frac{22}{5}=4\frac{2}{5}$.

4. If $\frac{1}{2}$ of a pie be cut into 2 equal pieces, what part of the whole pie is 1 piece? If cut into 3 equal pieces?

5. A girl, having $\frac{1}{2}$ of an orange, gave $\frac{1}{3}$ of it to her brother: what part of the orange did she give him.

ANALYSIS. — $\frac{1}{2}$ of an orange is equal to $\frac{3}{6}$: $\frac{1}{3}$ of $\frac{3}{6}$ of an orange is $\frac{1}{6}$. *Hence she gave him* $\frac{1}{6}$ *of* an orange.

ORAL, SLATE, AND BLACKBOARD EXERCISES.

MODEL.*	(1.)	(2.)	(3.)
$\frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$	$\frac{2}{3} \times \frac{4}{5}$	$\frac{6}{7} \times \frac{5}{8}$	$\frac{8}{9} \times$ [illegible]
$\frac{4}{7} \times \frac{3}{8} = \frac{3}{14}$	$\frac{6}{7} \times \frac{3}{8}$	$\frac{3}{9} \times \frac{6}{7}$	$\frac{7}{11} \times$ [illegible]
$\frac{8}{9} \times \frac{5}{7} = \frac{40}{63}$	$\frac{5}{9} \times \frac{7}{10}$	$\frac{5}{8} \times \frac{9}{10}$	$\frac{9}{10} \times$ [illegible]
$2\frac{1}{3} \times \frac{4}{7} = \frac{7}{3} \times \frac{4}{7} = 1\frac{1}{3}$	$\frac{4}{11} \times \frac{3}{8}$	$\frac{3}{4} \times \frac{5}{8}$	$\frac{8}{9} \times$ [illegible]
$2\frac{4}{5} \times 1\frac{2}{8} = \frac{14}{5} \times \frac{10}{8} = 3\frac{1}{2}$	$\frac{5}{13} \times \frac{9}{10}$	$\frac{6}{7} \times \frac{11}{12}$	$\frac{5}{8} \times$ [illegible]

DRILL CARD EXERCISES.—TABLE IV.

MULTIPLICATION OF FRACTIONS.

Exercise I. — Multiply **V** by 5. By 6. By 7.
II. — Multiply **W** by 8. By 9. By 10.
III. — Multiply **X** by 11. By 12. By 13.
IV. — Prefix **1** to **V**, and multiply by 2. By 3. By 4.
V. — Prefix **2** to **W**, and multiply by 4. By 5. By 6.
VI. — Prefix **3** to **X**, and multiply by 7. By 8. By 9.
VII. — What is **V** of **1**? **W** of **2**? **X** of **3**?
VIII. — What is **V** of **W**? **V** of **X**? **W** of **X**?
IX. — Prefix **1** to **V**, and multiply by $\frac{11}{8}$. By $\frac{9}{2}$. By $\frac{11}{1}$.
X. — Prefix **2** to **W**, and multiply by $\frac{11}{13}$. By $\frac{10}{11}$. By [illegible].
XI. — Prefix **3** to **X**, and multiply by $2\frac{3}{7}$. By $8\frac{2}{9}$. By $4\frac{3}{4}$.
XII. **1 — V + W + 2** × **X** = what?

ORAL AND WRITTEN EXERCISES.

15. If a pound of rice is worth \$$\frac{1}{9}$, what is $\frac{5}{7}$ of a pound worth? *W.* $6\frac{1}{2}$ pounds? $9\frac{2}{3}$ pounds?

16. *W.* How much grain can be put into $7\frac{1}{2}$ bags, if each bag holds $2\frac{1}{8}$ bushels? $2\frac{2}{7}$ bushels?

17. *W.* What will $4\frac{1}{3}$ pounds of sugar cost, at $8\frac{3}{4}$ cents per pound? At $9\frac{4}{5}$ cents? At $11\frac{5}{8}$ cents?

18. What will 114 pounds of tea cost, at \$1 per pound? At \$$\frac{1}{2}$ per pound? *W.* At \$$\frac{2}{5}$? At \$$\frac{9}{10}$? At \$$1\frac{3}{10}$?

19. What will 228 bushels of corn cost, at 1 dollar a *bushel?* At \$$1\frac{1}{2}$ per bushel? At \$$1\frac{1}{8}$?

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* *Change mixed numbers to improper fractions, and cancel when possible.*

WRITTEN EXERCISES.

0. What will $\frac{3}{4}$ of a pound of sugar cost at 19$\frac{1}{3}$ ts per pound?

)CESS.
$\frac{1}{3}$ cents.

BY CANCELLATION.

$$\frac{\overset{29}{\cancel{58}}}{\cancel{3}} \times \frac{\cancel{3}}{\underset{2}{\cancel{4}}} = 14\tfrac{1}{2}.$$

$\frac{1}{2}$ cents.

FORMULA. 19$\frac{1}{3}$ cents × $\frac{3}{4}$ = 14$\frac{1}{2}$ cents.

ANALYSIS.—If 1 pound costs 19$\frac{1}{3}$ cents, $\frac{3}{4}$ of a pound will cost $\frac{3}{4}$ of 19$\frac{1}{3}$ cents (or $\frac{1}{4}$ of 3 times 19$\frac{1}{3}$ cents),* which is 14$\frac{1}{2}$ cents.

1. What will $\frac{3}{4}$ of a bushel of corn weigh, if 1 hel weighs 56 pounds?

2. If a box of starch weighs 28$\frac{2}{3}$ pounds, how many ınds in 3 boxes? In $\frac{2}{3}$ of a box? In 13$\frac{2}{3}$ boxes?

3. How many yards in $\frac{3}{5}$ of 176$\frac{1}{3}$ yards? In $\frac{4}{7}$ of $\frac{5}{9}$ yards? In $\frac{5}{9}$ of 113$\frac{4}{5}$ yards?

4. If 1 yard of lace is worth 88$\frac{1}{4}$ cents, what are yards worth?

CESS.
88$\frac{1}{4}$
16$\frac{2}{3}$
76$\frac{1}{2}$
58$\frac{5}{6}$ val. $\frac{2}{3}$ yd.
32 } val. 16 yds.
8 }
70$\frac{5}{6}$ val. 16$\frac{2}{3}$ yds.

FORMULA. \$.88$\frac{1}{4}$ × 16$\frac{2}{3}$ = \$14.70$\frac{5}{6}$, value 16$\frac{2}{3}$ yards.

ANAL. STEPS.—1. *Find value $\frac{2}{3}$ yard.*
2. *Find value 16 yards.*

ANALYSIS.—(1.) $\frac{2}{3}$ of \$88$\frac{1}{4}$ = \$58$\frac{5}{6}$.
(2.) 16 times \$.88$\frac{1}{4}$ = \$14.12; the sum of \$.58$\frac{5}{6}$ and \$14.12 = \$14.70$\frac{5}{6}$.

5. If in 1 pane of glass there are 20$\frac{1}{4}$ square inches, v many square inches in 4$\frac{3}{4}$ such panes?

6. Find the weight of 3$\frac{1}{7}$ boxes of raisins, if 1 box ghs 67$\frac{1}{3}$ pounds?

7. If there are 90$\frac{1}{4}$ quarts of milk in a can, how ıy quarts in $\frac{2}{9}$ of the can?

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* *Either expression* may be used at the option of the teacher.

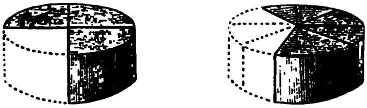
DIVISION OF FRACTIONS.

98. TO DIVIDE A FRACTION OR MIXED NUMBER B AN INTEGER.

PROBLEM. — (*a.*) I wish to divide $\frac{3}{4}$ of a cake equall among 3 boys: what should each boy receive?

(*b.*) I wish to divide $\frac{2}{3}$ of a cake in the same way what should each boy receive?

ILLUSTRATION.



$\frac{3}{4} \div 3 = \frac{1}{4}$. $\frac{2}{3} \div 3 = \frac{2}{9}$.

ANALYSIS. — (*a.*) If 3 bo are to receive $\frac{3}{4}$ of a cak each boy should receive on third of $\frac{3}{4}$, or $\frac{1}{4}$ of a cake.

(*b.*) Since 2 thirds cann be divided into 3 equal pa without a remainder, chan the $\frac{2}{3}$ to $\frac{6}{9}$ by cutting ea third into 3 equal parts.

If 3 boys are to receive $\frac{6}{9}$ of a cake, each boy should receive one-th of $\frac{6}{9}$, or $\frac{2}{9}$ of the cake.

PRINCIPLE. — (*a.*) I. *Dividing the numerator of fraction divides the value of the fraction.*

The *number* of parts are decreased, while their *size* remains tl same.

(*b.*) II. *Multiplying the denominator of a fractio divides the value of the fraction.*

The *number* of parts remain the same, while the *size* is decreased.

NOTE TO THE TEACHER. — The teacher, in explaining the abov principles, should refer to the corresponding illustrations, remembering that children can readily learn to *recite* explanations, and yet b utterly ignorant of their meaning.

ORAL EXERCISES.

1. What is 1 third of 3 apples? Of 6 apples? O *12 lemons*? Of 3 fifths of a pound? Of 6 eighths o *an ounce*? Of 12 sevenths of a gallon?

2. Divide 6 apples and 3 fifths of an apple into 3 equal parts. What is 1 third of $6\frac{3}{5}$ apples? What is 1 fifth of $10\frac{15}{16}$ oranges? Of $15\frac{5}{7}$ melons?

3. Divide 1 orange into 3 equal parts. Divide $\frac{1}{2}$ of an orange into 3 equal parts. In the same way divide $\frac{1}{3}$ of an orange. Divide $\frac{2}{3}$ of an orange. Divide $\frac{5}{8}$ of of an orange.

PROBLEM. — Divide $347\frac{2}{3}$ by 5.

PROCESS.

$$\begin{array}{r} 5)\underline{347\frac{2}{3}} \\ 69\frac{8}{15} \end{array}$$

ANALYSIS. — (1.) One-fifth of 347 is 69, with a remainder of 2 units.

(2.) 2 units are equal to $\frac{6}{3}$, which, added to $\frac{2}{3}$, equal $\frac{8}{3}$. One-fifth of $\frac{8}{3}$ is $\frac{8}{15}$, which annexed to $69 = 69\frac{8}{15}$.

RULE. — *To divide a fraction by an integer, divide the numerator, or multiply the denominator, by the integer.*

NOTE. — When the dividend is a mixed number, and less than the divisor, change it to an improper fraction, otherwise divide the integral number, and add the remainder, if any, to the fraction; then divide the fraction.

ORAL, SLATE, AND BLACKBOARD EXERCISES.

MODEL.	(1.)	(2.)	(3.)	(4.)
$\frac{2}{3} \div 4 = \frac{1}{6}$	$\frac{3}{7} \div 5$	$\frac{3}{5} \div 7$	$\frac{5}{9} \div 10$	$1\frac{1}{2} \div 3$
$\frac{5}{8} \div 2 = \frac{5}{16}$	$\frac{8}{9} \div 4$	$\frac{9}{11} \div 5$	$\frac{10}{11} \div 5$	$2\frac{1}{4} \div 4$
$\frac{6}{7} \div 5 = \frac{6}{35}$	$\frac{5}{11} \div 7$	$\frac{6}{7} \div 4$	$\frac{8}{9} \div 16$	$5\frac{3}{4} \div 6$
$\frac{8}{9} \div 2 = \frac{4}{9}$	$\frac{8}{11} \div 8$	$\frac{3}{8} \div 3$	$\frac{5}{12} \div 10$	$7\frac{1}{2} \div 5$

ORAL AND WRITTEN EXERCISES.

28. *W.* Divide $572\frac{4}{7}$ by 3. By 8. By 12. By 14.

29. *W.* Divide $210\frac{7}{9}$ by 3. By 5. By 10. By 21.

30. *W.* Divide $607\frac{5}{8}$ by 2. By 7. By 41. By 20.

31. If I gave $\frac{3}{7}$ of a dime for 11 boxes of matches, what did 1 box cost? *W.* If I gave $3\frac{5}{8}$ dimes?

ANALYSIS. — If 11 boxes cost $\frac{3}{7}$ of a dime, 1 box costs one-eleventh of $\frac{3}{7}$ of a dime, or, $\frac{3}{77}$ of a dime.

32. What will be the cost of 1 box of pencils, if 8 *boxes cost* \$$\frac{7}{8}$? Cost \$$\frac{5}{9}$? *W.* Cost \$$2\frac{3}{5}$? \$$1\frac{13}{15}$?

33. What will a lead pencil cost, if 9 cost $\$\frac{4}{5}$? $\$\frac{6}{7}$? *W.* $\$\frac{1}{11}$? $18\frac{7}{11}$ cents? $38\frac{4}{5}$ cents?

34. If 2 oranges cost $10\frac{1}{2}$ cents, what will 4 cost?

Formula. $10\frac{1}{2}$ cents $\div \cancel{2} \times \overset{2}{\cancel{4}} = 21$ cents.

Anal. Steps. — 1. *Find the cost of* 1 *orange.*
2. *Find the cost of* 4 *oranges.*

35. If 5 lemons cost $10\frac{2}{3}$ cents, what will be the cost of 3? Of 7? Of 5? *W.* Of 19? Of 211?

36. If 3 pounds of butter cost $37\frac{1}{2}$ cents, what will 2 pounds cost? *W.* 30 pounds? 119 pounds?

37. If 5 pounds of rice cost $31\frac{1}{2}$ cents, what will 2 pounds cost? 5 lbs.? *W.* 23 lbs.? 69 lbs.?

38. If 4 apples cost $\frac{5}{6}$ of a shilling, what will 6 apples cost? *W.* 47 apples? 129 apples?

39. If 3 pounds of sugar cost $21\frac{1}{3}$ cents, what will 2 pounds cost? *W.* What will $5\frac{2}{3}$ pounds cost? $11\frac{5}{7}$ pounds? $120\frac{4}{5}$ pounds?

40. If 9 pounds of ginger are worth $50\frac{1}{2}$ cents, what is the value of 5 pounds? *W.* $11\frac{2}{3}$ pounds? $37\frac{5}{7}$ pounds?

41. *W.* $112\frac{3}{4}$ yards of silk will make 9 dresses: how many yards are required for one of the dresses?

42. *W.* 9 pounds of sugar cost $\$1.37\frac{1}{2}$: what is the cost of 1 pound? 5 pounds? $6\frac{2}{3}$ pounds?

43. *W.* If 7 fish weigh $57\frac{1}{2}$ pounds, how much will 9 of equal size weigh?

44. *W.* What will $3\frac{3}{4}$ yards of lace cost, if 12 yards cost $\$13.82\frac{1}{8}$? If 7 yards cost $\$8.13\frac{1}{4}$?

Review. — What is a least common multiple? (76.) What is meant by cancellation? (77.) Upon what principle is cancellation based? What is a fraction? (79.) Illustrate by use of an object. What does the denominator of a fraction show? (80.) Which term of a fraction is the denominator? What is the use of *the denominator of a fraction?* Illustrate the meaning of $\frac{3}{4}$ by an object. $\frac{3}{8}$.

99. TO DIVIDE AN INTEGER OR MIXED NUMBER BY A FRACTION.

PROBLEM. — (*a.*) 48 are how many times $\frac{5}{7}$? (*b.*) $40\frac{5}{6}$ are how many times $\frac{8}{9}$?

PROCESSES.

(*a.*)	(*b.*)
48	$40\frac{5}{6}$
7	9
5)336	8)$360\frac{45}{6}$
$67\frac{1}{5}$	$45(\frac{45}{48})\frac{15}{16}$

ANALYSIS. — (*a.*) Since 1 unit $= \frac{7}{7}$, 48 units $= 48$ times $\frac{7}{7}$, or $\frac{336}{7}$: 5 sevenths are contained in 336 sevenths, $67\frac{1}{5}$ times: hence $48 \div \frac{5}{7} = 67\frac{1}{5}$.

(*b.*) Since 1 unit $= \frac{9}{9}$, $40\frac{5}{6}$ units $= 40\frac{5}{6}$ times $\frac{9}{9}$, or $360\frac{45}{6}$ *ninths;* and 8 *ninths* are contained in $360\frac{45}{6}$ *ninths* $45\frac{15}{16}$ times: hence $40\frac{5}{6} \div \frac{8}{9} = 45\frac{15}{16}$.

NOTE FOR THE TEACHER. — It will be observed that the above processes are but abbreviated methods of reducing the dividends and divisors to equivalent fractions having a common denominator, after which the division is performed.

RULE. — *To divide by a fraction, divide by the numerator, and multiply the quotient by the denominator, of the fraction.*

It is usually more convenient to multiply before dividing. When the divisor is a mixed number, change to an improper fraction. Do not change the dividend to an improper fraction, unless it is smaller than the divisor.

WRITTEN EXERCISES.

45. Divide 49 by $\frac{3}{7}$. By $\frac{6}{11}$. By $\frac{4}{11}$. By $\frac{9}{13}$.
46. Divide 186 by $1\frac{3}{4}$. By $2\frac{7}{11}$. By $8\frac{3}{4}$. By $9\frac{3}{5}$.
47. Divide $67\frac{3}{4}$ by $2\frac{3}{4}$. By $4\frac{5}{12}$. By $9\frac{6}{7}$. By $3\frac{4}{7}$.
48. Divide $483\frac{2}{13}$ by $3\frac{5}{7}$. By $9\frac{4}{7}$. By $11\frac{6}{7}$. By $2\frac{9}{13}$.

100. TO DIVIDE A FRACTION BY A FRACTION OR MIXED NUMBER.

ORAL EXERCISES.

1. How many halves in 1? One is how many times a half? How many quarters in 1? One is how many times a fourth? A fifth? A ninth? A seventh?

2. How many fourths in 2? 2 are how many times $\frac{1}{4}$? 3 are how many times $\frac{1}{5}$? $\frac{2}{3}$? $\frac{3}{6}$? $\frac{1}{7}$?

3. One is how many times $\frac{3}{4}$? $\frac{5}{6}$? $\frac{2}{7}$? $\frac{3}{5}$?

FORMULA. $1=\frac{4}{4}\div\frac{3}{4}=1\frac{1}{3}$.

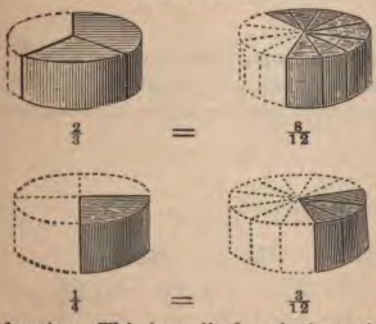
ANALYSIS.—One equals 4 fourths; and 3 fourths are contained in 4 fourths $1\frac{1}{3}$ times.

4. How many quarters in a half? How many times can a quarter be taken from a half?

5. How many sixths in an apple? How many sixths in a third of an apple? In $\frac{1}{5}$? In $\frac{3}{5}$?

PROBLEM.—Divide $\frac{2}{3}$ by $\frac{1}{4}$; or, $\frac{2}{3}$ are how many times $\frac{1}{4}$?

ILLUSTRATION.


$\frac{2}{3}$ = $\frac{8}{12}$
$\frac{1}{4}$ = $\frac{3}{12}$

ANALYSIS. $\frac{2}{3}=\frac{8}{12}$; $\frac{1}{4}=\frac{3}{12}$. 3 *twelfths* are contained in 8 *twelfths* $2\frac{2}{3}$ times: hence $\frac{2}{3}\div\frac{1}{4}=2\frac{2}{3}$.

FORMULA. $\frac{2}{3}\div\frac{1}{4}=\frac{8}{12}\div\frac{3}{12}=2\frac{2}{3}$.

NOTE FOR THE TEACHER. $\frac{2}{3}$ and $\frac{1}{4}$ must be changed to the same denomination before dividing. In practice, the numerator of each fraction must be multiplied by the denominator of the other fraction. This is really done by inverting the divisor, and multiplying.

RULE.—Change the integral or mixed numbers to improper fractions, invert the divisor, and multiply or cancel the terms as in the multiplication of fractions.

ORAL, SLATE, AND BLACKBOARD EXERCISES.

MODEL.	(1.)	(2.)	(3.)	(4.)
$\frac{1}{1} \div \frac{3}{4} = \frac{4}{3} = 1\frac{1}{3}$	$\frac{5}{6} \div \frac{3}{7}$	$1 \div \frac{3}{4}$	$6 \div \frac{3}{8}$	[illegible]
$1 \div \frac{3}{4} = \frac{4}{3} = 1\frac{1}{3}$	$\frac{5}{9} \div \frac{4}{11}$	$1 \div \frac{3}{11}$	$3 \div \frac{4}{7}$	[illegible]
$\frac{6}{1} \div \frac{5}{6} = \frac{36}{5} = 7\frac{1}{5}$	$\frac{2}{7} \div \frac{3}{4}$	$1 \div$ [illegible]	$5 \div \frac{5}{6}$	[illegible]
$6 \div \frac{5}{6} = \frac{36}{5} = 7\frac{1}{5}$	$\frac{8}{9} \div \frac{6}{7}$	$1 \div \frac{4}{7}$	$9 \div \frac{4}{7}$	[illegible]
$\frac{5}{7} \div \frac{2}{3} = \frac{15}{14} = 1\frac{1}{14}$	$\frac{4}{5} \div \frac{6}{7}$	$1 \div \frac{5}{13}$	$11 \div \frac{2}{3}$	[illegible]
$\frac{4}{3} \div \frac{3}{4} = \frac{16}{9} = 1\frac{7}{9}$	$\frac{3}{4} \div \frac{8}{9}$	$1 \div \frac{7}{11}$	$4 \div \frac{4}{9}$	[illegible]

ORAL AND WRITTEN EXERCISES.

49. Among how many persons can $\$\frac{3}{5}$ be distributed, if each receives $\frac{1}{5}$ of a dollar? $\$\frac{1}{8}$? $\$\frac{1}{20}$? $\$\frac{1}{50}$?

50. Among how many persons can $\$7\frac{2}{3}$ be distributed, if each receives $\$\frac{1}{2}$? $\$\frac{1}{3}$? $\$\frac{1}{4}$? $\$\frac{1}{10}$? *W.* $\$\frac{3}{5}$? $\$\frac{5}{9}$? $\$\frac{7}{11}$?

51. Among how many persons can $\$\frac{39}{40}$ be distributed, if each receives $\$\frac{1}{2}$? $\$\frac{1}{3}$? $\$\frac{2}{3}$? $\$\frac{3}{4}$? *W.* $\$\frac{5}{11}$? $\$\frac{7}{12}$? $\$\frac{13}{17}$?

52. How many times is $\frac{2}{3}$ of a dollar contained in \$1? In \$2? In \$5? *W.* In $\$7\frac{2}{3}$? In $\$13\frac{5}{7}$? In $\$123\frac{4}{11}$?

53. At $\frac{2}{3}$ of a cent apiece, how many oranges can be bought for 2 cents? For 5 cents? For $8\frac{2}{3}$ cents? *W.* For $437\frac{2}{3}$ cents? For \$6.82? For $\$4.53\frac{5}{8}$?

54. If there are 100 cents in a dollar, how many cents in $\$\frac{1}{2}$? $\$\frac{2}{3}$? $\$\frac{7}{10}$? $\$\frac{9}{10}$? *W.* $\$\frac{5}{11}$? $\$\frac{7}{9}$? $\$\frac{8}{17}$? $\$\frac{1}{13}$?

Review. — What is the use of the numerator of a fraction? (81.) Illustrate by the use of an object. What does the numerator show in $\frac{3}{4}$ of an apple? What is meant by a proper fraction? (82.) When is a fraction said to be improper? (83.) Which of the following fractions are proper, and which are improper, *two-thirds, three-thirds*, seven-fifths, five-sevenths?

PROCESS.

100 cents.
$\underline{\quad 5}$
$11\overline{)500}$
$\underline{\qquad}$
45+ cents.*

FORMULA. $\$1.00 \times \frac{5}{11} = 45+$ cents.

ANALYSIS. — Since there are 100 cents in $1, in $\frac{5}{11}$ of a dollar, there are $\frac{5}{11}$ of 100 cents (or $\frac{1}{11}$ of 5 times 100 cents), which is 45+ cents.

55. *W.* How many cents and mills in $\frac{8}{9}$ of a dollar? In $\$\frac{4}{5}$? In $\$\frac{6}{7}$? In $\$\frac{12}{13}$? In $\$\frac{6}{21}$?

FORMULA. 1000 mills $\times \frac{8}{9} = \$.888+$.

ANALYSIS. — Since there are 1000 mills in $1, in $\frac{8}{9}$ of a dollar there are $\frac{8}{9}$ of 1000 mills (or $\frac{1}{9}$ of 8 times 1000 mills), which is 888 mills = $.888.

NOTE. — For additional examples see drill cards.

DRILL CARD EXERCISES. — TABLE IV.

DIVISION OF FRACTIONS.

Exercise I. — Divide **V** by 5. By 6. By 7.
II. — Divide **W** by 8. By 9. By 10.
III. — Divide **X** by 11. By 12. By 13.
IV. — Prefix **1** to **V**, and divide by 1. By 2. By 3.
V. — Prefix **2** to **W**, and divide by 4. By 5. By 6.
VI. — Prefix **3** to **X**, and divide by 7. By 8. By 9.
VII. — 28 is how many times **V**? **W**? **X**?
VIII. — $200\frac{3}{5}$ is how many times **V**? **W**? **X**?
IX. — Prefix **1** to **V**, and divide by $\frac{11}{8}$. By $\frac{9}{2}$. By $\frac{11}{1}$.
X. — Prefix **2** to **W**, and divide by $\frac{11}{13}$. By $\frac{10}{11}$. By $\frac{21}{22}$.
XI. — Prefix **3** to **X**, and divide by $3\frac{5}{7}$. By $8\frac{3}{11}$. By $4\frac{5}{7}$.
XII. — 2 — **V** $\div$ 1 + **W** + 3 $\times$ **X** = what?

UNITED STATES CURRENCY.

XIII. — Change $**V** to cents and mills.
XIV. — Change $**W** to cents and mills.
XV. — Change $**X** to cents, mills, and tenths.

PRACTICAL PROBLEMS, ORAL AND WRITTEN.

NOTES. — (1.) For each of these problems, the pupil should prepare a clear and logical analysis.

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* *The sign* + affixed to answers denotes a remainder of too little value to be oticed in business transactions.

(2.) This analysis should be expressed in words, and also in a well-arranged arithmetical formula, which may be a visible plan of the solution.

(3.) The formula should be reduced by the simplest and shortest methods, cancelling when there is opportunity.

(4.) To secure *accuracy*, the teacher should require many analyses in writing, with their corresponding formulas: both should be carefully criticised with reference to simplicity of method, clearness of statement, and neatness of execution.

(5.) To secure *readiness*, the oral analyses of similar questions should be required, the formulas being written upon the blackboard.

56. *W.* 7 equal bins contain $421\frac{1}{5}$ bushels of salt: how much in each bin?

PROCESS.

$7)421\frac{1}{5}$

$60\frac{6}{35}$ bushels.

FORMULA. $421\frac{1}{5} \div 7 = 60\frac{6}{35}$ bushels, contents 1 bin.

ANALYSIS. — If 7 bins contain $421\frac{1}{5}$ bushels of salt, 1 bin contains 1 seventh of $421\frac{1}{5}$, or $60\frac{6}{35}$ bushels.

57. *W.* If 5 equal piles of coal contain $1941\frac{2}{3}$ tons, how many tons in each pile?

58. *W.* I filled $123\frac{4}{7}$ barrels of apples from 10 loads of apples: how many barrels in each load?

59. The aggregate weight of two men is $300\frac{2}{16}$ pounds: what is the average weight of each? What is the average weight of each, if the aggregate weight of 5 is $830\frac{2}{7}$ pounds? Of 12 is $1680\frac{2}{5}$ pounds?

60. If 87 cows cost $\$3870\frac{1}{5}$, what is their average cost?

61. What is the value of $\frac{5}{9}$ of a dollar?

PROCESS.

\$1.000

5

9)5.000

.555+

FORMULA. $\$1.000 \times \frac{5}{9} = \$555+$, value in cents and mills.

ANALYSIS. — Since in \$1 there are 1000 mills, in $\frac{5}{9}$ of a dollar there are $\frac{5}{9}$ of 1,000 mills (or $\frac{1}{9}$ of 5 times 1000 mills), which is 555 mills $= \$.555+$.

62. If 3 yards of calico cost 38$\frac{1}{2}$ cents, what does 1 yard cost? *W.* 5 yards? 6$\frac{3}{7}$ yards?

FORMULA. \$.385 ÷ 3 = cost 1 yard.
\$.385 ÷ 3 × 6$\frac{3}{7}$ = cost of 6$\frac{3}{7}$ yards.

63. A tract of 17 acres of land cost \$34: what was the price per acre? *W.* If it cost \$263$\frac{5}{7}$? If it cost \$215$\frac{4}{9}$?

WRITTEN EXAMPLES.

64. If 7 firkins of butter weigh 680$\frac{3}{4}$ pounds, what will $\frac{3}{4}$ of a firkin weigh? What will it be worth at 30$\frac{2}{3}$ cents per pound?

FORMULA. 680$\frac{3}{4}$ lbs. ÷ 7 = weight of 1 firkin.
680$\frac{3}{4}$ lbs. ÷ 7 × $\frac{3}{4}$ = weight of $\frac{3}{4}$ of a firkin.
\$.30$\frac{2}{3}$ × (680$\frac{3}{4}$ ÷ 7 × $\frac{3}{4}$) = cost of $\frac{3}{4}$ of a firkin, at 30$\frac{2}{3}$ cents.

65. Paid 25 laborers \$206$\frac{1}{4}$ for 6 days' or 1 week's work: what were the wages of each per day?

66. 16 car-loads of wood, of 3$\frac{5}{8}$ cords each, cost \$297$\frac{3}{4}$: what was the cost of 1 cord?

67. If 12 yards of silk are worth \$16.46$\frac{2}{3}$, what is the value of 1 yard? $\frac{1}{7}$ of a yard? $\frac{3}{11}$ of a yard? 2$\frac{5}{6}$ yards?

68. What cost 6$\frac{2}{3}$ barrels of flour, if 4$\frac{1}{3}$ barrels cost \$17.37$\frac{1}{2}$?

PROCESS.

13)\$17.375
1.336+
20
\$26.720+ = cost 6$\frac{2}{3}$ lbs.

ANAL. STEPS. — (1.) *Find cost $\frac{1}{3}$ barrel.*
(2.) *Find cost $\frac{20}{3}$ barrels.*

FORMULA. \$17.375 ÷ 13 × 20 = 26.72.

BY CANCELLATION.

$$\$17.375 \div \frac{13}{3} \times \frac{20}{3^*} = \$26.720+.$$

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* *SUGGESTION.* — When, in dividing one fraction by another, the denominators are *alike, they may be disregarded, since they would be cancelled in the process.*

69. If 17 baskets of strawberries cost $110\frac{1}{2}$ cents, what will $2\frac{4}{3}$ baskets cost?

70. What cost $1\frac{1}{2}$ bushels of wheat, if $46\frac{1}{3}$ bushels cost \$$82.30\frac{1}{4}$?

71. If $25\frac{2}{5}$ yards of flannel are worth \$$12.87\frac{2}{3}$, what is the value of $\frac{1}{2}$ a yard?

72. What is the cost of $\frac{1}{8}$ of a pound of tea, if $7\frac{2}{7}$ pounds cost \$$8\frac{2}{5}$?

73. If $36\frac{2}{3}$ acres of wheat yield 635 bushels, what is the yield of $5\frac{4}{5}$ acres?

74. What cost 56 lemons, at \$$1.12\frac{1}{2}$ a dozen?

75. Bought a barrel of sugar at $14\frac{1}{8}$ cents a pound. The gross weight was $256\frac{1}{2}$ pounds; weight of barrel, $13\frac{3}{4}$ pounds: what did I pay for the sugar?

76. Paid \$$34.11\frac{1}{8}$ for a barrel of sugar. Gross weight, $247\frac{3}{4}$ pounds; weight of barrel, $12\frac{1}{2}$ pounds: what was the cost of 100 pounds?

77. Bought 3 crocks of butter, weighing $25\frac{7}{16}$, $29\frac{3}{4}$, and $27\frac{1}{8}$ pounds; the empty crocks weigh $5\frac{3}{16}$, $5\frac{3}{4}$, and $5\frac{7}{8}$ pounds: what did the butter cost at $23\frac{3}{4}$ cents a pound?

78. Sold butter of first and second crocks at 26 cents: what did I receive?

79. Sold butter in third crock at 28 cents: what did I receive?

80. What did I make on the whole lot of butter?

81. If a man can saw $13\frac{1}{3}$ cords of wood in $\frac{1}{6}$ of a month, how many cords can he saw in $\frac{3}{4}$ of a month?

FORMULA. $13\frac{1}{3} \div \frac{1}{6} \times \frac{3}{4}$; or, $\frac{13\frac{1}{3}}{\frac{1}{6}} \times \frac{3}{4} = \frac{\overset{10}{\cancel{40}} \times 6 \times 3}{\cancel{3} \times \cancel{4}} = 60.$

ANAL. STEPS.—*(1.) Find what he can saw in 1 month.*
(2.) Find what he can saw in $\frac{3}{4}$ of a month.

82. If a bird can fly $10\frac{3}{4}$ miles in $\frac{1}{3}$ of an hour, how far can it fly in $2\frac{1}{2}$ hours?

83. If $\frac{2}{5}$ yard of velvet costs $74\frac{1}{4}$ cents, how much must be given for $13\frac{2}{5}$ yards?

84. If $\frac{3}{5}$ of a steeple casts a shadow $83\frac{2}{3}$ feet long, how long is the shadow cast by $\frac{5}{9}$ of it?

MISCELLANEOUS PROBLEMS. — REVIEW.

NOTE. — Should these problems be considered too difficult, the class may omit them until Decimals and Denominate Numbers have been finished.

N.B. — The final results should always be given in dollars and cents, instead of fractions of a dollar.

85. How many yards of muslin can be bought for 57 cents, at $9\frac{1}{2}$ cents per yard?

FORMULA. 57 cents $\div \frac{19}{2}$ cents $= 6$ (yards).

ANALYSIS. — If 1 yard costs $9\frac{1}{2}$ cents ($\frac{19}{2}$), as many yards can be bought for 57 cents as $9\frac{1}{2}$ cents are contained times in 57 cents, or 6 yards.

86. How many yards of tape can be bought for 11 cents, at $2\frac{3}{5}$ cents per yard? *W.* At $1\frac{5}{8}$ cents? At 9[illegible] cents?

87. How many yards of silk, at $97\frac{2}{3}$ cents per yard, can be bought for $97\frac{2}{3}$ cents? For 2 times $97\frac{2}{3}$ cents? For $2\frac{1}{2}$ times $97\frac{2}{3}$ cents? *W.* For \$1? For \$5.25? For \$6.75?

88. At $\frac{1}{10}$ of a dollar per quart, how many quarts of vinegar can be bought for $\$\frac{1}{3}$? For $\$\frac{5}{9}$? *W.* For $\$16\frac{3}{11}$? For $\$27\frac{5}{8}$?

FORMULA. $\$\frac{1}{3} \div \$\frac{1}{10} = 3\frac{1}{3}$ (quarts).

ANALYSIS. — *If* $\$\frac{1}{10}$ will buy 1 quart, $\frac{1}{3}$ of a dollar will buy *as many quarts as* $\$\frac{1}{10}$ is contained times in $\$\frac{1}{3}$, or $3\frac{1}{3}$ quarts.

89. At $\$2\frac{1}{4}$ per yard, how many yards of carpet can be purchased for $\$5\frac{1}{2}$? For $9? *W.* For $\$17\frac{5}{11}$? For $\$186\frac{8}{9}$?

90. If 3 oranges cost $10\frac{1}{2}$ cents, what will 4 cost?

ANAL. STEPS. — (1.) *Find cost of* 1 *orange.*
(2.) *Find cost of* 4 *oranges.*

91. What will 8 lemons cost, if 5 cost $10\frac{2}{3}$ cents? *W.* If 9 cost $29\frac{5}{8}$ cents? If 27 cost $75\frac{4}{7}$ cents? If 47 cost $\$2.25\frac{1}{2}$?

92. What will $7\frac{2}{3}$ pounds of butter cost, if 5 pounds cost $\$1.25\frac{2}{3}$? If 8 pounds cost $\$2.87\frac{1}{2}$? If 95 pounds cost $\$27.49\frac{3}{4}$?

93. If 5 pounds of rice cost $31\frac{1}{2}$ cents, what will 2 pounds cost? *W.* 8 pounds? $11\frac{1}{2}$ pounds? $113\frac{5}{7}$ pounds?

94. If 4 apples cost $\frac{5}{6}$ of a shilling, what will 1 apple cost? 6 apples? *W.* 37 apples? 387 apples?

95. What will 3 apples cost, if 1 apple costs $\frac{5}{6}$ of a cent? If 2 apples cost $1\frac{2}{3}$ cents? If $\frac{1}{2}$ of an apple costs $\frac{7}{8}$ of a cent? *W.* If $\frac{2}{3}$ costs $\frac{13}{14}$ of a cent? If $\frac{7}{8}$ costs $1\frac{3}{4}$ of a cent?

96. What will $\frac{1}{3}$ of a pound of sugar cost, if 1 pound costs $16\frac{2}{3}$ cents? *W.* If 8 pounds cost $\$1.25\frac{1}{2}$? If $7\frac{1}{2}$ pounds cost $99\frac{3}{8}$ cents? If $21\frac{3}{4}$ pounds cost $\$12.10\frac{3}{4}$?

97. Change $\dfrac{\frac{2}{3}}{\frac{2}{5}}$ to a simple fraction.

SUGGESTION. — Since the value of a fraction is the quotient of the numerator divided by the denominator, the above expression $= \frac{2}{3} \div \frac{2}{5}$.

NOTE. — Fractions having a fractional quantity for either numerator or denominator, or both, are sometimes called "*complex fractions.*" They *are simply the* fractional expression for the division *of fractions.* $2\frac{2}{5} \div \frac{7}{8}$ may be written $\dfrac{2\frac{2}{5}}{\frac{7}{8}}$.

98. Change $\frac{\frac{7}{8}}{\frac{11}{12}}$ to a simple fraction.

99. Find the value of $\frac{\frac{14}{15}}{\frac{21}{55}}$.

100. Reduce $\frac{3\frac{7}{8}}{5}$ to its simple fractional expression.

101. Change $\frac{7}{\frac{11}{13}}$ to a simple fraction.

102. Reduce $\frac{\frac{5}{8}}{\frac{3}{7}} \times 1\frac{7}{20}$.

Formula. $\frac{5}{8} \div \frac{3}{7} \times 1\frac{7}{20} = \frac{5}{8} \times \frac{7}{3} \times \frac{27}{20} = \frac{63}{32} = 1\frac{31}{32}$.

PROBLEMS CONTAINING NOT MORE THAN THREE ELEMENTARY QUESTIONS.

103. If $2\frac{1}{5}$ chests of tea cost \$$63\frac{5}{7}$, what will $6\frac{2}{3}$ chests cost?

Anal. Steps. — (1.) *Find the cost of* 1 *chest:* $63\frac{5}{7} \div 2\frac{1}{5} = 28\frac{74}{77}$ = cost of 1 chest.

(2.) *Find the cost of* $6\frac{2}{3}$ *chests:* $28\frac{74}{77} \times 6\frac{2}{3} = 193\frac{17}{231}$; or, \$193.073+, cost of $6\frac{2}{3}$ chests.

Formula. $63\frac{5}{7} \div 2\frac{1}{5} \times 6\frac{2}{3} = \frac{446}{7} \times \frac{5}{11} \times \frac{20}{3} =$ \$$193\frac{17}{231}$.

104. If $2\frac{1}{2}$ bushels of apples cost 7 shillings, what will $\frac{1}{7}$ of a bushel cost? *W.* $\frac{5}{9}$ of a bushel. $6\frac{5}{8}$ bushels? $143\frac{5}{7}$ bushels?

105. If a peck of corn is worth $15\frac{3}{4}$ cents, what are 3 pecks worth? 4 pecks, or a bushel? *W.* $2\frac{1}{2}$ bushels? $38\frac{4}{7}$ bushels?

106. If 3 pecks of rye are worth \$$\frac{7}{8}$, what is 1 peck worth? *W.* What are 4 pecks, or 1 bushel, worth? 3 bushels? $29\frac{2}{3}$ bushels.

Review. — Describe a mixed number. (84.) Illustrate by objects. Give *Principles I.* and *II.* (85.) How are fractions written? How is the value of a *fraction affected* by multiplying both terms by the same number? (86.) When *are fractions said* to be expressed in the lowest terms? (87.)

107. If 2 pecks of barley are worth $\$\frac{8}{11}$, what is 1 bushel worth? *W.* $5\frac{1}{2}$ bushels? $27\frac{3}{4}$ bushels? $346\frac{7}{8}$ bushels?

108. If 4 pounds of sugar cost 50 cents, what will 2 pounds cost? *W.* 100 pounds, or 1 cwt.? 5 cwt.? $7\frac{3}{9}$ cwt.?

109. If a piece of meat weighs 100 pounds, or 1 cwt., what will $\frac{1}{2}$ of it weigh? *W.* $\frac{3}{4}$ of it? $\frac{5}{9}$ of it? $\frac{6}{7}$ of it?

110. A barrel of flour weighs 196 pounds: how much will $\frac{1}{2}$ of a barrel weigh? *W.* $\frac{5}{8}$ of a barrel? 16 barrels? $23\frac{1}{3}$ barrels?

111. A bushel of wheat weighs 60 pounds: what will 2 bushels weigh? *W.* $\frac{2}{3}$ of a bushel? $46\frac{3}{5}$ bushels?

112. 1 rod of wire fence costs 98 cents, what will be the cost of 2 rods? *W.* Of $89\frac{1}{2}$ rods? Of 320 rods, or 1 mile? Of 2 miles? Of $63\frac{1}{2}$ miles?

113. 1 acre of land is worth $\$30\frac{2}{3}$: at the same rate, what are 2 acres worth? *W.* $\frac{2}{3}$ of an acre? $3\frac{4}{5}$ acres? 640 acres, or 1 section? $5\frac{1}{2}$ sections?

114. 1 cubic foot of stone is worth $2\frac{1}{2}$ cents, what are 3 cubic feet worth? *W.* $16\frac{2}{3}$ cubic feet? $24\frac{3}{4}$ cubic feet, or 1 perch? 128 cubic feet, or 1 cord? $3\frac{2}{3}$ cords?

115. A man's age is $47\frac{1}{2}$ years; his son is $\frac{1}{2}$ as old: how old is his son? *W.* His wife is $\frac{5}{8}$ as old: how old is she?

116. *W.* A merchant has $\$300.25$ in the bank, and $\frac{3}{5}$ as much in his safe: how much has he in his safe? How much has he in both?

REVIEW. — Give the analysis for reducing an improper fraction to a mixed number. *(90.)* What is meant by a common denominator? (92.) Can fractions having different denominators be added or subtracted? (93.) Why?

117. 20 is 5 times a certain number: what is $\frac{1}{2}$ of it? *W.* 200 is $6\frac{2}{3}$ times a certain number: what is $\frac{3}{5}$ of it? (101$\frac{1}{4}$ is $\frac{6}{7}$ of a certain number: what is $\frac{3}{5}$ of it?) 671$\frac{2}{3}$ is $\frac{5}{7}$ of a certain number: what is $3\frac{3}{4}$ times the number?

118. 60 is $\frac{1}{2}$ of a certain number: what is $\frac{1}{3}$ of it? *W.* 60 is $\frac{3}{4}$ of a certain number: what is $\frac{5}{8}$ of it? 60 is $3\frac{1}{2}$ times a certain number: what is $\frac{4}{5}$ of it?

119. Matthew had $\frac{5}{8}$ of a bushel of peaches. He sold $\frac{3}{4}$ of them: how many did he sell? *W.* How many had he left? Had he sold $\frac{2}{5}$ of a bushel, how many would he have had left?

120. *W.* Helen paid \$$\frac{5}{6}$ for a geography, and \$$\frac{3}{4}$ for a reader: how much more did she give for the one than the other? How much for both? How much had she left of a \$10 bill?

121. Alexander has \$$9\frac{7}{8}$: how much more must I give him to make \$10? *W.* To make \$$13\frac{1}{2}$? To make \$$37\frac{5}{7}$?

122. Robert earns \$$7\frac{3}{4}$ a week; Joseph earns \$$1\frac{1}{4}$ less than Robert: how much does Joseph earn? *W.* How much do both earn? How much do they lack of earning \$20?

123. Ruth will be 21 years old in $3\frac{5}{12}$ years: how old is she now? Maude is $18\frac{7}{12}$ years old: what is the sum of their ages?

124. A boy sold $\frac{3}{4}$ of his nuts: what part had he remaining?

ANALYSIS.—If a boy sold $\frac{3}{4}$ of his nuts, he must have remaining the difference between $\frac{3}{4}$ and $\frac{4}{4}$ (the whole) of his nuts, or $\frac{1}{4}$.

125. If he sold what he had left for 25 cents, for how

REVIEW. — *Give the analysis of a short example in addition of fractions. (93.) Give the analysis of a short example in subtraction of fractions. (94).*

much could he have sold the whole at the same rate? How much would that lack of \$5?

126. *W.* A gambler lost in play $\frac{1}{4}$ and $\frac{2}{3}$ of his money: what part of the whole did he lose? What part had he left? If the amount left was \$125.25, how much had he at first?

127. *W.* Bought $7\frac{2}{3}$ yards of cloth at one time, and $4\frac{5}{7}$ yards at another: what is the cost of the whole at \$$2\frac{3}{7}$ per yard?

128. *W.* Bought 37 yards of muslin; kept $9\frac{5}{8}$ yards; sold the remainder at \$$1\frac{3}{5}$: how much did I get for it?

129. *W.* A boy spent \$$1\frac{5}{8}$ for a book, \$$2\frac{3}{4}$ for $\frac{3}{5}$ of a ream of paper, and had \$$1\frac{2}{3}$ left: how much had he at first?

130. John wishes to pay \$$\frac{18}{25}$ for a pair of skates; he has $\frac{3}{5}$ of a dollar: how much more does he need? *W.* How much would he need to buy 3 pairs of such skates? One dozen pairs?

131. By mistake a man paid \$$\frac{1}{2}$, which was $\frac{1}{4}$ of a dollar too much: how much ought he to have paid?

132. *W.* By mistake a man paid \$$14\frac{7}{12}$, which was $5\frac{1}{10}$ dollars too much: how much ought he to have paid?

133. *W.* A boy by mistake subtracted $\frac{7}{8}$ instead $\frac{8}{7}$: was his answer too large, or too small, and how much? What would have been the condition of his answer, had he subtracted $\frac{4}{5}$? $2\frac{1}{7}$? $\frac{1}{11}$?

134. *W.* A man bought eggs for \$$3\frac{8}{9}$, and sold them for \$$3\frac{9}{10}$: did he make, or lose, and how much?

135. *W.* A man bought $13\frac{2}{3}$ dozen of eggs at $\frac{3}{10}$ of a

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REVIEW.—Tell how fractions are added. (93.) Tell how they are subtracted. (94.) Tell how fractions are multiplied by an integer. (95.) Tell how a number is multiplied by a fraction. (96.) How do you take the fractional part of a number.

dollar a dozen, and sold them at $\$\frac{3}{11}$: did he make, or lose, in the transaction, and how much?

136. *W.* If $\$2\frac{5}{8}$ will pay a woman for working $2\frac{1}{2}$ days, how much will pay for $5\frac{1}{2}$ days' work? For 6 days', or 1 week's work? For 2 weeks'? For $5\frac{5}{6}$ weeks'?

137. If 2 barrels of beef are worth $\$14\frac{4}{9}$, what is 1 barrel worth? *W.* What are 10 barrels? $12\frac{2}{3}$ barrels?

138. *W.* If $5\frac{1}{2}$ pounds of sugar cost $\$\frac{3}{5}$, what will $3\frac{2}{3}$ pounds cost? What will $\frac{5}{8}$ of a pound cost? $\frac{7}{13}$ of a pound? 250 pounds?

PROBLEMS CONTAINING NOT MORE THAN FOUR ELEMENTARY QUESTIONS.

139. If 100 oranges cost $\$3.20\frac{2}{3}$, what will be the cost of $5\frac{1}{2}$ boxes, each containing 250 oranges?

FORMULA. $\$3.20\frac{2}{3} \div 100 \times 250 \times 5\frac{1}{2} = \$3.20\frac{2}{3} \times \frac{1}{\underset{2}{\cancel{100}}} \times \frac{\overset{5}{\cancel{250}}}{1} \times \frac{11}{2}$ $= \$44.09+.$

PROCESS.

$$\begin{array}{r} 2)\$3.20\frac{2}{3} \\ \hline 1.60\frac{1}{3} \\ 5 \\ \hline 8.01\frac{2}{3} \\ 11 \\ \hline 2)88.18\frac{1}{3} \\ \hline \$44.09\frac{1}{6}; \\ \text{or, } \$44.09+ \end{array}$$

ANAL. STEPS. — (1.) *Find cost of* 1 *orange.*
(2.) *Find cost of* 250 *oranges, or* 1 *box.*
(3.) *Find cost of* $5\frac{1}{2}$ *boxes.*

SUGGESTION. — Cancel the common factor 50.

140. If 2 pounds of indigo cost $\$5\frac{2}{3}$, how much will 1 pound cost? *W.* $\frac{5}{9}$ of a pound? $13\frac{5}{7}$ pounds?

REVIEW. — *How is a fraction multiplied by a fraction or mixed number?* (97.) *How is a fraction divided by an integer?* (98.) *How is a mixed number divided by a fraction?* (99.) Give the rule for dividing a fraction by a fraction. (100.)

141. If I buy 8 eggs for 25 cents, and sell 7 eggs for 25 cents, will I make, or lose, and how much on each egg? *W.* How much on each dozen? How much on 25 dozen? How much on a barrel containing $80\frac{5}{12}$ dozen? On 5 barrels of the same size?

142. *W.* $7\frac{1}{2}$ pounds of coffee cost $\$1.87\frac{1}{2}$: what will a bag cost weighing $63\frac{2}{3}$ pounds? What will 5 bags of the same size cost?

143. *W.* 7 pounds of cheese are worth $87\frac{1}{2}$ cents: what is a cheese worth weighing $45\frac{1}{2}$ pounds? How much is that a ton of 2,000 pounds?

144. (*a.*) 6 is two times what number? (*b.*) $\frac{2}{3}$ of what number?

FORMULA. (*a.*) $6 \div 2 = 3$. (*b.*) $6 \div \frac{2}{3} = 9$.

ANALYSIS.—(*b*). If 6 is $\frac{2}{3}$ times a certain number, the number must contain as many units as $\frac{2}{3}$ is contained times in 6, or 9 (times or units).

145. 25 bushels of apples are twice a certain quantity: how many bushels in the quantity? *W.* 138 bushels are 3 times what quantity? How many bushels in the quantity, if 43 bushels are 5 times the quantity? If they are $\frac{1}{2}$ of the quantity? $\frac{2}{3}$? $\frac{7}{11}$?

146. What is the monthly salary of a clerk, if $60 is 3 times his salary? If it is $\frac{1}{2}$ of it? If it is $\frac{3}{4}$ of it? *W.* If it is $\frac{7}{11}$ of it?

147. 7 is $\frac{3}{4}$ of what number? $\frac{1}{2}$ is $\frac{1}{2}$ of what number? $\frac{2}{3}$ is $\frac{2}{3}$ of what number? *W.* $6\frac{5}{11}$ is $\frac{5}{8}$ of what number? $\frac{8}{9}$ is 3 times what number? $4\frac{5}{7}$ is $8\frac{2}{7}$ times what number?

148. What is the value of my horse, if $40 is twice its value? Is $\frac{1}{2}$ of its value? *W.* Is $\frac{7}{9}$ of its value? Is $\frac{11}{23}$ *of its value?*

149. I have $320\frac{1}{5}$ acres of land: how much will be left after selling $\frac{3}{5}$ of it?

150. I had $34.26, and lost $\frac{2}{7}$ of it: how much was left?

151. Five hundred dollars is $\frac{2}{3}$ of the money I had in a bank: what was the amount?

152. Five hundred dollars is $\frac{3}{7}$ of the money I have in my safe: what is the amount of money in the safe?

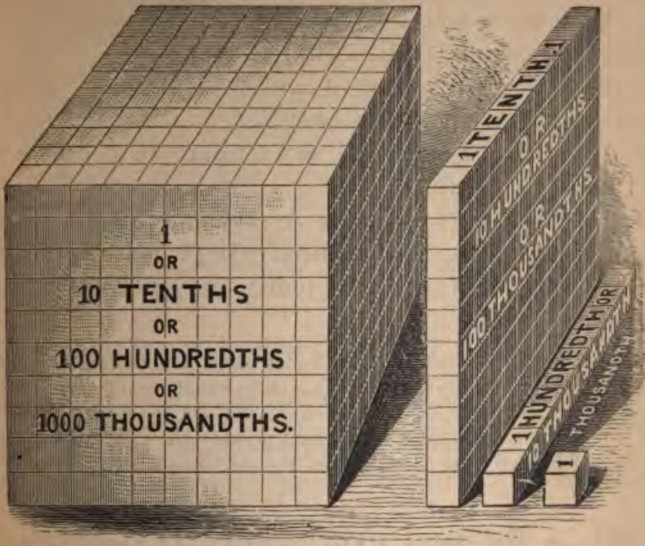
153. Twenty-five dollars is $2\frac{3}{10}$ times the money I gave to a poor man: how much did he receive?

154. A man has $4\frac{6}{7}$ bushels of potatoes, which is $\frac{5}{9}$ of the quantity that he planted: how many did he plant?

ANALYSIS SECTION VIII.

Fractions.

1. ILLUSTRATION.
2. DEFINITIONS.
3. TERMS.
    - *How written.*
4. REDUCTION.
    - *a. To Higher Terms.*
    - *b. To Lowest Terms.*
        - Illustration of each.
        - Analysis.
        - Principle.
        - Rule.
    - c. *Of Integers or Mixed Numbers to Improper Fractions.*
    - d. *Of Improper Fractions to Integral or Mixed Numbers.*
    - *e. To Common Denominators.*
        - Illustration of each.
        - Analysis.
        - Rule.
5. ADDITION OF FRACTIONS.
    - *a. Illustration.*
    - *b. Principle.*
    - *c. Analysis.*
    - *d. Rule.*
6. *SUBTRACTION* OF FRACTIONS.
    - *a. Illustration.*
    - *b. Analysis.*
    - *c. Rule.*
7. MULTIPLICATION OF FRACTIONS.
    - *a. Fractions by Integers.*
    - *b. Mixed Numbers by Integers.*
        - Illustration of each.
        - Analysis.
        - Principle.
        - Rule.
    - *c. Integers by Fractions.*
    - *d. Fractions by Fractions.*
        - Illustration of each.
        - Analysis.
        - Rule.
8. DIVISION OF FRACTIONS.
    - *a. Fractions by Integers.*
    - *b. Mixed Numbers by Integers.*
        - Illustration of each.
        - Analysis.
        - Principle.
        - Rule.
    - *c. Integers by Fractions.*
    - *d. Mixed Numbers by Fractions.*
        - Illustration of each.
        - Analysis.
        - Rule.
9. CONCRETE PROBLEMS.
    - a. Formula.
    - b. Analysis.



SECTION IX.

DECIMAL FRACTIONS.

ILLUSTRATIONS.

1. If a block of wood is divided into ten equal parts, what is *one* of the parts called? What are two of the parts called? 5 of the parts? 7 of the parts?

2. If 1 *tenth* of a block of wood is divided into ten equal parts, what is one of the parts called? 3 of the parts? 6 of the parts.

3. If 1 *hundredth* of a block of wood is divided into

ten equal parts, what is one of the parts called? 4 of the parts? 8 of the parts?

4. How many *tenths* in a unit? In 2 units? In 5 units? In 20 units? In 45 units?

5. How many *tenths* in 1 unit? How many hundredths? How many thousandths?

6. How many *hundredths* in a tenth? In 3 tenths? In 1 unit? In 5 units?

7. How many *thousandths* in 1 hundredth? In 3 hundredths? In 5 hundredths? In 1 tenth? In 6 tenths? In 8 tenths? In 1 unit? In 7 units?

When a unit is divided into ten equal parts, and each fractional part into ten other equal parts, &c., these subdivisions are called respectively, *tenths, hundredths, thousandths, &c.* As each subdivision is one-tenth as large as the part divided, the parts are called *Decimal Fractions,* from *decem,* which signifies ten.

101. *A Decimal Fraction, or Decimal,* is a fraction whose denominator is ten, or some power of ten, as, $\frac{1}{10}$, $\frac{11}{100}$, $\frac{43}{1000}$.

102. *A Mixed Decimal* consists of an integer and a decimal; as, $4\frac{3}{10}$, $8\frac{59}{1000}$.

103. *Decimals* may be written like a common fraction, with the denominator expressed; as, $\frac{3}{10}$, $\frac{5}{100}$, $\frac{25}{1000}$; or with the denominator indicated by placing a period before the decimal at the right of units; as, .3, .05, .025.

104. *U. S. Currency* is written in like manner; dollars occupying the place of units, and the cents and mills being considered as hundredths and thousandths, as seen by the following

NUMERATION TABLE.

INTEGERS.							DECIMALS.					
Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	***Units.***	*Tenths.*	*Hundredths.*	*Thousandths.*	*Ten-Thousandths.*	*Hundred-Thousandths.*	*Millionths.*
1	1	1	1	1	1	**1**.	1	1	1	1	1	1

U. S. CURRENCY NUMERATION TABLE.

DOLS.				CTS.		M.
Thousands.	Hundreds.	Tens.	***Units.***	*Tenths.*	*Hundredths.*	*Thousandths.*
1	1	1	1.	1	1	1

NOTE. — It will be observed that figures in places equidistant at the right or left of *units* have corresponding names; as, *tens* correspond to *tenths*, *hundreds* to *hundredths*, *thousands* to *thousandths*, &c.

Decimals are written and read as integral numbers; the denomination being shown by the place of the right-hand figure. At the right of units, the

1st place shows	*tenths*; as,	.3	=	$\frac{3}{10}$.
2d " "	*hundredths*; as,	.03	=	$\frac{3}{100}$.
3d " "	*thousandths*; as,	.003	=	$\frac{3}{1000}$.
4th " "	*ten-thousandths*; as,	.0003	=	$\frac{3}{10000}$.
5th " "	*hundred-thousandths*; as,	.00003	=	$\frac{3}{100000}$.
6th " "	*millionths*; as,	.000003	=	$\frac{3}{1000000}$.

NOTE. — The above table should be so committed to memory, that the pupils can write any fraction *instantly*.

EXERCISES IN READING DECIMAL NUMBERS.

The integral part of a mixed decimal number is read as a simple number: the fractional part is read as a simple number of the denomination of the right-hand figure. *Thus, 432.327* is read, four hundred and thirty-*two, and three* hundred and twenty-seven thousandths

Read the following numbers: *—

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)
.2	4.	3.14	2.1	3.14	4.70	51.03	1.634
41.1	6.3	2.21	4.6	.03	20.01	5.	2.504
3.4	14.2	.47	.5	5.	.03	70.71	8.101
7.1	.8	.19	8.23	.6	25.1	50.05	5.001
2.8	.7	5.27	5.1	8.10	1.83	2.02	20.020
1.5	11.8	4.51	.27	.58	10.10	41.41	5.005

(9.)	(10.)	(11.)	(12.)	(13.)
30.201	4.037	30.020	1.413	1.305
75.005	.067	5.005	1.27	.4
93.505	.004	50.005	50.037	6.103
40.076	.4	80.08	2.143	4.004
30.300	30.	5.011	.087	.6
100.001	8.010	500.501	.002	8.008

Note. — Also read the above as dollars, cents, and mills.

(14.)	(15.)	(16.)	(17.)
413.4021	403.413	50000.001	413.407012
41.0341	4.4021	5000.0001	4001.300010
5000.001	30.204101	500.00001	4.301
41.0213	3.012401	50.0001	400.300901
4.4021	$2.401\frac{2}{3}$	5.00001	30.410801
412.4134	$3.1241\frac{4}{5}$	.500001	.600001
21.4012	$4.3101\frac{1}{2}$	.050001	4.300101
412.4302	$400.4001\frac{3}{7}$	.00501	5.800001

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* The following exercises should be read aloud by the pupils at recitation. The class should be still further practised, if necessary, by reading similar exercises from the blackboard, until they can read quickly and accurately.

The teacher should then, with the books closed, dictate the same exercises for *the class to copy.* When they have finished, let them exchange slates, and let *the teacher read the* lesson from the book, requiring each pupil to note his neigh-[illegible] *errors by* drawing a line under the numbers incorrectly written.

105. NOTATION AND ADDITION OF DECIMALS.

WRITTEN EXERCISES.

Write and add,	PROCESS.
5 and 2 *tenths*,	5.2
3 and 6 *tenths*,	3.6
41 and 1 *tenth*,	41.1
25 and 14 *hundredths*,	25.14
1 and 7 *thousandths*.	1.007
	76.047

ANALYSIS. — The analysis and rule the same as in simple numbers.

1. Add,

4 and 3 *tenths*,
5 and 4 *tenths*,
7,
1 and 3 *tenths*,
6 and 7 *tenths*.

2. Add,

6 and 21 *hundredths*,
14 and 6 *hundredths*,
5 and 7 *hundredths*.
8 and 23 *hundredths*,
5 and 74 *hundredths*.

3. Add,

5 and 4 *tenths*,
6 and 3 *hundredths*,
5 *hundredths*,
5 *tenths*,
46 and 1 *hundredth*.

4. Add,

6 and 5 *tenths*,
47,
40 and 7 *tenths*,
47 *hundredths*,
2 and 5 *tenths*.

5. Add,

5 and 21 *hundredths*,
8 and 8 *hundredths*,
21 and 8 *tenths*,
7 and 3 *tenths*,
46 and 2 *hundredths*.

6. Add,

41 and 1 *thousandth*,
5 and 31 *thousandths*,
89 *thousandths*,
47 and 371 *thousandths*.

7. Add,

26 and 4 *tenths*,
4 and 5 *hundredths*,
41 and 5 *thousandths*,
6 and 32 *thousandths*,
4 and 32 *hundredths*.

8. Add,

25 and 16 *hundredths*,
17 and 5 *thousandths*,
1 and 27 *hundredths*,
327 *thousandths*,

REVIEW What is the difference between a composite and a prime number? (70, 71.) Can a prime number have a factor or divisor? Can a composite number *have a factor or divisor?* (72.) Give an example. What is meant by *greatest common divisor?* (74.)

9. Add,

3 *ten-thousandths,*
41 *ten-thousandths,*
321 *ten-thousandths,*
6002 *ten-thousandths,*

10. Add,

5 and 3 *tenths,*
5 and 3 *ten-thousandths,*
6 and 3 *thousandths,*
600 and 3 *thousandths,*
6 and 33 *hundredths.*

11. Add,

4 and 46 *hundred-thousandths,*
2 and 11 *hundred-thousandths,*
59 *hundred-thousandths,*
70 *hundred-thousandths.*

12. Add,

27 and 3 *tenths,*
27 and 3 *hundredths,*
27 and 3 *ten-thousandths,*
27 and 3 *hundred-thousandths.*

13. Add,

1 and 37 *hundredths,*
5 and 27 *hundred-thousandths,*
1 and 11 *thousandths,*
1 and 11 *ten-thousandths,*
3 and 5 *tenths.*

14. Add,

6 and 3 *tenths,*
11 and 47 *hundredths,*
5 *hundred-thousandths,*
500 *thousandths,*

NOTE. — The teacher should dictate examples similar to the above, until the pupils are ready and accurate in notation.

106. SUBTRACTION OF DECIMALS.

From 97 *hundredths* take 37 *thousandths.*

ANALYSIS AND RULE. — The same as in simple numbers.

PROCESS.

$$\begin{array}{r} .97 \\ .037 \\ \hline .933 \end{array}$$

15. From 63 and 4 *hundredths* take 4 and 3 *tenths.*

16. From 25 take 5 and 3 *hundredths.*

17. From 547 *ten-thousandths* take 447 *hundred-thousandths.*

18. From 495 *thousandths* take 459 *millionths.*

19. From 1 take 4701 *millionths.*

20. Take 5 and 3 *thousandths* from 25.

21. Take 33 *thousandths* from 33 *hundredths.*

22. Take 3 *thousandths* from 3000.

23. Take 101 *millionths* from 101000000. ✓

24. Take 79 *ten-thousandths* from 79000.

MISCELLANEOUS WRITTEN EXAMPLES.

25. What is the difference between 59 and .59? Between 3.7 and .37? Between 37 and .037?

26. John is 28.4 years old; Edward is 30.94 years old: what is the sum of their ages? What is the difference of their ages?

27. A man received $37.43 for corn, and $274.07 for wheat: how much did he receive for both? How much more did he receive for the wheat than for the corn?

28. From a cask containing 32.5 gallons of wine, 20.47 gallons were drawn: how much remained? How many gallons would three such casks hold?

29. Two men build 118 rods of stone fence; one built 63.50 rods: how much did the other build? How much more did one build than the other?

30. Two men earned $195.73; one earned $86.80; how much did the other earn? How much more did one earn than the other?

31. What is the sum of $\frac{2}{10}$, $\frac{7}{10}$, $\frac{25}{100}$, $\frac{6}{100}$, $\frac{37}{100}$, and $\frac{2}{100}$?

32. $2.4 + \frac{1}{100} + \frac{3}{100} + \frac{2}{10} - \frac{83}{100} =$ how many?

33. $5 - \frac{3}{10} + \frac{4}{10} + 6 - \frac{46}{100} + 3 =$ how many?

34. $4.1 + \frac{6}{10} + \frac{120}{1000} + \frac{4}{100} + \frac{5}{100} =$ how many?

35. $5.02 + \frac{3}{100} - \frac{6}{10} + \frac{41}{100} - \frac{1}{1000} =$ how many?

36. $6.3 + \frac{4}{100} + 5.4 - \frac{6}{1000} + \frac{1}{10} =$ how many?

REVIEW. — How can a composite number be resolved into prime factors? (73.) What is meant by a common multiple? (75.) What by a least common multiple? (76.) What is the use of cancellation? (77.) Upon what principle does it depend? (78.) Give an illustration. Give an illustration of what is meant by a fraction. (79.) Define a fraction. (79.) What does the denominator of a fraction show? (80.) The numerator of a fraction? (81.) What is a proper fraction? (82.) An improper fraction? (83.) A mixed number? (84.) Give the principles. (85.) How are fractions written?

FELTER'S

ARITHMETICAL DRILL CARDS.

TABLE IV.							TABLE V.	
FRACTIONS.							UNITS. Dec. Point. 10ths. 100ths. 1000ths. 10000ths. 100000ths. 1000000ths.	
	3	2	1	V	W	X	5.834709	
A	472	38	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{3}{1}$	A
B	604	59	6	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{50}{1}$	B
C	103	16	8	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{9}{10}$	C
D	504	20	9	$\frac{1}{11}$	$\frac{1}{12}$	$\frac{1}{13}$	$\frac{20}{10}$	D
E	300	47	4	$\frac{2}{3}$	$\frac{2}{5}$	$\frac{2}{7}$	$\frac{63}{10}$	E
F	412	32	1	$\frac{2}{9}$	$\frac{2}{11}$	$\frac{2}{13}$	$\frac{25}{100}$	F
G	755	93	3	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{3}{7}$	$\frac{300}{100}$	G
H	411	51	7	$\frac{3}{8}$	$\frac{3}{10}$	$\frac{3}{11}$	$\frac{208}{100}$	H
I	640	80	6	$\frac{4}{5}$	$\frac{4}{7}$	$\frac{4}{9}$	$\frac{732}{1000}$	I
K	832	21	8	$\frac{4}{11}$	$\frac{4}{13}$	$\frac{4}{15}$	$\frac{8000}{1000}$	K
L	253	73	5	$\frac{5}{6}$	$\frac{5}{7}$	$\frac{5}{8}$	$\frac{8021}{1000}$	L
M	414	86	2	$\frac{5}{9}$	$\frac{5}{11}$	$\frac{5}{12}$	$\frac{2143}{10000}$	M
N	674	13	1	$\frac{6}{7}$	$\frac{6}{11}$	$\frac{6}{13}$	$\frac{105}{10}$	N
O	305	14	8	$\frac{7}{8}$	$\frac{7}{9}$	$\frac{7}{10}$	$\frac{6134}{100}$	O
P	501	73	3	$\frac{7}{11}$	$\frac{7}{12}$	$\frac{7}{13}$	$\frac{43}{1000}$	P
Q	244	89	7	$\frac{8}{9}$	$\frac{8}{11}$	$\frac{8}{13}$	$\frac{1009}{100}$	Q
R	106	67	1	$\frac{9}{10}$	$\frac{9}{11}$	$\frac{9}{13}$	$\frac{5}{100}$	R
S	243	45	6	$\frac{10}{11}$	$\frac{10}{13}$	$\frac{10}{17}$	$\frac{2}{100000}$	S
T	182	38	2	$\frac{11}{12}$	$\frac{11}{13}$	$\frac{11}{14}$	$\frac{70}{1000000}$	T
U	100	72	3	$\frac{12}{13}$	$\frac{12}{17}$	$\frac{12}{19}$	$\frac{82000}{10}$	U

N.B. —*For* answers to the problems on the above drill card, see ☞.

DRILL CARD EXERCISES. — TABLE V.

ADDITION AND SUBTRACTION.

ADDITION.

Exercise I. — 3 numbers.
II. — 4 "
III. — 5 "
IV. — 6 "

Exercise V. — 7 numbers.
VI. — 8 "
VII. — 9 "
VIII. — 10 "

SUBTRACTION.

Take each of the decimals in Table V. from the following numbers as minuends: —

Exercise 1. — 93.416 minuend.
II. — 95.021 "
III. — 100.0003 "
IV. — 301 "

Exercise V. — 702.5016 minuend.
VI. — 450.021 "
VII. — 700 "

Ex. VIII. — Find the difference between 54.6 and A, B, C, &c.
IX. — Find the difference between 2.01613 and A, B, C, &c.
X. — Find the difference between .20153 and A, B, C, &c.
XI. — Find the difference between .000913 and A, B, C, &c.
XII. — Find the difference between .10101 and A, B, C, &c.
XIII. — Find the difference between A and B, B and C, C and D, &c.

MISCELLANEOUS WRITTEN EXAMPLES.

37. $\frac{8}{1000} + 4.3 + \frac{9}{100} - \frac{6}{1000} + \frac{1}{10} =$ how many?

38. $2.4 + 3.4 - \frac{6}{100} + \frac{3}{1000} - \frac{5}{1000} =$ how many?

39. $\frac{5}{100} + 6.03 - \frac{2}{1000} + \frac{1}{10} - .02 =$ how many?

40. $\frac{2}{10} - .02 + 4.3 - \frac{6}{1000} + 3 =$ how many?

41. $\frac{81}{1000} - .02 + .5 - \frac{4}{1000} + 6.01 =$ how many?

42. $8.3 - .002 + 5.004 - .43 =$ how many?

43. $\frac{5}{10} + \frac{8}{1000} - \frac{3}{10} + 5.3 - \frac{6}{1000} =$ how many?

44. What is the sum of $\frac{4}{10} + \frac{8}{1000} + \frac{2}{10000} + \frac{1}{100000} + \frac{2}{1000}$ and $\frac{5}{10}$?

45. What is the sum of $\frac{5}{10} + .0002 + \frac{15}{10000} + \frac{6}{100000} + \frac{43}{1000}$ and 3?

Review. — *What is meant by reducing a fraction to its lowest terms?* (87.) *Upon what principle does it depend?* (88.)

MULTIPLICATION OF DECIMALS.

ORAL EXERCISES.

1. How many are 3 times 8 tenths? 12 times 8 tenths? 9 times 11 hundredths? 7 times 6 thousandths?

Formula. $.8\times 3=2.4$.

Analysis. — 3 times 8 tenths are 24 tenths, equal to 2 units and 4 tenths.

2. How many are 4 times 8 cents? 7 times 11 mills? 9 times 5 tenths? 6 times 12 hundredths?

3. How many are 3 times 1 tenth? 2 hundredths? 12 hundredths? 5 thousandths? 25 thousandths? 6 ten-thousandths? 4 millionths? 12 hundred-thousandths?

4. What is $\frac{1}{10}$ of 3? $\frac{2}{10}$ of 3? .4 of 3? .7 of 3? .9 of 6?

Formula. $3\times\frac{1}{10}=3\times .1=\frac{3}{10}$ or .3. $3\times\frac{2}{10}=3\times .2=\frac{6}{10}$ or .6.

Analysis. — Since 1 tenth of 3 equals $\frac{3}{10}$ of 1, or .3, $\frac{2}{10}$ of 3 equals 2 times .3, or .6.

5. What is $\frac{7}{10}$ of 8? .7 of 9? .9 of 6? .4 of 12?

6. What is $\frac{1}{100}$ of 3? $\frac{2}{100}$ of 3? .05 of 7? .08 of 9?

7. What is $\frac{1}{1000}$ of 3? $\frac{2}{1000}$ of 3? .006 of 5? .012 of 8?

8. What is $\frac{2}{10}$ of 1? $\frac{2}{10}$ of $\frac{1}{10}$? $\frac{2}{10}$ of $\frac{6}{10}$? $\frac{7}{10}$ of $\frac{8}{10}$?

Formula. $\frac{2}{10}$ of $1=\frac{2}{10}$, or .2 $\frac{2}{10}$ of $.1=\frac{2}{10}$ of $\frac{1}{10}=\frac{2}{100}$, or .02.

Analysis. — Since 2 tenths of 1 = 2 tenths, 2 tenths of 1 tenth = 2 hundredths = .02.

Review. — How are integral numbers changed to improper fractions? (89.) How are mixed numbers changed to improper fractions? How are improper fractions reduced to integral or mixed numbers? (90.) Illustrate what is meant by a common denominator of two or more fractions. (92.) What axioms are used in the addition and subtraction of fractions? (93.) Give the analysis of a on example. Give the analysis of an example in the subtraction of fractions. (94.)

9. What is $\frac{2}{10}$ of 3? Of .3? Of .7? Of .6? Of .03? Of .08? Of .09?

10. What is .5 of 8? Of .6? Of .9? Of .03? Of .08?

11. What is $\frac{3}{100}$ of 1? Of 6? Of .2? Of .8? Of .08?

Problem. — Multiply 2.43 by 3.4.

Formula. $\frac{243}{100} \times \frac{34}{10} = \frac{8262}{1000}$, or 8.262.

PROCESS.

$$\begin{array}{r} 2.43 \\ 3.4 \\ \hline 972 \\ 729 \\ \hline 8.262 \end{array}$$

Analysis. — By the rule for multiplication of fractions, the numerators must be multiplied together for a new numerator, and the denominators for a new denominator.

As the denominator of a decimal fraction is always a power of ten, the denominator of the result will be expressed decimally by the sum of the factors of ten in each denominator: hence the following

107. Rule. — *Multiply as in simple numbers, and point off from the right of the product as many figures for decimals as there are decimal places in both factors.*

DRILL CARD EXERCISES. — TABLE V.

MULTIPLICATION.

Note. — Use only such portions of these exercises as the class may need.

Multiply each decimal in Table V. by the following multipliers: —

Exercise I. — 2 multiplier.
II. — 25 "
III. — .2 "
IV. — 4.2 "
V. — .08 "
VI. — 3.07 "
VII. — .29 "
VIII. — .006 "
IX. — .407 "
X. — 6.001 "

Exer. XI. — 2.002 multiplier.
XII. — 20.01 "
XIII. — 700.07 "
XIV. — 709.09 "
XV. — 5000 "
XVI. — 200.002 "
XVII. — 3.0004 "
XVIII. — .03002 "
XIX. — 30.09 "
XX. — .0006 "

XXI. — *A* × *B*, B × C, C × D, D × E, E × F, F × G, &c.
XXII. — *A* × *C*, B × D, C × E, D × F, E × G, &c.

ORAL AND WRITTEN EXERCISES.

46. How many are 3 times 1 tenth? 8 tenths? 6 hundredths? 25 hundredths? *W.* 4.3? 25.03?

47. How many are 9 times 2 tenths? 11 hundredths? 7 thousandths? *W.* 328 thousandths? 674 hundredths?

48. How many are 8 times 9 hundredths? 7 ten-thousandths? 11 hundred-thousandths? 9 millionths? *W.* 473 millionths? 203 hundred-thousandths?

49. What is 6 tenths of 3? Of 8 tenths? Of 11 hundredths? Of 12 thousandths? *W.* Of 27 thousandths? Of 103 hundred-thousandths?

50. What is 8 hundredths of 7? Of .7? Of .09? Of .011? *W.* Of 6.03? Of .4034?

51. *W.* Multiply 30.003 by 8. By .08. By .12.

52. What is 2 thousandths of 6? Of .007? Of 9 millionths? *W.* Of 34.67? Of 4.0003? Of 7043 hundred-thousandths?

53. What will 7 barrels of sugar cost, at $20 a barrel? At $30? *W.* At $28.25? At $32.37?

54. What is the value of 30 pounds of sugar, at 6 cents per pound? At 8 cents? *W.* At 8.2 cents? At 8$\frac{1}{2}$ cents? At 7$\frac{5}{7}$ cents?

55. At $30 a barrel, what is the value of 2 barrels of sugar? Of 3 tenths of a barrel? Of .6? Of .03? *W.* Of 2.8 barrels? Of .025?

56. At 2\frac{1}{2}$ a box, what is the cost of 2 boxes of raisins? Of .3 of a box? *W.* Of 2.37? Of .81?

Review.—How are fractions multiplied by integers? (95.) Illustrate by an *example.* How are integers multiplied by fractions? (96.) Illustrate by an example. How are fractions multiplied by fractions? (97.) Illustrate by an example. How are fractions divided by integers? (98.)

57. *W.* A man bought 19.5 barrels of cider, at $2.20 a barrel; he sold it all for $28: did he make, or lose, and how much?

58. *W.* A man bought .3 of a barrel of cider, at $2¾ a barrel; he sold what he purchased for $2⅔: did he make, or lose, by the transaction? X

59. *W.* A boy bought .75 of a bushel of nuts, at $2 a bushel; he sold them again for $2: did he make, or lose, and how much?

60. *W.* Another boy bought 1.75 bushels of nuts, at $2 a bushel; he sold them again for $2: did he make, or lose, and how much?

61. *W.* A girl had a ten dollar bill; she bought .38 of a box of raisins, at 2\frac{7}{9}$ a box: how much money had she left? +

62. $4.3 \times \frac{5}{8} + 3.47 -- .08 =$ what?

63. $43.24 - \overline{2.4 \times 6\frac{1}{2}} + 40.74 =$ what?

64. $(2.01 - .003) \times (1 + \frac{3}{4}) =$ what?

65. $\overline{7 \times \frac{1}{5}} - (3 \times .3) + 12\frac{1}{2} =$ what?

66. $(\overline{6 \times .3} - \overline{.6 \times 3}) \times 8.401 =$ what?

67. $(\overline{\frac{4}{5} \times .7} - \overline{.8 \times .7}) \times 4.2 \times 3 =$ what?

DIVISION OF DECIMALS.

ORAL EXERCISES.

1. How many tenths in a unit? In 2 units? In 7 units?

2. In 1 unit how many tenths? How many hundredths? How many thousandths? How many ten-thousandths?

3. How many tenths in 1 half of a unit? In 1 fourth? In 1 fifth?

REVIEW.—*How are integers divided by fractions?* (99.) Illustrate by an example. *How are fractions divided by fractions?* (100.) Illustrate by an example.

4. In 1 fourth of a unit, how many tenths? Ho many hundredths? How many thousandths? Ho many millionths?

FORMULA. 10 tenths ÷ 4 = $.2\frac{1}{2}$.
100 hundreds ÷ 4 = .25.

ANALYSIS. — Since in 1 unit there are 10 tenths, in 1 fourth o unit there are 1 fourth of 10 tenths, or $2\frac{1}{2}$ tenths.

5. Divide .8 by 4. .14 by 7. 1.4 by 7. 2.8 by 4.

6. What is 1 fifth of 2.5? Of 35? Of .35? Of 1.0

7. What is 1 eighth of 24? Of 4.8? Of $4\frac{8}{10}$? (.72?

8. What is 1 twelfth of 72? Of 7.2? Of $7\frac{2}{1}$ Of $9\frac{6}{10}$?

108. PRINCIPLE. — *Any order divided by the sa order gives a quotient of units.*

Decimals, like common fractions, *must* be changed to like denor nations, before one fraction can be divided by another; and the qu tient must be units, or parts of a unit: hence the place of the decin point can be determined by inspection, by reducing the divisor a dividend to like denominations, and placing an accent in the divide over the place of the decimal point in the quotient, annexing ciphe if necessary; thus, —

ILLUSTRATIONS.

(1.)	(2.)	(3.)	(4.)	(5.)
4.7)4.7′	.007)2.000′	4)6.08	.2).0′8	40.2).0′08(
1	285.71+	1.52	.4	.000

REVIEW. — What is meant by a decimal fraction? (101.) What is a mix decimal? (102.) How are decimals written? (103.) In how many ways can dec mals be written? How does the notation of United States currency differ fro decimals? (104.) What is the first place at the right of the units? The second *The third? The fourth?* The first at the left of the units' place? The second *The third at the right?* The third at the left? The fourth at the right? At th *left? How are decimals added?* (105.) How is subtraction of decimals performed *(106.)*

PROBLEM. — Divide 22.848 by 3.4.

PROCESS.

```
3.4)22.8'48(6.72
    204
    ───
     244
     238
     ───
       68
       68
      ───
```

ANALYSIS. — (1.) For convenience, write the numbers as in simple division.

(2.) 3.4 equals 34 tenths; 22.8 equals 228 tenths; 34 tenths are contained in 228 tenths 6 times; which write in the quotient as units. The remainder of 24 tenths is equal to 240 hundredths.

(3.) 240 hundredths and 4 hundredths equal 244 hundredths; 34 tenths are contained in 244 hundredths 7 tenths times; which write in the quotient. The remainder, 6 hundredths, is equal to 60 thousandths.

(4.) 60 thousandths and 8 thousandths equal 68 thousandths; 34 tenths are contained in 68 thousandths 2 hundredths times; which write in the quotient.

To determine the decimal point by inspection: —

109. RULE. — I. *Make the decimal places in the dividend equal to those in the divisor,* write the quotient of these figures as units. Place the decimal point, and continue dividing as in simple division.*

Some may prefer the following: —

RULE. — II. *Make the number of decimal places in the dividend at least equal to those in the divisor.*

Point off as many decimals in the quotient as the decimal places in the dividend exceed those in the divisor, prefixing ciphers to the quotient, if necessary.

REVIEW. — Any order divided by the same order gives a quotient of what order? (108.) Units divided by units give what? Give the rule for the multiplication of decimals. (107.) Give the rule for the division of decimals. (109.)

* Annex ciphers to the dividend, if necessary.

BLACKBOARD EXERCISES.

Determine the decimal point in the quotient of each of the following: —

ILLUSTRATIONS.

(1.)	(2.)	(3.)	(4.)	(5.)
1)82	1).21	100).360	.1)7.5′	.0001).0930′
82	.21	.0036	75	930

(1.)	(2.)	(3.)	(4.)
82 ÷ 1	47 ÷ .1	.31 ÷ 1.	.34 ÷ .1
74 ÷ 10	63 ÷ .01	.86 ÷ 10.	.65 ÷ .01
57 ÷ 100	27 ÷ .001	.57 ÷ 100.	.72 ÷ .001
32 ÷ 1000	53 ÷ .0001	.43 ÷ 1000.	.82 ÷ .0001

WRITTEN EXERCISES.

The answers to the following examples should be correct to five decimal places. Annex ciphers to the dividend, if necessary.

Note. — The sign + annexed to the quotient shows that the division is not completed.

68. 3.41 ÷ 4
69. 20.1 ÷ .6
70. 91.4 ÷ 7
71. 13.4 ÷ .08
72. 134 ÷ .6
73. 2 ÷ 8
74. 7 ÷ .08
75. 5 ÷ .3
76. 4 ÷ .007
77. 2 ÷ .0005
78. 3 ÷ .6
79. 21 ÷ 2.1
80. .21 ÷ 21
81. .21 ÷ .21
82. .21 ÷ .021
83. 5 ÷ 3
84. 6 ÷ 9
85. 8 ÷ 4
86. .3 ÷ 3
87. 20 ÷ .02

DRILL CARD EXERCISES. — TABLE V.

DIVISION.

Divide each decimal in Table V. by the following divisors: —

Exercise I. — 2 divisor.
II. — .04 "
III. — .009 "
IV. — .007 "
V. — .0006 "
VI. — .0008 "
VII. — 20 "
VIII. — 25 "
IX. — 4.2 "
X. — .13 "

Exercise XI. — .014 divisor.
XII. — .0028 "
XIII. — 400 "
XIV. — 403 "
XV. — 12.7 "
XVI. — 1.87 "
XVII. — 8.93 "
XVIII. — .721 "
XIX. — .0713 "
XX. — .00207 "

Exercise XXI. — A ÷ B, B ÷ C, C ÷ D, D ÷ E, &c.
XXII. — A ÷ C, B ÷ D, C ÷ E, D ÷ F, &c.

ORAL AND WRITTEN EXERCISES.

88. If a man can cut 2.3 cords of wood in 1 day, how many cords can he cut in 2 days? In 3 days? *W.* In 47 days? In 123 days?

89. If I give $1.2 for a pair of gloves, how much will 3 pairs cost? 4 pairs? *W.* 58 pairs? 137 pairs?

90. $2\frac{3}{10}$ dollars will buy 1 pair of shoes: at that rate, how many dollars will it take to purchase 3 pairs? 5 pairs? *W.* 27 pairs? 2883 pairs?

91. One barrel of flour is worth $5.34: at that rate, what are 2 barrels worth? *W.* 2.5 barrels worth? .63 of a barrel? 42.73 barrels?

92. If it takes .4 of a bushel of apples to make a gallon of cider, how many gallons can be made from

---

REVIEW. — *What is a fraction?* (79.) A denominator? (80.) The numerator of *a fraction?* (81.) *A proper fraction?* (82.) An improper fraction? (83.) A mixed *number?* (84.)

.8 of a bushel? 1.6 of a bushel? *W.* 45.8 bushels? 49.47 bushels?

93. If 8 bushels of apples cost $4, what will one bushel cost? 2 bushels? *W.* 5.2 bushels? 7.57 bushels?

94. *W.* If 80.2 pounds of starch can be made from 2.1 bushels of corn, how much starch can be made from 1 bushel? From 7.5 bushels?

FORMULA. (1.) 80.2 lbs. $\div$ 2.1 = (?) lbs.
(2.) (?) lbs. $\times$ 7.5 = (?) lbs.

95. *W.* If 3 cubic feet of water weigh 187.5 lbs., how much will 9 cubic feet weigh? 37.42 cubic feet?

110. TO CHANGE COMMON FRACTIONS TO DECIMAL.

ORAL EXERCISES.

1. What is $\frac{1}{2}$ of 8? $\frac{2}{3}$ of 5? $\frac{5}{9}$ of 11? $\frac{8}{13}$ of 12?

2. How many tenths in a unit? How many hundredths? How many thousandths? How many millionths?

3. How many tenths in $\frac{3}{4}$ of a unit? or, change $\frac{3}{4}$ to tenths.

ANALYSIS. — $\frac{3}{4}$ of a unit = 3 $\div$ 4, or one-fourth of 3 units. In one unit there are 10 tenths, and in $\frac{3}{4}$ of a unit there are $\frac{1}{4}$ of 3 times 10 tenths, or $7\frac{1}{2}$ tenths.

4. Express $\frac{3}{4}$ of 1 in an equivalent decimal.

PROCESS. FORMULA. $\frac{3}{4} = 3.00 \div 4 = .75$

$$\begin{array}{r} 4)\underline{3.00} \\ .75 \end{array}$$

ANALYSIS. — 3 units equal 300 hundredths: $\frac{1}{4}$ of 300 hundredths equal 75 hundredths.

Since a fraction indicates the division of the numerator by the denominator, if the numerator is reduced to the required denomination, and divided by the denominator, the result will be an equivalent decimal expression: hence the following

RULE. — *Add ciphers to the numerator, and divide by the denominator, as in the division of decimals.*

BLACKBOARD EXERCISES.

Change each of the following fractions to tenths, hundredths, and thousandths.

The sign +, placed after a decimal, shows that there is a remainder; thus, $\frac{5}{7} = .714\frac{2}{7}$, or .714 +.

MODEL.	(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)	(9.)
$\frac{3}{4} = .7\frac{1}{2}$	$\frac{8}{9}$	$\frac{6}{7}$	$\frac{8}{9}$	$\frac{1}{4}$	$\frac{2}{7}$	$\frac{4}{9}$	$\$\frac{6}{13}$	$\$\frac{3}{10}$	$\$\frac{5}{11}$
$\frac{4}{5} = .8$	$\frac{7}{8}$	$\frac{5}{8}$	$\frac{4}{7}$	$\frac{1}{7}$	$\frac{2}{13}$	$\frac{4}{15}$	$\$\frac{7}{10}$	$\$\frac{4}{7}$	$\$\frac{6}{11}$
$\$\frac{6}{7} = \$.857+$	$\frac{5}{11}$	$\frac{3}{4}$	$\frac{8}{11}$	$\frac{1}{10}$	$\frac{3}{7}$	$\frac{5}{8}$	$\$\frac{7}{13}$	$\$\frac{4}{13}$	$\$\frac{7}{9}$
$\$\frac{4}{11} = \$.363+$	$\frac{4}{9}$	$\frac{2}{3}$	$\frac{4}{15}$	$\frac{1}{13}$	$\frac{3}{11}$	$\frac{5}{12}$	$\$\frac{8}{13}$	$\$\frac{5}{7}$	$\$\frac{7}{12}$

WRITTEN EXERCISES.

96. Change $\frac{2}{41}$ to a decimal of four places.

FORMULA. $2 \div 41 = .0487+$.

97. Reduce $\frac{5}{8}$ to a decimal, $\frac{6}{11}$, $\frac{4}{7}$, $\frac{9}{11}$.

Find the decimal equivalents, correct to four places, for each of the following fractions: —

MODEL.	(1.)	(2.)	(3.)	(4.)	(5.)
$\$\frac{1}{2} = \$.50$	$\$\frac{1}{6}$	$\frac{4}{7}$	$\frac{8}{9}$	$\frac{2}{27}$	$\frac{5}{137}$
$\$\frac{1}{3} = .3333+$	$\$\frac{1}{7}$	$\frac{3}{9}$	$\frac{7}{74}$	$\frac{6}{57}$	$\frac{9}{672}$
$\$\frac{1}{4} = .25$	$\$\frac{2}{7}$	$\frac{4}{11}$	$\frac{8}{37}$	$\frac{2}{324}$	$\frac{10}{681}$
$\$\frac{3}{8} = .375$	$\$\frac{3}{8}$	$\frac{6}{7}$	$\frac{9}{95}$	$\frac{8}{117}$	$\frac{2}{1001}$

NOTE. — Those fractions whose denominators are composed of prime factors other than 2 or 5, when changed to a decimal, repeat one figure, or a series of figures; as, $\frac{1}{3} = .333+$, $\frac{1}{7} = .142857+$, &c. These figures are sometimes marked with a dot over the recurring figure, or series of figures, and are called "repetends."

111. TO CHANGE DECIMALS TO COMMON FRACTIONS.

ORAL EXERCISES.

1. 5 tenths of an apple is what part of an apple expressed in its lowest terms?

FORMULA. $.5 = \frac{5}{10} = \frac{1}{2}$.

2. Express $\frac{4}{10}$ of an orange in its lowest terms. 8 tenths of a lemon. 25 hundredths. 50 hundredths.

3. What is the value of .4 expressed by a common fraction in its lowest terms? Of .25? Of .6? Of .28? Of .06? Of .75? Of .44? Of .16? Of .15?

4. Give the common fraction equivalent to each of the following: —

MODEL.	(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
$.04 = \frac{1}{25}$	.6	.04	.38	.15	.06	.012	.085
$.8 = \frac{4}{5}$	.8	.08	.28	.60	.05	.885	.012
$.05 = \frac{1}{20}$	.9	.25	.35	.15	.03	1.755	5.058
$2.4 = 2\frac{2}{5}$	.5	.30	.75	.12	.2	6.855	1.025

5. Change each of the following number of cents and mills to the fraction of a dollar: —

MODEL.	(1.)	(2.)	(3.)	(4.)	(5.)
\$.36 = \$$\frac{9}{25}$	\$.125	\$1.10	\$.20	\$.875	\$.004
$.25 = \frac{1}{4}$	1.25	6.90	.55	1.375	.008
$2.12 = 2\frac{3}{25}$	.18	.12	.06	.605	.002
$1.375 = 1\frac{3}{8}$	.40	.15	.88	.005	.104

ORAL AND WRITTEN EXERCISES.

PROBLEM. — Reduce .75 to an equivalent common fraction.

FORMULA. $.75 = \frac{75}{100} = \frac{3}{4}$.

98. Reduce .5 to a common fraction. Reduce .05.

99. What common fraction is equal to .25? To .75? To .55? To .125? To .185?

NOTE. — When the decimal is a repetend, as, $\frac{1}{3} = .\dot{3}$, $\frac{1}{11} = .0\dot{9}$, *these repetends* can be changed to a fraction by writing for a denominator as many *nines* as there are figures in the repetend; as, $.\dot{3} = \frac{3}{9} = \frac{1}{3}$, $.0\dot{9} = \frac{9}{99} = \frac{1}{11}$, $.2\dot{3} = .2\frac{3}{9} = .2\frac{1}{3}$.

100. Change each of the following repetends to fractions: —

MODEL.	(1.)	(2.)	(3.)	(4.)
$.\dot{3} = \frac{3}{9} = \frac{1}{3}$	$.\dot{2}\dot{7}$	$.\dot{7}$	$.4\dot{3}\dot{3}\dot{3}$ +	$.\dot{1}4285\dot{7}$
$.\dot{6} = \frac{6}{9} = \frac{2}{3}$	$.\dot{8}$	$.\dot{5}\dot{3}$	$.9\dot{6}\dot{6}\dot{6}$ +	$.\dot{2}8571\dot{4}$
$.4\dot{0}\dot{9} = .4\frac{9}{99} = .4\frac{1}{11}$	$.\dot{5}$	$.\dot{2}\dot{3}$	$.\dot{1}11\dot{1}$ +	$.\dot{4}135\dot{4}$
$.6\dot{1} = .6\frac{1}{9} = .6\frac{1}{9}$	$.\dot{4}\dot{5}$	$.\dot{3}\dot{6}$	$.\dot{7}77\dot{7}$ +	$.\dot{4}44\dot{4}$ +

101. Reduce $.33\dot{3}$ to a common fraction. Reduce $.16\frac{2}{3}$.

FORMULA. $.33\dot{3} = \frac{33\frac{1}{3}}{100} = \frac{100}{3} \div 100 = \frac{100}{300} = \frac{1}{3}$.

$.16\frac{2}{3} = \frac{16\frac{2}{3}}{100} = \frac{50}{3} \div 100 = \frac{50}{300} = \frac{1}{6}$.

ANALYSIS. — $.33\dot{3}$, written with a denominator, equals $\frac{33\frac{1}{3}}{100}$. The numerator $33\frac{1}{3} = \frac{100}{3}$; and $\frac{100}{3}$ divided by $100 = \frac{100}{300}$, or $\frac{1}{3}$: hence the following

RULE. — *Write the denominator under the decimal, erase the decimal point, and reduce the fraction to its lowest terms.*

NOTE. — Repetends must be changed to fractions, and the fractions to improper fractions, before dividing by the denominator.

102. Write common fractions for each of the following decimals: —

MODEL.

$.25 = \frac{25}{100} = \frac{1}{4}$

$.12\frac{1}{2} = \frac{12\frac{1}{2}}{100} = \frac{25}{200} = \frac{1}{8}$

$.4\dot{4} = \frac{4\frac{4}{9}}{100} = \frac{40}{900} = \frac{2}{45}$

$.09\dot{0}\dot{9} = \frac{9\frac{9}{99}}{100} = \frac{900}{9900} = \frac{1}{11}$

(1.)	(2.)	(3.)	(4.)
.875	.15	$.33\dot{3}$	.24
.42	.015	$.6\frac{2}{3}$	.35
.55	.025	$.7\dot{7}$	.004
.75	.2	$.88\dot{8}$	$.\dot{4}\dot{5}$
.45	.8	.24	$.25\dot{3}$
.755	.28	.63	.74
$.12\frac{1}{2}$	$.37\frac{1}{2}$	$.\dot{4}$	.28

REVIEW. — *Give principles.* (85.) Give rule for reducing a mixed number to an improper fraction. (89.) For reducing an improper fraction to an integral or mixed number. (90.)

What is the value of each of the following, expressed in decimals?

103. $1.13 \times 5 \div 7$.
104. 3.7 lbs. $\div 8 \times 2.5$.
105. $\overline{\$1.75 \times .3} \div 5.7$.
106. $(\$4.75 + .27) \div 90$.
107. $\overline{4.3 \text{ bbls.} \times \frac{8}{10}} + 2.5$ bbls.
108. $7.1 \times 8.2 - \overline{2\frac{3}{5} \div 7.4}$.
109. $\overline{4.25 \div .4} - (4\frac{3}{4} + 5.2)$.
110. $(\overline{1\frac{4}{7} \times 5.3} + \overline{2\frac{3}{7} \div .0\frac{5}{7}}) \times .1\dot{3}$.
111. $(8.\dot{3} + 5.\dot{4}) \times 8\frac{3}{8} \div 7.3\frac{1}{4}$.
112. $\overline{\frac{8}{10} + 3.4} \times \overline{6 + \dot{5}0\dot{4}} \div 1.\dot{4}7\dot{4}$.

112. *To find the cost when price per* 100 *or* 1000 *is given.*

WRITTEN EXAMPLES.

What is the cost of 234 feet lumber, at \$2.70 per hundred feet? At \$29.30 per thousand?

NOTE.—When the mills in the answer equal or exceed 5, *add* 1 cent; when less than 5, reject them.

FORMULA. (1.) $\$2.70 \times \overline{234 \div 100} = \$6.32$.
(2.) $\$29.30 \times \overline{234 \div 1000} = 6.86$.

PROCESS.

\$2.70 cost C.* feet.
2.34 No. of C. feet.
\$6.32

ANALYSIS.—(1.) 234 feet equals 2.34 hundred feet.

(2.) Since 100 feet costs \$2.70, 2.34 hundred feet will cost 2.34 times \$2.70, or \$6.318.

REVIEW.—Define common denominator. (92.) Give the rule for the addition of fractions. (93.) Give the rule for the subtraction of a fraction from an integer. (94.) A fraction from a fraction. A fraction from a mixed number. A mixed number from a mixed number. Give a rule for multiplying a fraction by an integer. (95.) A mixed number by an integer. Give the rule for multiplying an integer by a fraction. (96.) An integer by a mixed number. Give the rule for multiplying a fraction by a fraction. (97.) A mixed number by a fraction. A mixed number by a mixed number.

* *In business transactions*, C. is sometimes used for hundreds, M. for thousands, and T. for *tons*, or two thousand pounds.

NOTE. — As any number can be divided by 10, 100, or 1000 by removing the decimal point to the left, the first step of the analysis may be omitted in the process.

113. What will 574 feet of boards cost, at $5.90 per C.?

114. What will 7243 feet of lumber cost, at $63.20 per M.?

115. At $8.59 per M., what will 8900 bricks cost? 2750 bricks? 300 bricks?

116. At $5.25 per C., what will 340 pounds of lead cost? 47 pounds? 9 pounds?

117. At $3.25 per M., what will 450 envelopes cost? 50 envelopes? 58,700 envelopes?

118. What is the freight on 834 pounds of plough-castings, at 1.37\frac{1}{2}$ per 100 pounds? At $.95$\frac{2}{3}$ per C.?

119. What will 583 lath cost, at $.75$\frac{2}{3}$ per C.?

120. What will 950 shingles cost, at 2.17\frac{1}{2}$ per thousand?

MISCELLANEOUS EXAMPLES FOR REVIEW.

WRITTEN EXERCISES.

Find the cost of the following items expressed in dollars, cents, and mills.

@ means by the pound, yard, &c.

121 $47\frac{1}{2}$ pounds rice, @ $5\frac{1}{4}$ cents.

122. $73\frac{7}{8}$ yards calico, @ $18\frac{1}{4}$ cents.

123. $89\frac{1}{4}$ quarts beans, @ $7\frac{2}{3}$ cents. 6.93¢

124. $67\frac{5}{8}$ yards lace, @ $9\frac{3}{4}$ cents.

125. $112\frac{1}{2}$ yards cloth, @ $\$1\frac{3}{7}$. 160.706

REVIEW. — Give the rule for dividing a fraction by an integer. (98.) A mixed number by an integer. Give the rule for dividing an integer by a fraction. (99.) An integer by a mixed number. Give the rule for dividing a fraction by a fraction. (100.) A mixed number by a fraction. A mixed number by a mixed number.

126. $413\frac{1}{2}$ bushels oats, @ $66\frac{1}{3}$ cents. 274.284
127. $6\frac{1}{2}$ bushels apples, @ $174\frac{1}{2}$ cents.
128. $5\frac{1}{2}$ quarts nuts, @ $19\frac{3}{4}$ cents.
129. 415 pounds starch, @ $4\frac{1}{7}$ cents.
130 $2\frac{3}{5}$ yards silk, @ \$$1\frac{1}{8}$.
131. 417 pounds honey, @ $18\frac{3}{4}$ cents.
132. $97\frac{3}{4}$ pounds coffee, @ \$$\frac{3}{5}$.
133. 24.2 bushels apples, @ \$$\frac{8}{9}$. 21.505
134. $97\frac{5}{8}$ yards calico, @ $11\frac{1}{2}$ cents.
135. $410\frac{7}{8}$ pounds cheese, @ $15\frac{1}{2}$ cents.
136. $73\frac{4}{5}$ tons coal, @ \$$12\frac{3}{5}$.
137. 81.3 pounds veal, @ $19\frac{3}{4}$ cents.
138. $23\frac{2}{3}$ pecks peaches, @ $37\frac{1}{2}$ cents. 8874
139. 4210.5 pounds wool, @ $49\frac{1}{2}$ cents.
140. 742.6 pounds cotton, @ $24\frac{2}{3}$ cents.
141. 2173 pounds pork @ \$$5\frac{2}{7}$ per C.
142. $4113\frac{1}{2}$ pounds butter, @ \$$27\frac{1}{2}$ per hundred weight?
143. $751\frac{1}{2}$ pounds lard, @ \$8.70 per C.
144. 5350 shingles, @ \$2.50 per M.
145. 7400 bricks, @ \$$9\frac{1}{4}$ per M.
146. 5234 feet timber, @ \$35 per M.

147. A banker received, on Jan. 1, cash to the amount of \$30.40, and paid out \$25.70; Jan. 2, received 18 dollars, \$5, \$20.50, paid out \$7.30, \$6.95, \$3.47; Jan. 3, received \$4.20, \$923, paid out \$9.20, \$6.35: how much did he receive on the three days? How much did he pay out? Did he pay out more, or less, than he received? and how much?

148. On Jan. 4, I bought $3\frac{1}{2}$ bushels apples @ 95 cents, 6 pecks of beans @ 90 cents, and sold $4\frac{3}{4}$ bushels potatoes @ 87 cents, 24 cabbages @ 10 cents, 3 pecks

apples @ 28 cents; on Jan. 5, I bought 35 bushels potatoes @ 60 cents, and sold 5 cabbages @ 10 cents, 5 pecks apples @ 28 cents, 3 quarts beans @ 12 cents: how much did I buy? How much did I sell? Did I sell more, or less, than I bought? and how much?

TEST QUESTIONS FOR EXAMINATION.

70. What is a composite number? Name the composite numbers less than 20. Are the following numbers prime, or composite, 27, 36, 40, 41, 71, 21, 83, 63, 55, 67, 57, 17, 77? Give the factors of each of the composite numbers. What composite number is formed by the multiplication of 2 and 3? 7 and 2? 3, 5, and 7?

71. What is a prime number? How does it differ from a composite number? Give an example of each. Is 6 a prime number? Why? Is 12? Why? Is 7? Why?

72. What is a factor, or divisor? What is a factor of 14? How many factors has 14? When is a factor said to be prime? Give the prime factors of 4, 8, 24.

73, 74. What is the greatest common divisor, or factor, of two or more numbers? Give two numbers that have a common factor, and name the factor. Name the common factor of 4 and 2. Of 10 and 15. Name the greatest common factor of 8 and 12. Illustrate by an example the method of finding the greatest common factor of three numbers. Give the analysis.

75. What is a common multiple of two or more numbers? What is a common multiple of 3 and 4? Of 2 and 7? Of 5 and 10?

76. What is a least common multiple? What is the least common multiple of 3 and 4? Of 6 and 9? Of 2 and 7? Give the rule for finding the least common multiple. Illustrate by an example.

77. What is cancellation? Illustrate by an example.

78. Upon what principle does it depend? Is a number changed by being both multiplied and divided by the same number? How is a multiplier known in arithmetic? ANS.—*By the sign of multiplication placed before it.* How is the divisor known? ANS.—*By the sign of division before it.* When a sign of division is placed before a parenthesis, is a part, or all, the numbers within it used as a divisor? ANS.—*All of them.* What are the numbers considered when placed below a horizontal line, as in the fractional sign of division? ANS.—*They are considered as divisors.*

79. What is a fraction? What is meant by $\frac{1}{4}$, $\frac{2}{3}$, $\frac{3}{4}$? Upon what does the size of the equal parts depend? Which is the larger, 4ths or 10ths? 3ds or 5ths?

80. What does the denominator show? To what does the denominator give a name?

81. What does the numerator show? In the following fractions, $\frac{2}{3}$, $\frac{4}{5}$, $\frac{3}{8}$, $\frac{5}{9}$, which is the denominator? Which the numerator?

82. What is meant by a proper fraction? Give an example.

83. Give an example of an improper fraction. Why so called? How does a proper fraction differ from an improper fraction?

84, 85. What is a mixed number? What sign is understood between the integer and the fraction?

How are fractions usually written? Give an example. What other method is sometimes used? Give an example.

What is meant by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$? What by $\frac{2}{3}$, $\frac{3}{4}$? When no definite number is expressed, what is considered the unit of a fraction? Ans. — 1.

86-88. What is meant by reducing a fraction to lower terms? To its lowest terms? Give the analysis of an example.

Upon what principle does it depend? Is the value of the fraction changed?

89. What is meant by changing integral or mixed numbers to improper fractions? Give the analysis of an example.

90. What is meant by changing improper fractions to integral or mixed numbers? Give the analysis of an example.

92. What is meant by reducing fractions to a common denominator? Is the value changed?

When are fractions said to have a common denominator? Give an example. Give the analysis of an example. Give a rule for reducing fractions to like denominators. Upon what principle does it depend? What does the least common multiple of the denominators of the several fractions indicate? Ans. — *I indicates the least common denominator of those fractions.*

93. What axioms are used in addition and subtraction of fractions? Upo what two axioms are the operations based?

Like fractions are what? Can fractions of different units be added or sub tracted? Could $\frac{3}{4}$ of a barrel be taken from $\frac{3}{4}$ of a pound? Could $\frac{2}{3}$ of a poun of flour be taken from $\frac{2}{3}$ of a bushel of potatoes? Could $\frac{5}{7}$ of a pound be take from $\frac{8}{9}$ of a pound? Could $\frac{2}{7}$ of an apple be added to $\frac{1}{3}$ of an apple?

94. Give the analysis of the method of finding the difference between two fractions. Give the rule.

95. How may a fraction be multiplied by an integer? How may a mixed number be multiplied by an integer? Why does multiplying the numerator multiply the fraction? Why does dividing the denominator by the integer multiply the fraction? Which method should be preferred? Should a mixed number ever be changed to an improper fraction before multiplying by an integer? Illustrate by an example, and give an analysis of the process.

96, 97. How are integers multiplied by fractions? How are mixed numbers multiplied by fractions? How are fractions multiplied by fractions? How fractions multiplied by mixed numbers? To what is multiplying by a fraction, Ans. — *To dividing by an integer equal to the denominator* *number is less than the divisor.* Illustrate by an example, and give an analysis of the process.

99, 100. How is an integer divided by a fraction? How is a fraction divided by a fraction? How is a mixed number divided by a fraction? How are mixed numbers divided by mixed numbers? What must be done to mixed numbers before dividing? Give a rule for dividing by a fraction or mixed number. Write an analysis of the process of an example in the division of fractions.

General Questions. In how many ways can fractions be reduced? Give rules for each. In how many ways can a fraction be multiplied? In how many ways can a fraction be divided?

DECIMAL FRACTIONS.

101. What is a decimal fraction? What is meant by a power of ten? Give an example of a decimal fraction. Of a common fraction.

102. What is a mixed decimal? What is placed between the integer and the decimal? How do the orders increase in the decimal scale? How do they decrease? Illustrate what is meant by a tenfold ratio.

103. How may decimals be written? Are decimals ever written like common fractions? Can they be added, subtracted, multiplied, and divided like common fractions? Can they be added to, or subtracted from, common fractions? Give an example.

104–106. How do the orders increase in U. S. Currency? How does the notation differ from decimals? To what do dimes correspond? Cents? Mills? Tenths of mills? Dollars? How is dollars' place indicated? How is units' indicated? How many hundredths make a tenth? How many tenths a unit? How many hundredths make a unit? How many cents make a dollar? How many thousandths make a unit? How many mills make a dollar? How many ten-thousandths make a unit? How many tenths of a mill make a dollar?

What is the denomination of the first decimal place? The third? The second? The sixth? The fourth? The fifth? How do the addition and subtraction of decimals differ from the addition and subtraction of integers?

107. Give the rule for the multiplication of decimals. Will the result be affected by alternating the factors?

108. A decimal of any order divided by the same order gives a quotient of what order? Units by units gives what? Tenths by tenths? Tens by tens? Hundredths by hundredths?

109. What is the rule for the division of decimals? How is the units' place determined? What must be done when the divisor is of an order less than that of the dividend? When the dividend is of an order less than the divisor?

110. What is the rule for changing common fractions to decimals? When will the decimal figure, or series of figures, repeat?

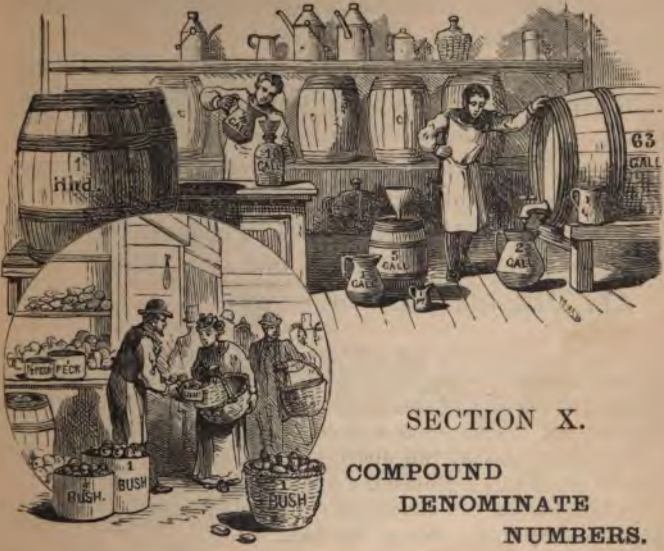
111. What is the rule for changing a decimal to a common fraction? When the decimal contains a fraction, what must be done?

112. When the price per 100 or 1000 is given, how can the cost be found? Give rule. Give the analysis.

ANALYSIS SECTION IX.

Decimal Fractions.

1. ILLUSTRATION.
2. DEFINITIONS.
3. NOTATION.
4. ADDITION AND SUBTRACTION.
    Axioms.
5. MULTIPLICATION.
    Rule.
6. DIVISION.
    (*a.*) *Principle.*
    (*b.*) *Rule.*
7. REDUCTION.
    (*a.*) *Common Fractions to Decimals.*
        Rule.
    (*b.*) *Decimals to Common Fractions.*
        Rule.
8. ABSTRACT PROBLEMS.
    (*a*) *Analysis.*
    (*b*) *Rule.*
9. CONCRETE PROBLEMS.
    Analysis.



SECTION X.

COMPOUND DENOMINATE NUMBERS.

IN buying and selling, the quantity of different articles is shown by counting, weighing, or measuring: thus some articles are bought and sold by the dozen or hundred; some, by the quart or gallon; some, by the peck or bushel, &c. The names — as foot, mile, gallon, or pound — used to designate certain quantities are called **denominations**; hence 4 yards, 9 pounds, are said to be *denominate numbers*. When a number representing a quantity is expressed in two or more denominations, it is said to be a *compound denominate number;* as, 4 yards, 2 feet, 5 inches.

Those denominations in a compound number, which express by comparison the greater amount, are said to be the *higher denominations;* those which express the less amount are said to be the *lower denominations:* thus a pound is a higher denomination than an ounce.

When numbers of a higher denomination are changed to a lower, without altering their values, the reduction is said to be *descending;* when changed from a lower to a higher, the reduction is said to be *ascending*.

113. *A Compound Denominate Number* is one expressing more than one denomination; as, 1 pound and 7 ounces.

114. *The Reduction of Compound Numbers* is the process of changing the denomination without altering the value; as, 1 gallon = 4 quarts or 8 pints.

115. *Reduction Descending* changes the number to a lower denomination; as, miles to inches.

116. *Reduction Ascending* changes the number to a higher denomination; as, inches to miles.

117. MEASUREMENT OF SPACE.

Space is extension in any direction: it may have three dimensions, or measurements, — length, breadth, and thickness. A *Solid*, or a *Volume*, is a portion of space which has three dimensions, — length, breadth, and thickness; as, a block, or a book.

A Surface has but two dimensions, — length and breadth. The leaf on which this is printed is a solid: the page is the surface.

A Line has but one dimension, — length. The boundary of a surface is a line; as, the edge of this page.

LINEAR MEASURE.

118. *The Measures of Length* are Long Measure and Surveyors' Measure. They are used for measuring distance.

TABLES OF LINEAR MEASURE.

LONG MEASURE.

12 inches (*in.*) = 1 foot (*ft.*).
3 feet = 1 yard (*yd.*).
5½ yards
(11 half yds.) } = 1 rod (*rd.*).
16½ feet
rods = 1 mile (*mi.*).

SURVEYORS' MEASURE.

7.92 inches = 1 link (*lk.*).
25 links = 1 rod (*rd.*).
100 links
4 rods } = 1 chain (*ch.*).
80 chains = 1 mile (*mi.*).

The following denominations are sometimes used: —

4 inches = 1 hand. Used to meas. height of horses.
3 feet = 1 pace. Used for stepping off distances.
6 feet = 1 fathom. Used to measure depths at sea.
1⅐ statute miles (nearly) = 1 geographic mile, or nautical knot.
60 geographic miles = 1 degree of latitude on equator.

TABLE OF EQUIVALENTS.

$$1\text{ mile} = \begin{cases} 80\text{ chains.} \\ 320\text{ rods.} \\ 1760\text{ yards.} \\ 5280\text{ feet.} \end{cases}$$

$$1\text{ chain} = \begin{cases} 4\text{ rods.} \\ 66\text{ feet.} \\ 100\text{ links.} \end{cases}$$

1 yard = 36 inches.

1 meter = 39.37 inches, or 1 yard, 3⅜ inches.

NOTES. — (1.) The surveyors' chain contains 100 links, and measurements are given in chains and links; thus 12.34 = 12 chains and 34 links.

(2.) Railroad engineers use a chain containing 100 links, each 1 foot long; and they record their measurements in miles and feet.

(3.) The yard, half-yard, quarter, eighth, and sixteenth are used in measuring cloth.

REDUCTION DESCENDING.

ORAL EXERCISE.

1. How many inches in 3 feet?

ANALYSIS. — Since in 1 foot there are 12 inches, in 3 feet there are 3 times 12 inches, or 36 inches.

2. How many yards in 3 rods?

ANALYSIS. — Since in 1 rod there are 5½ yards, in 3 rods there are 3 times 5½ yards, or 16½ yards. 3 times 5 yards are 15 yards; 3 times 1 half-yard are 3 half-yards, equal to 1½ yard; 15 yards plus 1½ yard equals 16½ yards.

3. How many inches in 5 feet? In 4 feet?

4. How many half-yards in 1 rod? In 2 rods?

FELTER'S

ARITHMETICAL DRILL CARDS.

TABLE VI.

STEPS.*	I.	II.	III.	IV.	V.	VI.
Linear Meas.	*ml.*	....	*rd.*	*yd.*	*ft.*	*in.*
	ml.	....	*rd.*	......	*ft.*	*in.*
	ml.	*ch.*	*rd.*	......	*lk.*	*in.*
	ml.	*ch.*	......	......	*lk.*	*in.*
Square Meas.	*sq. ml.*†	*A.*	*sq. rd.*	*sq. yd.*	*sq. ft.*	*sq. in.*
	sec.†	*A.*	*sq. ch.*	*sq. rd.*	*sq. lk.*	*sq. in.*
	sec.†	*A.*	*sq. ch.*	......	*sq. lk.*	*sq. in.*
Cubic Meas.	*cd.*	*cu. ft.*	......	......	......	*cu. in.*
	pch.	*cu. ft.*	......	......	......	*cu. in.*
	cu. yd.	*cu. ft.*	......	......	......	*cu. in.*
Fluid Meas.	*gal.*†	*qt.*	*pt.*†	*gi.*	......	*cu. in.*
	gal.	....	......	......	......	*cu. in.*
	cong.†	....	*O.*†	*f*℥.	*f*ʒ.	*m.*
Dry Meas.	*bu.*	....	......	......	......	*cu. in.*
	bu.	*pk.*	*qt.*	*pt.*	......	*cu. in.*
Troy Wt.	*lb.*	*oz.*	*pwt.*	......	*gr.*	......
	lb.	....	......	......	*gr.*	......
Apoth. Wt.	*lb.*	℥.	ʒ.	℈.	*gr.*	......
Avoirdu. Weight.	*T.*	*cwt.*	*lb.*	*oz.*	*gr.*	......
	T.	....	*lb.*	......	*gr.*	......
	T.	....	*lb.*	*oz.*	*gr.*	......
Time.	*yr.*	*mo.*	*da.*	*hr.*	*min.*	*sec.*
	yr.	*W.*	*da.*	*hr.*	*min.*	*sec.*
	yr.	....	*da.*	*hr.*	*min.*	*sec.*
Circular and Ang. Meas.	*cir.*	*quad.*†	*deg.*	*min.*	*sec.*	......
	cir.	*r. a.*†	*deg.*	*min.*	*sec.*	......
	cir.	*S.*	°	′	″	......
Paper.	*B.*	*bun.*	*rm.*	*qr.*	*sheet*	......
Currency	$	*ct.*	*m.*	......	......	mills
	F.	....	......	......	......	mills
	F.	*ct.*	*m.*	......	......	mills
	£	....	......	......	......	mills
	£	*S.*	*d.*	*far.*	......	mills

	V	W	X	Y	Z
A	2	20	127	4019	$\frac{1}{2}$
B	1	17	105	3806	$\frac{2}{5}$
C	3	86	420	2914	$\frac{3}{7}$
D	5	35	900	8637	$\frac{4}{9}$
E	7	42	109	5189	$\frac{5}{11}$
F	2	63	207	7933	$\frac{6}{13}$
G	9	87	350	2380	$\frac{7}{15}$
H	8	67	765	6734	$\frac{8}{17}$
I	4	95	903	2981	$\frac{9}{19}$
K	6	43	508	9171	$\frac{10}{21}$
L	10	23	343	2986	$\frac{11}{100}$
M	7	60	627	3941	$\frac{12}{1000}$
N	11	25	895	7243	$\frac{3}{10}$
O	4	37	546	8907	$\frac{14}{10}$
P	2	80	387	6000	$\frac{15}{100}$
Q	12	14	216	5900	$\frac{46}{10}$
R	9	28	950	3009	$\frac{7}{100}$
S	4	79	221	1957	$\frac{98}{100}$
T	5	75	333	9192	$\frac{9}{1000}$
U	3	40	575	8000	$\frac{20}{100}$

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* Indicates the number of steps in changing one denomination to another.

† Those denominations of the same weight or measure, in the same column, having different names, and marked with a dagger, are of the same value.

DRILL CARD EXERCISES. — TABLE VI.

Prepare the following Drill Card Exercises on slate or paper at seats, to be read as a class-exercise.

Exercise I. — Change **V** miles to chains. (Twenty examples.)
II. — Change **V** miles to rods.
III. — Reduce **V** rods to yards.

Directions. — Copy the number opposite each letter as miles, and reduce it to chains. Arrange the exercises as follows: —

Exercise I.	Exercise II.	Exercise III.
A. 2 mi. = 160 ch.	A. 2 mi. = 640 rds.	A. 2 rds. = 11 yds.
B. 1 " = 80 "	B. 1 " = 320 "	B. 1 " = 5½ "
C. 3 " = 240 "	C. 3 " = 960 "	C. 3 " = 16½ "
&c. to **U**.	&c. to **U**.	&c. to **U**.

N. B. — Under each table, the teacher should dictate Drill Card Exercises, similar to the following, until the pupils are prompt and accurate in their work.

REDUCTION DESCENDING.	REDUCTION ASCENDING.
Ex. I. — Reduce **W** rds. to feet. II. — Reduce **V** ml. to yds. III. — Reduce **W** ch. to feet.	Ex. IV. — Reduce **X** feet to rds. V. — Reduce **Y** in. to miles. VI. — Reduce **X** ch. to ml.
VII. — Change **V** sq. ml. to acres.	X. — Change **Y** sq. in. to sq. feet.
VIII. — Change **W** sec. to acres.	XI. — Change **Y** sq. yds. to acres.
IX. — Change **V** acres to sq. rods.	XII. — Change **X** sq. yds. to sq. rods.
XIII. — Reduce **V** cds. to cu. ft.	XVI. — Reduce **Y** cu. in to cu. feet.
XIV. — Reduce **V** pch. to cu. ft.	XVII. — Reduce **X** cu. ft. to cords.
XV. — Reduce **V** cu. yds. to cu. inches.	XVIII. — Reduce **W** cu. ft. to cu. yds.
XIX. — Change **V** gals to gills. XX. — Change **W** gal. to cu. in.	XXII. — Change **X** pts. to gals. XXIII. — Change **Y** cu. in. to gallons.
XXI. — Change **V** Cong. to m.	XXIV. — Change **X** gills to

EXAMPLES FOR PRACTICE.

ORAL AND WRITTEN.

1. How many yards in 426 rods?

PROCESS.

2)426(rds.)
$5\frac{1}{2}$ yds.
213 "
2130 "
2343 "

ANALYSIS.—Since in 1 rod there are $5\frac{1}{2}$ yards, in 426 rods there are 426 times $5\frac{1}{2}$ yards, or 2343 yards.

ANALYSIS (Process).—426 times $\frac{1}{2}$ yard = 426 half-yards = to 213 yards. 426 times 5 yards = 2130 yards, which, added to 213 yards = 2343 yards.

2. *W.* Reduce 75 rods to yards. To feet.

3. *W.* How many feet in 673 rods? In 783 yards?

How many feet in 2 yards of railroad-track? In 2 rods? *W.* In 26 miles?

PROCESS.

STEP I.
26 (mi.)
320 rds.
520 "
78
8320 (rds.)

STEP II.
8320 (rds.)
$5\frac{1}{2}$ yds.
4160 "
41600 "
45760 (yds.)

STEP III.
45760 (yds.)
3 ft.
137280 "

ANAL. STEPS.—(1.) *Change miles to rods.*
(2.) *Change rods to yards.*
(3.) *Change yards to feet.*

ANALYSIS.—(1.) Since in 1 mile there are 320 rods, in 26 miles there are 26 times 320 rods; or, 8320 rods.

(2.) Since in 1 rod there are $5\frac{1}{2}$ yards, in 8320 rods there are 8320 times $5\frac{1}{2}$ yards; or, 45760 yards.

(3.) Since in 1 yard there are 3 feet, in 45760 feet there are 45760 times 3 feet; or, 137280 feet.

4. How many inches in 8 feet? In 2 yards? *W.* In 643 feet? In 52 yards?

5. How many feet in 1 rod? In 1 rod and 1 yard? *W.* In 14 rods? In 23 rods and 6 yards?

6. How many feet in 1 rod of railroad track? In 2

REVIEW.—Define a unit. (1.) A number. (2.) A concrete number. (3.) A *denominate* number. (4.) An abstract number. (5.) Arithmetic treats of what? (6.) *What is notation?* (7.) Name the methods in use. Which method uses? (8.) *For* what purpose used?

rods? *W.* In 320 rods? In 7 miles? In 3 miles and 1 rod?

7. *W.* How many ties will it take for 24 miles of track, if every 3 feet requires one tie?

8. How many yards of carpet will it take to cover a hall 18 feet long, if the width requires 2 breadths? *W.* If the hall is 156 feet long, and the width requires 3 breadths? What will the carpet cost, at $\$1\frac{3}{4}$ per yard?

9. *W.* From New York to Binghamton is 215 miles: how many railroad-ties will it take to lay the track, if they are placed 3 feet apart? What will they cost, at 8 cents each?

10. *W.* There is a door-yard 2 rods long and 2 rods wide: how many feet of fence will it take to go around it? If it is 4 rods long and 2 rods wide? If it is 12 rods long and 8 rods wide?

11. *W.* There is a barn-yard 6 rods square: how many feet around it? How many boards $16\frac{1}{2}$ feet long will it take to fence it, if the fence is 4 boards high? What will the boards cost, at 12 cents each?

12. *W.* I have a room 24 feet long by 15 feet wide: how many feet of paper bordering will it take to go around the walls? How many yards will it make? What will it cost, at 3 cents a yard?

13. *W.* How many panels of fence, $16\frac{1}{2}$ feet each, will it take to fence a lot 26 chains, 25 links long, and 12 chains and 50 links wide? How much will it cost, at $1.87 per panel?

REVIEW.—What is the effect of repeating a letter in the Roman notation? (8.) Of placing a letter of less value before one of greater value? Of placing a letter of less value after one of greater value? Of placing a dash over a letter, or combination of letters? How does the Arabic notation differ from the Roman? (9.) How many characters are used? How is the number fifteen expressed?

14. *W.* My farm is 23 chains, 50 links wide: how many steps across it, if each step measures 3 feet?

REDUCTION ASCENDING.

ORAL EXERCISES.

1. How many feet in 14 inches? In 25 inches?

FORMULA. 14 inches ÷ 12 inches = 1 (foot) + 2 inches.

ANALYSIS. — Since 12 inches = 1 foot, 14 inches = as many feet as 12 inches are contained times in 14 inches, which is once, with a remainder of 2 inches.

2. How many yards in 6 feet? In 9 feet? In 18 feet? In 24 feet?

3. How many yards in 14 feet? In 22 feet? In 13 feet? In 25 feet?

4. How many half-yards in 3 yards? In 17 yards? In 18 yards? In 7 yards?

5. How many rods in 11 half-yards? In 22 half-yards? In 33 half-yards?

6. How many rods in 648 inches?

FORMULA. (1.) 648 inches ÷ 12 inches = 54 (feet).*
(2.) 54 feet ÷ 3 feet = 18 (yards).
(3.) 2 half-yards × 18 (yards*) = 36 half-yards.
(4.) 36 half-yards ÷ 11 half-yards = 3 (rods*) + 1½ half-yards.

ANALYSIS. — (1.) Since 12 inches = 1 foot, 648 inches = as many feet as 12 inches are contained times in 648 inches; or, 54 feet.

(2.) Since 3 feet = 1 yard, 54 feet = as many yards as 3 feet are contained times in 54 feet; or, 18 yards.

(3.) Since 1 yard = 2 half-yards, 18 yards = 18 times 2 half-yards; or, 36 half-yards.

(4.) Since 11 half-yards (5½ yards) = 1 rod, 36 half-yards = as many rods as 11 half-yards are contained times in 36 half-yards; or, 3 rods and 3½ yards; or, 3 rods, 1 yard, 1 foot, 6 inches.

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* The numbers having the denomination enclosed in parentheses are used as abstract in the analysis.

7. How many rods in 252 inches? In 1008 inches? In 39 feet? In 76 feet?

8. How many rods in 80 chains? In 50 chains? In 70 chains? In 90 chains?

9. How many rods in 75 links? In 87 links?

ORAL AND WRITTEN EXERCISES.

PROBLEM. — How many miles in 14267 feet?

PROCESS.

STEP I. 3 ft.)14267 ft.
4755 (yds.) + 2 ft.
2 h. yds. **STEP II.**
STEP III. 11 h. yds.)9510 h. yds.
320 rds.)864 (rds.) + (6 h. yds.) 3 yds. **STEP IV.**
2 mi. + 224 rds.
14267 ft. = 2 mi. 224 rds, 3 yds. 2 ft.

ANAL. STEPS. — (1.) *Change feet to yards.*
* (2.) *Change yards to half-yards.*
(3.) *Change half-yards to rods.*
(4.) *Change rods to miles.*

ANALYSIS. — (1.) Since 3 feet = 1 yard, 14267 feet = as many yards as 3 feet are contained times in 14267 feet; or, 4755 (yds.) with 2 feet remainder.

(2.) Since 1 yard = 2 half-yards, 4755 yards = 4755 times 2 half-yards; or, 9510 half-yards.

(3.) Since 11 half-yards ($5\frac{1}{2}$ yds.) = 1 rod, 9510 half-yards = as many rods as 11 half-yards are contained times in 9510 half-yards; or, 864 (rds.) with (6 h. yds.) 3 yards remainder.

(4.) Since 320 rods = 1 mile, 864 rods = as many miles as 320 rods are contained times in 864 rods; or, 2 (mi.) with a remainder of 224 rods.

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* Pupils who have studied fractions may, in this and all similar cases, divide by the *mixed number*, as in division of fractions.

PROOF.

STEP I.	2 mi. + 224 rds. + 3 yds. + 2 ft.	
	320	(rds.)
	640	"
	224	"
	2)864	(rds.)
STEP II.	$5\frac{1}{2}$	(yds.)
	432	(yds.)
	4320	"
	4752	"
	3	"
	4755	(yds.)
STEP III.	3	(ft.)
	14265	(ft.)
	2	"
	14267	"

ANAL. STEPS. — (1.) *Change miles to rods, and add rods.*

(2.) *Change rods to yards, and add rods.*

(3.) *Change yards to feet, and add feet.*

ANALYSIS. — NOTE TO TEACHER. — The teacher should rigidly hold the pupils to a correct analysis of each step, and not allow them, by carelessness of expression, to fall into the fallacy of multiplying rods by yards, &c. The denominations are affixed to the numbers to aid the memory : the analysis alone determines which are used as abstract.

15. How many feet in 24 inches? In 36 inches? In 72 inches? In 39 inches? *W.* In 367 inches? In 872 inches?

16. How many yards in 6 feet? In 36 inches? In 47 inches? In 25 feet? In 27 feet? *W.* In 2347 feet? In 9673 inches?

17. How many rods in $16\frac{1}{2}$ feet? In 33 feet? In 11 yards? *W.* In 27 yards? In 2473 feet? In 138 yards?

18. *W.* At 30 cents a foot, what will 13 yards 2 feet of wire railing cost? At 40 cents? At $1.13?

19. *W.* At 88 cents a rod, what will 80.75 chains of fence cost? 63.25 chains? 47.50 chains?

20. *W.* What will it cost to carpet a room 51 feet long, if it takes 4 breadths of carpet to cover the floor, at $1.75 per yard? At $2.25 per yard?

21. What will 5 feet 9 inches of wire railing cost, at $1.50 per foot?

PROCESS.

12)$1.500 × 5 = $7.500 cost of 5 feet.
.125 × 9 = 1.125 cost of 9 in.
$8.625 total cost.

Anal. Steps.—(1.) *Find cost of* 5 *feet.*
(2.) *Find cost of* 1 *inch.*
(3.) *Find cost of* 9 *inches.*
(4.) *Find total cost.*

Analysis.—(1.) Since 1 ft. costs $1.50, 5 ft. will cost 5 times as much; or, $7.50.

(2.) Since 12 inches (1 ft.) costs $1.50, one inch will cost one-twelfth as much; or, $.125.

(3.) Since 1 inch costs $.125, nine inches will cost 9 times as much; or, $1.125.

(4.) The total cost will be the sum of $7.50 and $1.125; or, $8.625.

22. What will 4 yards and 1 foot of brass wire cost, at 12 cents a yard? *W.* What will 17 yards 2 feet cost, at 18 cents a yard?

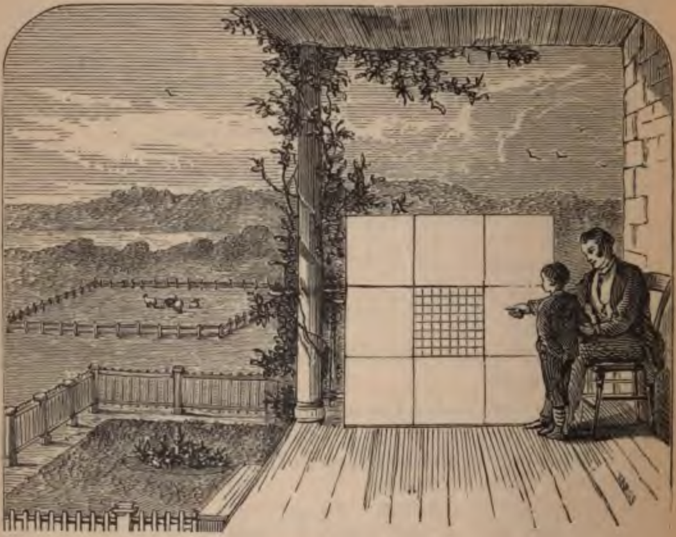
23. At 36 cents a yard, what will 2 feet of cloth cost? *W.* What will 7 yards, 6 inches cost? 14 yards, 2 feet, 6 inches? 13 yards, 2 feet?

24. *W.* At $1.80 a rod, what will 7 rods and 2 yards of stone fence cost? 4 rods and 4 yards?

25. *W.* What will it cost to fence a piece of land 18.24 chains long and 11.38 chains wide, at $1.10 per rod?

Review.—What is meant by the simple value of a figure? (10.) By the local value of a figure? (11.) *What is numeration?* (12.) What is the use of ciphers *in writing a number? Repeat* the numeration table by periods from right to *left. From left to right.*

SQUARE MEASURE.



If you will take a piece of paper, like the one on the board in the picture, one *foot* square, and divide it into 12 rows of squares, with 12 squares in each row, making 144 pieces of equal size, each piece will contain just one *square inch*. It will take 9 square feet of paper to cover the surface of the board, which contains a *square yard*. The grass-plat in the door-yard is 30¼ times as large as the surface of the board, and contains one *square rod*. The field in which you see the sheep is 160 times as large as the grass-plot, and contains one *acre of land*.

119. ***Square Measure*** is used for calculating areas of land, roofing, flooring, paving, painting, &c.

Review.—How may units of any order make one of the next higher? (13.) How does a change of one place to the right affect the value of a digit? How does a change of one place to the left affect its value?

SURFACE MEASURE.

144 square inches (*sq. in.*) = 1 square foot (*sq. ft.*).
9 square feet = 1 square yard (*sq. yd.*).
$30\frac{1}{4}$ square yards (121 quarter square yards) = 1 square rod or perch (*sq. rd.* or *pch.*).
160 square rods = 1 acre (*A.*).

SURVEYORS' SQUARE MEASURE.

625 square links (*sq. lk.*) = 1 square rod or perch (*sq. rd.* or *pch.*).
16 square rods = 1 square chain (*sq. ch.*).
10 square chains = 1 acre (*A.*).
640 acres = 1 section (*sec.*).
36 sections = 1 township (*T.*)

NOTE. — A section is a piece of land a mile square, and is equal to a square mile.

TABLE OF EQUIVALENTS.

1 A. = 160 sq. rds. = 4840 sq. yds. = 43560 sq. ft. = 6272640 sq. in.
1 sq rd. = $30\frac{1}{4}$ " = $272\frac{1}{4}$ " = 39204 "
1 sq. yd. = 9 " = 1296 "

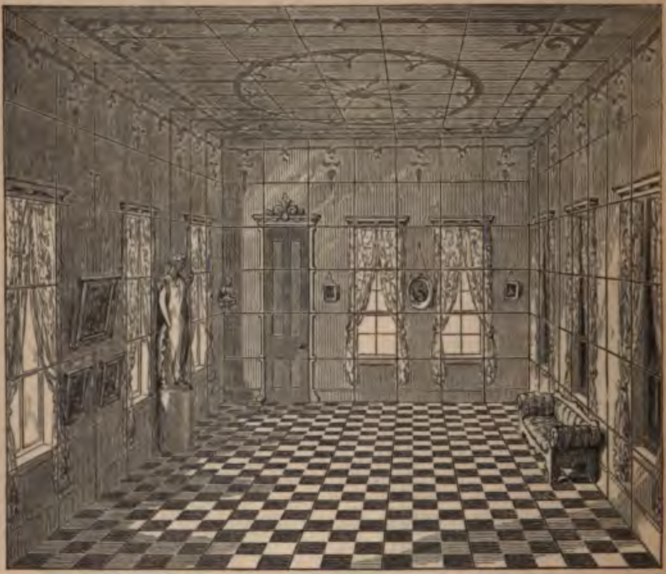
SLATE AND BLACKBOARD EXERCISES.

(See Drill Card, Table VI.)

Exercise. I. — Reduce **V** acres to square rods.
II. — Reduce **V** square rods to square yards.
III. — Reduce **V** square yards to square feet.
IV. — Reduce **V** square feet to square inches.
V. — Reduce **W** square rods to square chains.
VI. — Reduce **W** square chains to acres.
VII. — Reduce **W** acres to sections.
VIII. — Reduce **W** sections to townships.

Prepare the above exercises in the same manner as on page 205, the results *to be read* as a class-exercise.

ANALYSIS OF SQUARE MEASURE.



1. In the above illustration is seen a room 24 feet long, 21 feet wide, and 18 feet high: what is the size of the room in yards? How many square yards in the first row on the right side of the room? How many square yards in the entire floor?

ANALYSIS.—Since in 1 row (of square yards) there are 8 square yards, in 7 rows there are 7 times 8 square yards; or, 56 square yards.

2. How many square yards in the right hand wall? How many square yards in the left hand wall? How many in the end wall? How many in the ceiling? How many walls in a room? How many floors? How many ceilings?

3. How many square yards in the two side walls? How many in the two end walls? How many in all the walls taken together?

4. Since there are 9 tiles one foot square in each square yard, how many tiles in one row of square yards? How many in the entire floor?

ANALYSIS. — (1.) Since in 1 square yard there are 9 tiles, in 8 square yards (1 row) there are 8 times 9 tiles; or, 72 tiles.

(2.) Since in 1 row of 8 yards there are 72 tiles, in 7 rows (the entire floor) there are 7 times 72 tiles; or, 504 tiles.

120. TO FIND THE AREA OF A SQUARE OR PARALLELOGRAM: —

RULE. — *Multiply the length by the breadth, and the product will be the area.*

NOTE. — Usage, and the want of any other equally terse form of language, sanction this rule. Logically it is not true, any more than multiplying deer by grain will produce buckwheat. If the argument of the above paragraphs is thoroughly understood, the form of the rule can do no harm.

Artificers usually estimate their work —

1. By the square foot; as in stone-cutting.
2. By the square yard; as in papering, lathing, plastering, paper-hanging, &c.
3. By the 100 square feet; as in flooring, roofing, siding, slating, &c.
4. By the thousand bricks, or by the square yard; as in bricklaying.

WRITTEN EXERCISES.

26. How many square inches in a cardboard 5 inches long and 3 inches wide?

FORMULA. 5 square inches × 3 (rows) = 15 square inches.

ANALYSIS. — Since in one row (5 inches long) there are 5 square inches, in 3 rows (3 inches wide) there are 3 times 5 square inches; or, 15 square inches.

REVIEW. — *Define a rule.* (14.) Recite the rule for notation. (15.) Recite the *rule for numeration.* (16.) Define addition. (17.)

27. How many square inches in a board 26 inches long by 9 inches wide? 40 inches by 13 inches?

28. How many square feet in a plat of ground 75 feet by 100? 85 by 76 feet? 67 by 86 feet?

29. How many square inches in a board 108 inches long by 12 inches wide? How many square feet?

30. How many square feet in a pavement 156 feet long and 8 feet wide? How many square yards?

31. How many square feet in the side wall of a room 15 feet long and 8 feet high? How many square yards?

32. How many square feet in the floor of a room 36 feet long and 18 feet wide? How many square yards?

33. How many square feet in the ceiling of a room 81 feet long and 18 feet wide? How many square yards?

34. How many square feet in the two end walls of a room 25 feet wide and 8 feet high? How many square yards?

35. How many square yards in the four walls of a room 44 feet long, 18 feet wide, and 11 feet high?

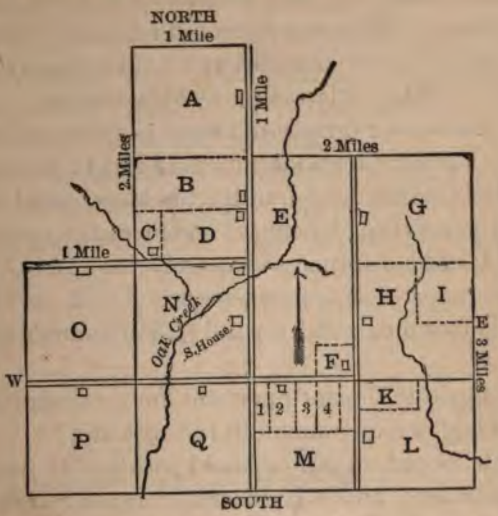
36. How many square yards in the ceiling and walls of a room 26 feet long, 18 feet wide, and 12 feet high?

37. How much will it cost to lay the tiles on the floor of a room 24 feet long and 21 feet wide, at 50 cents per square yard? (See illustration.)

38. How many square yards of paper will it take to paper the walls of the same room, the ceiling being 18 feet in height? What will it cost, at 18 cents per square yard, no allowance being made for windows or doors?

Review.—*Define* "the sum" in addition. (18.) Describe the sign of addition, *and illustrate the use.* (19.) Describe the sign of equality, and give its use. (20.) *Describe the dollar-sign, and give its use.* (21.) What is an axiom? (22.) Write *the analysis of a* short example in addition. (28.)

39. What will it cost to paint the ceiling of the same room, at $1.14 per square yard?



40. The above is the map of Oakdale school district. A.'s farm lies in the square in the northern part. The north line is one mile long, and the east line is 80 chains: how many acres in his farm? What is it worth, at $\$33\frac{1}{3}$ per acre?

41. B.'s farm is 80 chains long, and contains 320 acres: how wide is it? What is it worth, at $35.20 per acre?

42. D. had a farm of the same shape and size as A.; but he sold to B. 320 acres, the north half: how wide was the remainder? He then sold to C. a piece 40 chains long and 80 rods wide, from the west part of what remained: how many acres did C. buy? How many acres did D. sell in all? How much did he receive for it, at $15.75 per acre?

43. E. has the largest farm in the district: it was 2 miles long and 1 mile wide before he sold to F. a piece of land 35 rods long and 8 chains wide from the south-east corner. How many acres did he sell? How many acres had he before he sold any? How many acres had he left? What is it worth, at $10 per acre.

44. G. owns a farm containing a section of land situated in a square, for which he paid $18.42 per acre. He paid 1928 dollars for building his house and barn, and $0.87½ per rod for fencing it: what was the entire cost?

45. H. had a farm in shape and size like G.; but he sold the north-east quarter-section to I., at $22⅓ per acre. How much did he sell? How much should he receive?

46. I. paid 87 cents per rod for enclosing his land with a hedge: how much did it cost him?

47. K. bought a strip of land south of H., which was 160 rods long and 45 rods wide, located in the north-west part of the south-east section of the district: how much will it cost him to fence it, at 96 cents per rod? How much is L.'s land worth, at $31.35 per acre, including buildings?

48. M. owns a section of land, and he wishes to divide the north half into 5 equal parts: how wide a front will each piece have on the road at the north? How many acres will each contain? What will the first be worth, at $17 per acre? The second, at $42.30 per acre? The third, at $14 per acre? The fourth and fifth, at $9.75 per acre? What is the whole north half worth? What is the south half worth, at $11 per acre? He

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REVIEW.—Define subtraction. (31.) The subtrahend. (32.) The minuend. *(33.) The remainder.* (34.) Describe the minus-sign. (35.) Give the axioms. (36.) *Give the analysis of a short problem.* (37.)

aid $3000 for the whole farm: is the present value nore or less than the cost, and how much? What is he gain or loss?

49. N. owns a section, and O. owns the section directly west: how much will it cost each to surround his land with fence at 90 cents per rod, supposing each to make is half only of the line* fences?

50. How many acres in the entire district?

51. The school district is assessed as follows: —

Mr.	A.	at	$7	per	acre.	Mr.	I.	at	$8	per	acre.
"	B.	"	$8\frac{1}{2}$ [†]	"	"	"	K.	"	$8\frac{1}{2}$	"	"
"	C.	"	9	"	"	"	L.	"	5	"	"
"	D.	"	$6\frac{3}{4}$	"	"	"	M.	"	$9\frac{1}{2}$	"	"
"	E.	"	$4\frac{1}{2}$	"	"	"	N.	"	7	"	"
"	F.	"	11	"	"	"	O.	"	7	"	"
"	G.	"	$9\frac{1}{3}$	"	"	"	P.	"	$7\frac{1}{2}$	"	"
"	H.	"	$9\frac{7}{8}$	"	"	"	Q.	"	4	"	"

What is the assessed value of the land of the entire district?

52. What is the average assessed value of the district per acre?

SUGGESTION. — Divide the entire valuation by the number of acres in the district.

53. What will a board 6 feet 3 inches (75 in.) long, and 7 inches wide, cost, at $.288 a square foot?

FORMULA. (1.) 6 ft. 3 in. = 75 in.; 75 sq. in. $\times$ 7 = 525 sq. in.
(2.) 288 mills $\div$ 144 = 2 mills.
(3.) 2 mills $\times$ 525 = $1.05.

54. *W.* At $1.30 per square yard, what will it cost to pave a walk 105 feet long and 7 feet wide? 47 feet long by 9 feet wide?

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* Road fences are not considered line fences.

[†] *Change the fractions to cents and mills.*

55. *W.* What will it cost to paint the floor of a room 35 feet long and 18 feet wide, at 40 cents per square yard? To paint a floor 55 feet long by 20 feet wide, at the same price?

56. *W.* What will it cost to plaster the walls and ceiling of a room 18 feet long, 16 feet wide, and 9 feet high, at $1.20 per square yard?

57. *W.* What will a piece of land 28 chains long, and 5 chains wide, cost, at $14.72 per acre?

58. How many square feet in the floor of a room 16 feet long and 8 feet wide? *W.* 13 feet long and 17 feet wide? 25 feet long and 17 feet wide?

59. How many square feet in the ceiling of a room 9 feet long and 8 feet wide? *W.* 14 feet long and 12 feet wide? 14 feet long and 11 feet wide?

60. *W.* How many square feet in the end walls of a room 25 feet long, 15 feet wide, and 8 feet high? 22 feet long, 11 feet wide, and 9 feet high?

61. *W.* How many square feet in the side walls of a room 17 feet long, 13 feet wide, and 9 feet high? In a room 14 feet long, $12\frac{2}{3}$ feet wide, and 7 feet high?

62. *W.* How much will it cost to plaster the walls and ceiling of a room 24 feet long, 15 feet wide, and 9 feet high, at 80 cents per square yard?

63. *W.* What will it cost to floor a room 16 feet long, and 12 feet wide, and $9\frac{2}{3}$ feet high, at $12.50 per square of 100 feet?

64. What will it cost to paint the outside of a house 46 feet long, 27 feet wide, and 20 feet high, at $1.20 per square yard?

CUBIC MEASURE.

121. *Cubic Measure* is used in estimating the contents of solids; as, wood, masonry, the capacity of cellars, cisterns, &c.

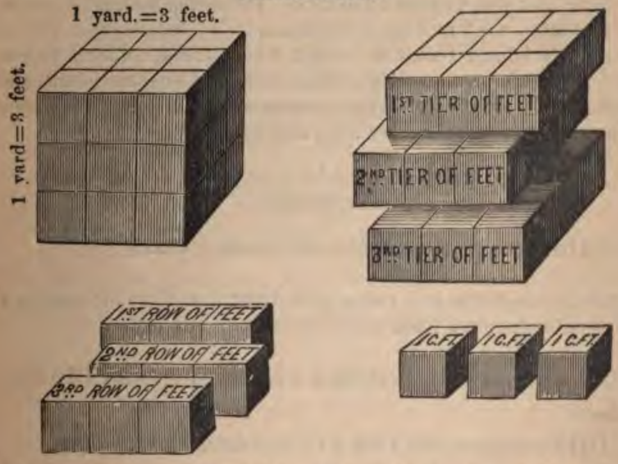
TABLE OF CUBIC MEASURE.

1728 cubic inches (*cu. in.*) = 1 cubic foot (*cu. ft.*).
27 cubic feet = 1 cubic yard (*cu. yd.*).
128 cubic feet = 1 cord (*cd.*).
$24\frac{3}{4}$ cubic feet = 1 perch of stone (*pch.*).

NOTE. — 99 quarter cubic feet make 1 perch of stone.

ANALYSIS OF CUBIC MEASURE.

ILLUSTRATION.



ANAL. STEPS. — (1.) *Find the number of rows of cu. ft. in* 1 *cu. yd.*

ANALYSIS. — Since there are 3 *rows* of cubic feet in 1 *tier* of feet, in 3 *tiers*, there are 3 times 3 *rows*, which is 9 *rows*.

(2.) *Find the number of cubic feet in* 1 *cubic yard.*

ANALYSIS. — Since in 1 row there are 3 cubic feet, in 9 rows, there *are 9 times 3 cubic feet,* which is 27 cubic feet.

122. To find the solid contents of a body, we have the following

RULE. — *Multiply the length by the breadth, and that product by the thickness.*

NOTES. — (1.) A load of earth contains a cubic yard, and weighs about 3250 pounds. A cubic foot of water weighs 62½ pounds.

(2.) Railway and transportation companies estimate light freight by the number of cubic feet it occupies; but heavy freight is estimated by weight.

(3). A pile of wood 4 feet wide, 4 feet high, and 8 feet long, contains 1 cord; and a cord-foot is 1 foot in length of such a pile.

(4.) A brick is usually 8 inches long, 4 inches wide, and 2¼ inches thick.

(5.) Sawed lumber, as joists, plank, and scantlings, are now generally bought and sold by board measure. The dimensions of a foot of board measure are 1 foot long, 1 foot wide, and 1 inch thick. Thus a plank 12 feet long, 1 foot wide, and 2 inches thick, contains 24 feet board measure, or as much as two boards of the same length and breadth, and 1 inch thick. But boards that are less than 1 inch thick are measured as if they were an inch thick.

ORAL EXERCISES.

1. How many cubic feet in 2 cubic yards?

ANALYSIS. — Since in 1 cubic yard there are 27 cubic feet, in 2 cubic yards, there are 2 times as many; or, 54 cubic feet.

2. How many cubic inches in 1 cubic foot and 19 cubic inches?
3. How many cubic feet in 2 cords of wood?
4. How many cubic feet in 1 cord and 17 cubic feet?
5. How many perch of stone in 99 cubic feet?

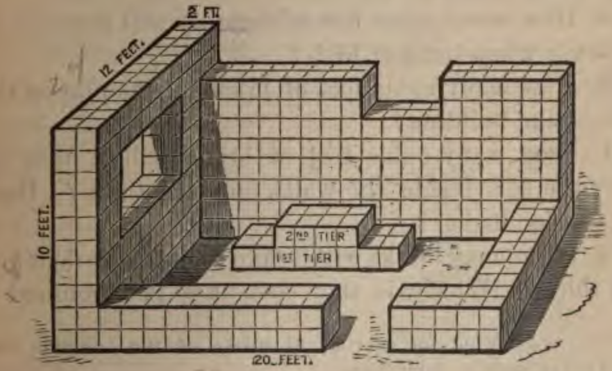
REVIEW. — *Give the use of the parenthesis.* (39.) Define multiplication. (40. *The multiplicand.* (41.) The multiplier. (42.) The product. (43.) Which are *factors?* (44.) Describe the sign of multiplication, and illustrate its use. (45.) Giv *the axiom.* (46.) Give the principles. (47.)

6. How many cubic inch blocks will it take to fill a mortise one foot long, 3 inches wide, and 6 inches deep?

7. How many quarter cubic inch blocks will it take to fill a mortise one inch each way? One foot each way?

8. How many cubic feet in a block of stone 5 feet long, 3 feet wide, and 3 feet thick?

8. What is the difference in volume between a cube 5 inches on each edge, and 5 cubic inches?


2 FT.
12 FEET.
10 FEET.
2ND TIER
1ST TIER
20 FEET.

In the above illustration will be seen a part of the walls and foundation of a building 20 feet long, 12 feet wide, and 10 feet high, the walls being 2 feet thick.

WRITTEN EXERCISES.

65. How many cubic feet in the left-hand wall?

Formula (1.) 12 cubic feet × 2 = 24 cubic feet.
(2.) 24 cubic feet × 10 = 240 cubic feet.

Analysis. — (1.) Since in one row (12 feet long) there are 12 cubic feet, in two rows (2 feet thick), there are 2 times as many, or 24 cubic feet.

(2.) Since in 1 tier (12 feet long by 2 feet thick) there are 24 cubic feet, in 10 tiers (10 feet high), there are 10 times as many, or 240 cubic feet.

NOTE.—In estimating the contents of walls or masonry, no allowance is made for windows, doorways, or corners. All measurements are understood as being outside measurements, unless otherwise specified.

66. How many cubic feet in the highest side wall?

67. How many cubic feet in the foundation of the lowest end wall? In the foundation of the lowest side wall, making a deduction for the doorway?

68. How many cubic feet in the walls of the building as they now stand?

69. How many cubic feet of masonry will there be in the walls when ten feet high?

70. How many cubic feet of bricks in the pile on the floor of the building?

71. How many cubic feet of bricks will it take to cover the floor inside the walls, one foot thick? How many cubic inches?

72. How many bricks will it take to make the floor one foot thick? What will they cost, at $5 per thousand?

NOTE.—Allowing for mortar, 21 bricks of the common size $(8 \times 4 \times 2\frac{1}{4})$ make a cubic foot.

73. How many bricks will it take to build the walls as they now stand? What will they cost, at $7.30 per thousand?

74. How many bricks will it take to build the walls 10 feet high? How much will it cost, including labor, at $11.20 per thousand?

REVIEW.—Write the analysis of a short example in multiplication. (48.) How may a number be multiplied by 10, 100, 1000, &c? (49.) Give table of U.S. currency. (50.) Give the definition of division. (51.) Of dividend. (52.) Of divisor. (53). Of quotient. (54.) Of remainder. (55.) Give use of the sign. (56.) Of the fractional sign. (57.) How does short division differ from long division? (58, 59.)

75. How many perch of stone would be required to make the walls 10 feet high?

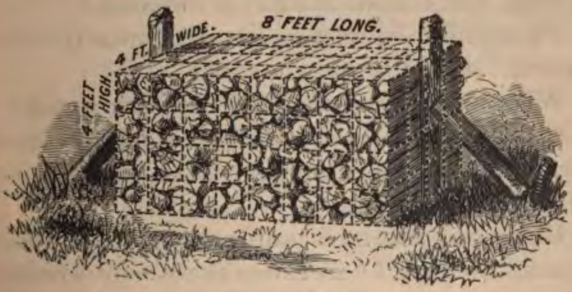
SUGGESTION. — Change the cubic feet to quarter cubic feet; then quarter cubic feet to perch.

76. Suppose the walls completed 12 feet high, how many perch of stone would they contain?

77. Suppose the walls to be built 10 feet high, how many cubic feet of wood would it take to fill the inside of the building level with the top?

78. How many cubic inch blocks would it take to fill it?

79. 231 cubic inch blocks are equal in volume to a gallon of water: how many gallons of water will fill a cistern of the same size?


4 FT. WIDE.
8 FEET LONG.
4 FEET HIGH.

CORD OF WOOD.

Arrange 128 cubic foot blocks into a pile 8 feet long, 4 feet wide, and 4 feet high, and it will represent the shape and size of a *cord.* One foot of the length of such a pile is called a *cord foot,* and contains 16 cubic feet. 8 cord feet make a cord.

REVIEW. — Give the axiom used in division. (60.) Name the principles. (61.) Write the analysis of an example in short division. (62.) How may a number be divided by 10, 100, 1000, &c.? (63.) Write the analysis of a short example in long division. (64.) Give the table of U. S. currency. (65.) Give the table of aliquot parts. (66.) Of what are coins made? (67.) What can you say about the coins? Of what *does paper money consist?* (68.) What is a bill, or invoice? (69.) Which *is the debtor? which the creditor?* What is an account?

80. How many cords of bark in a pile 12 feet long, 12 feet high, and 8 feet wide?

ANAL. STEPS. — (1.) *Find number of cubic feet in the pile.*
(2.) *Find number of cords.*

81. How many cords in a pile of wood 36 feet long, 4 feet wide, and 3 feet high? What will it be worth, at $3.75 per cord?

82. How much will a load of wood 16 feet long, 6 feet wide, and 3 feet high, cost, at $5.25 per cord?

83. Give the cost of a load of wood 13 feet long, 5 feet wide, 5 feet high, at 8\frac{3}{4}$ per cord?

84. There is a barn 40 feet long, 30 feet wide, and 15 feet high: how many feet of 2 inch plank will it take to floor it?

85. What will it cost, at $25 per thousand?

86. The rafters are 20 feet long: how many feet of boards will it take to cover the roof?

87. What will they cost, at $19.25 per thousand?

88. How many feet of boards will it take to enclose the barn, not including the gables? and what will it cost, at $23.33 per thousand?

89. How many feet of boards will it take to enclose the gables, it being 11 feet from the plate on which the rafters rest to the peak of the roof? and what will it cost, at $25 per thousand?

SUGGESTION. — Observe that the gables form two halves of a space 15 feet on one side, and 11 feet on the other.

REVIEW. — Define a compound denominate number. (113.) What is the reduction of denominate numbers? (114.) What is reduction descending? (115.) Reduction ascending? (116.) What is meant by space? (117.) By a solid? By a surface? By a line? What are the measures of length? (118.) For what are they used? Give the table of long measure. Give the table of surveyors' measure. How many links in a surveyor's chain? How long are they? How does it differ from the chain used by railroad engineers? How many half-yards make a yard? How many quarters? How many eighths? For what is this measure used?

90. How much will it cost to shingle the above barn, ach shingle covering 6 square inches, at $1.75 per housand shingles.

91. What is the estimated cost of the lumber of the bove building?

LIQUID MEASURE.

123. *Liquid Measure* is used for measuring liquids.

TABLE.

gills (*gi.*) = 1 pint (*pt.*).
pints = 1 quart (*qt.*).
quarts = 1 gallon (*gal.*).

TABLE OF EQUIVALENTS.

gal. *qt.* *pt.* *gi.*
1 = 4 = 8 = 32
1 = 2 = 8

PINT.

GILL.

QUART.

GALLON.

NOTE. — In calculating the capacity of cisterns, a hogshead is estimated to hold 63 gallons.

ORAL AND WRITTEN EXERCISES.

92. How many gills in 3 pints? In 2 quarts? In 2 gallons? In 1 gallon and 3 quarts? In 3 quarts and 1 pint?

93. How many gallons in 12 quarts? In 24 quarts? In 72 gills? *W.* In 463 quarts? In 463 gills?

94. How many pints in 13 quarts of berries? How much are they worth at 5 cents a pint?

95. *W.* How many quarts of milk in 43 gallons? How much is it worth at 9 cents a quart?

REVIEW. — For what is square measure used? (119.) Give the table of surface measure. Of surveyors' square measure. What is meant by a section of land? Give the *rule for finding* the area of a square or parallelogram. (120.) How do *artificers estimate their work?*

96. *W.* I have three cows, each gives 4 gallons of milk a day: how much milk will they yield in a week? How much would it be worth at 8 cents a quart?

DRY MEASURE.

124. *Dry Measure* is used for measuring grain, fruit, and for estimating the capacity of boxes, bins, corn-cribs, &c.

PECK.

QUART.

PINT.

BUSHEL.

TABLE.

2 pints (*pt.*) = 1 quart (*qt.*).
8 quarts = 1 peck (*pk.*).
4 pecks = 1 bushel (*bu.*).

TABLE OF EQUIVALENTS.

bu. *pks.* *qts.* *pts.*
1 = 4 = 32 = 64
1 = 8 = 16

TABLE OF COMPARISON.

1 gal. liquid mea. = 231 cu. in.
1 qt. " = 58 "
1 qt. dry measure = 67+ "
1 litre = 61 "

1 bushel = 2150 cu. in
1 peck = 537½ "
8 qts. liq. mea. = 462
1 hectolitre = 6102+ "

NOTES—(1.) A bushel measure is 18½ inches in diameter, and 8 inches deep: it contains 2150.42 cubic inches.

(2.) In measuring grain, seeds, peas, beans, and small fruit, the measure must be *even* full. In measuring large fruits,—as apples, pears; coarse vegetables, as potatoes, beets, and other bulky articles,—the measure must be *heaping* full. Four heaped measures should equal five even measures. Five gills liquid measure make about one pint dry measure.

REVIEW.—Give the use of cubic measure. (121.) Write out the table. How many steps in changing cords to cubic inches? Cubic inches to cords? Give the rule for finding the contents of a cubic body. (122.) About how many pounds does a cubic yard of earth weigh? How much does a cubic foot of water weigh? What are the dimensions of a cord of wood?

ORAL AND WRITTEN EXERCISES.

97. How many pints in a peck? In 3 pecks? *W.* In 75 pecks? In 47 bushels?

98. In 80 quarts how many pints? How many pecks? How many bushels?

99. At 6 cents a pint, what will be the cost of a peck of chestnuts? *W.* of 1 bushel? Of 138 bushels?

100. *W.* I bought a bushel of strawberries for $6.40, and sold them at 40 cents a quart: what did I gain?

101. *W.* I bought a barrel of apples, containing 2 bushels and 3 pecks, and sold them at the rate of 15 cents for 2 quarts: how much did I receive for them?

102. *W.* A grocer paid $5.75 for a barrel of potatoes containing 2 bushels and 2 pecks, and sold them at the rate of 20 cents for 2 quarts: did he make, or lose, and how much? Would he have made, or lost, if he had sold them at the rate of 20 cents for 3 quarts?

103. *W.* How many 3 pint bottles can be filled from 10 gallons of vinegar? How many pint bottles? How many gill cups and pint cups, each an equal number?

104. What will 2 gallons and 1 quart of wine cost, at $2 per gallon? *W.* At $2.12 per gallon? At $2.38?

105. At 10 cents a quart, what will 1 gallon, 1 pint, of vinegar cost? *W.* 21 gallons and 3 quarts? 47 gallons, 1 quart?

WRITTEN EXERCISES.

106. Change 47 quarts to bushels.

107. What will a bushel and a peck of wheat cost at $2 a bushel? At $2.37½?

REVIEW.—*How are joists, planks, and scantlings generally sold?* (122.) *What is meant by board measure?*

108. What will 3 bushels and 3 quarts of clover-seed cost, at $10 per bushel?

PROCESS.

cost 1 bu.
4)$10.000 × 3 = $30.000
8)2.500
.312 × 3 (qts.) = .936
$30.94

ANAL. STEPS.* — (1.) *Find cost of* 3 *bush.*
(2.) *Find cost of* 1 *pk.*
(3.) *Find cost of* 1 *qt., then* 3 *qts.*
(4.) *Find cost of* 3 *bush. and* 3 *qts.*

109. What will 9 bushels, 3 pecks, of pears cost, at $2.50 per bushel? At $3.43 per bushel?

110. What will 39 gallons, 2 pints, of molasses cost, at $1.05 per gallon? At $1.37½?

111. What will 11 gallons, 1 quart, 3 gills, of wine cost, at $2.25 per gallon? At $3.13½?

112. How many bushels in a box 12 feet long, 8 feet wide, and 4 feet deep?

PROCESS.

(1.) 12 × 8 × 4 = 384 cu. ft.
(2) 1728 cu. in. × 384 = 663552 cu. in.
(3.) 663552 cu. in. ÷ 2150 = 308 bu. 2 pks.

ANAL. STEPS. — (1.) *Find the cubic contents.*
(2.) *Change to cubic inches.*
(3.) *Reduce to bushels, pecks, &c.*

ANALYSIS (STEP 3). — Since 2150 cubic inches = 1 bushel, 663552 cubic inches (contents of box) = as many bushels as 2150 cubic inches are contained times in 663552 cubic inches; or, 308 bushels, with a remainder of 1352 cubic inches. Since 537 cubic inches = 1 peck, 1352 cubic inches = as many pecks as 537 are contained times in 1352 cubic inches; or, 2 pecks +. (The reduction may be carried to quarts and pints in the same way.)

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* *We much prefer* this method of solving problems of this class, because *of its brevity*, the simplicity of its analysis, and the avoidance of *troublesome fractions.*

113. How many bushels will a box 5 feet long, 3 feet wide, 3 feet deep, contain? 8 feet long, 4 feet wide, and 3 feet deep? 18 inches long, 12 inches wide, and 10 inches deep?

114. I have a corn-crib 26 feet long, 8 feet wide, and 10 feet high: how many bushels will it hold?

115. My wagon-box is 11 feet long, 6 feet wide, and 22 inches deep: how many bushels of shelled corn will it contain?

116. How many gallons of water will a circular tank or cistern contain, 5 feet in diameter, and 8 feet deep?

NOTE. — The distance across a cistern or tank, through the centre, is called the *diameter*. To find the number of cubic inches in a circular cistern or tank, use the following

RULE. — *Multiply the diameter in feet by itself, this product by* 1357, *and the result by the depth.*

FORMULA. (1.) $5 \times 5 \times 1357 \times 8 = 271400$ cu. in.
(2.) 271400 cu. in. $\div$ 231 cu. in. $= 1174 +$ gal.

ANAL. STEPS. — *(1.) Find the contents in cubic inches.*
(2.) Reduce the cubic inches to gallons, quarts, &c.

117. How many cubic inches in a circular cistern 4 feet in diameter and 7 feet deep? How many gallons will it hold?

118. How many gallons will a circular tank hold 4 feet across the top and 4 feet deep? If 8 feet across the top and 7 feet deep?

119. A railroad tank, the average diameter of which is 13 feet, and depth 12, contains how many gallons?

In addition to the above, the teacher should dictate a number of problems in estimating the contents of boxes, tanks, and cisterns.

APOTHECARIES' WEIGHT AND MEASURE.

125. Apothecaries' Weight and Fluid Measure are used in preparing prescriptions; but drugs and medicines are bought and sold by avoirdupois weight.

TABLE OF WEIGHTS.

20 grains (*gr.*) = 1 scruple (*sc.*, or ℈).
3 scruples = 1 drachm (*dr.*, or ʒ).
8 drachms = 1 ounce (*oz.*, or ℥).
12 ounces = 1 pound (*lb.*, or ℔.).

TABLE OF FLUID MEASURE.

60 minims (*m.*) = 1 fluid drachm (f. ʒ).
8 fluid drachms = 1 fluid ounce (f. ℥).
16 fluid ounces = 1 pint (O.).
8 pints = 1 gallon (*Cong.*).

NOTE.—(1). The pint (O.) and gallon (Cong.) are the same as in liquid measure.

(2.) Apothecaries' fluid measure is treated out of its logical order, for the sake of brevity and convenience.

REVIEW.—For what is liquid measure used? (123.) Write the table. How *many steps* in changing gallons to gills? In changing gills to gallons? Which is *reduction ascending*? Which reduction descending? In calculating the capacity of *cisterns*, how many gallons is a hogshead estimated to hold?

120. How many pills of 3 grains each can be made from an ounce of quinine? How many of 4 grains each?

121. Three pints of the essence of peppermint, costing a dollar a pint, were put in ounce phials, and sold at 15 cents each. How much was made by the sale?

TROY WEIGHT.

126. *Troy Weight* is used in weighing gold, silver, coins, jewels, &c.

TABLE.

24 grains (*gr.*) = 1 pennyweight (*pwt.*).
20 pennyweights = 1 ounce (*oz.*).
12 ounces = 1 pound (*lb.*).

NOTE. — A carat is a weight of about 3.2 grains, and is used by jewellers to weigh diamonds. The term *carat* is also used to denote the fineness of gold. When gold contains 18 parts pure gold and 6 parts alloy, which is usually silver and copper, it is said to be 18 carats fine. Gold 14 carats fine contains 14 parts pure gold and 10 parts alloy, &c.

122. What will a gold chain weighing 12 pennyweights cost, at 80 cents per pennyweight? At 67 cents?

123. What will a gold chain weighing one ounce and 7 pennyweights cost, at 75 cents per pennyweight?

NOTE. — For examples in oral and written reduction, use Drill Card.

AVOIRDUPOIS WEIGHT.

127. *Avoirdupois Weight* is used for weighing all common goods.

TABLE.

16 ounces (*oz.*) = 1 pound (*lb.*).
100 pounds = 1 hundred wt. (*cwt.*).
1 cental (*c.*).
20 hundred wt. = 1 ton (T.).

TABLE OF COMPARISON.

1 lb. Troy = 5760 grs. Troy
1 lb. apoth. = 5760 " "
1 lb. avoir. = 7000 " "
1 kilogram = 15432 "

128. Nearly all the States have adopted the weights in the following

TABLE.

196 lbs. = 1 barrel of flour.
200 “ = 1 “ beef, pork, fish.
280 “ = 1 “ salt.
32 “ = 1 bushel of oats.
56 “ = 1 “ corn or rye.
60 “ = 1 “ wheat, beans, peas.
14 “ = 1 “ bluegrass-seed.
60 “ = 1 “ clover-seed.
48 “ = 1 “ barley.
56 “ = 1 “ flaxseed.
44 lbs. = 1 bushel hemp-seed.
60 “ = 1 “ potatoes.
45 “ = 1 “ timothy-seed.
57 “ = 1 “ onions.
28 “ = 1 “ apples, pears (dry).
70 “ = 1 “ corn in ear.
52 “ = 1 “ buckwheat.
50 “ = 1 “ salt.
80 “ = 1 “ bituminous coal.

ORAL AND WRITTEN EXERCISES.

124. How many ounces in 2 pounds? In 5 pounds? *W.* In 75 pounds? In 647 pounds?

125. How many pounds in 5 hundred weight? In 7 cwt.? In 9 cwt.? *W.* In 13 cwt. and 46 lbs.? In 12 cwt. 37 lbs.?

126. How many hundred weight in 3 tons? *W.* In 5 T. 7 cwt.? In 14 T. 13 cwt.?

127. How many ounces in 4 lbs. 6 oz.? *W.* In 27 lbs. 12 oz.?

128. *W.* How many hundred weight in 347 lbs.? In 476 lbs.? In 2467 ounces? In 41 T.?

129. *W.* What will 913 lbs. of flour cost, at \$4.25 per hundred weight?

PROCESS.

$$\begin{array}{lr} 100)\,\underline{\$4.250}\times 9\text{ cwt.} = & \$38.25 \\ \quad .042\times 13\text{ lbs.} = & \underline{.546} \\ & \$38.80 \end{array}$$

Anal. Steps.—(1.) *Find cost of 9 cwt.*
(2.) *Find cost of 13 lbs.*
(3.) *Find cost of 9 cwt. 13 lbs.*

NOTE.—When the mills equal or exceed 5, add one to cents.

REVIEW.—For what is dry measure used? (124.) Write the table. How many *steps in* changing pints to bushels? In changing bushels to pints? How many *cubic inches* in a gallon of liquid measure? In a quart? In a quart dry measure? *In a liter?* In a bushel? In a peck? In 8 quarts liquid measure? In a *hectoliter?*

130. *W.* What will be the cost of a cask of sugar weighing 657 pounds, at $7\frac{1}{2}$ cents per pound? At \$8.385 per cwt.?

131. *W.* What will 5 lbs. 3 oz. of tea cost, at \$$1.87\frac{1}{2}$ per pound? At \$1.53 per pound? At \$2 per pound?

132. *W.* What will 274 pounds of butter cost, at \$43.42 per cwt.? At \$28.97 per cwt.? At \$35.46 per cwt.?

133. *W.* How many bushels of flaxseed in 470 pounds? What will it be worth, at \$1.83 per bushel?

134. *W.* Change 3478 lbs. of oats to bushels. Of corn. Of wheat. Of barley. Of potatoes. Of onions. Of salt.

135. *W.* In 439 pounds of rye, how many bushels? Of beans? Of clover-seed? Of corn in ear? Of hemp-seed?

136. *W.* What cost 7 lbs. 6 oz. rice, at \$.08 per pound?

ANAL. STEPS.—(1.) *Find cost of* 7 *lbs.*
(2.) *Find cost of* 1 *oz.*
(3.) *Find cost of* 6 *oz.*

137. *W.* What cost 2 lbs. 8 oz. of spice, at $37\frac{1}{2}$ cents per pound?

138. *W.* What cost 4 lbs. 4 oz. honey, at 38 cents per pound?

139. *W.* What cost 9 lbs. 5 oz. butter, at 55 cents per pound?

140. *W.* Add cost of 12 lbs. 13 oz. butter, at 20 cents per lb.; 11 lbs. 5 oz. rice, at 11 cents per lb.; 7 lbs. 3 oz. sugar, at 15 cents per lb.

REVIEW.—Give the dimensions of a bushel measure. (124.) The measure should be even full in measuring what grain and fruits? When should it be *heaping full? How many gills make a pint liquid measure? A pint dry measure?*

141. *W.* Add cost of 3 lbs. 6 oz. pepper, at 38 ce per pound ; 4 lbs. 3 oz. spice, 40 cents per lb. ; 18 2 oz. cinnamon, at 80 cents per lb.

142. *W.* Add cost of 5 lbs. 7 oz. cloves, at $1.80 lb. ; 9 lbs. 11 oz. dates, at $.37½ per lb. ; 4 lbs. 8 honey, at 35 cents per lb.

143. *W.* Add cost of 1438 lbs. flour, at $6 per cw 418 lbs. beef, at $18 per bbl. ; 186 lbs. pork at per bbl.

144. *W.* Add cost of 327 lbs. fish, at $14 per bbl. ; lbs. bluegrass-seed, at $4.30 per bu. ; 58 lbs. beans $2.25 per bu.

145. *W.* Add cost of 75 lbs. clover-seed, at $9.50 bu. ; 119 lbs. onions, at $1.75 per bu. ; 812 lbs. potat at $1.05 per bu.

MEASURE OF TIME.

129. *Time* is the measure of duration.

DIVISION OF THE YEAR.

Season.	Name of months.	No. of days.	Abbreviations.
Winter,	1. January,	31	Jan.
	2. February,	28 or 29	Feb.
Spring,	3. March,	31	—
	4. April,	30	—
	5. May,	31	—
Summer.	6. June,	30	—
	7. July,	31	—
	8. August,	31	Aug.
Autumn,	9. September,	30	Sept.
	10. October,	31	Oct.
	11. November,	30	Nov.
Winter,	12. December,	31	Dec.
		365 or 366	

Review. — How can gallons liquid measure be reduced to bushels dry m ure? (124.) Ans. — *By changing gallons to cubic inches, then the cubic inche bushels.* How can gallons be reduced to liters? How can liters be reduce *quarts,* liquid measure? To quarts, dry measure?

TABLE.

60 seconds (*sec.*) = 1 minute (*min.*).
60 minutes = 1 hour (*hr.*).
24 hours = 1 day (*da.*).
7 days = 1 week (*wk.*).
4 weeks 2 days }
30 days } = 1 month (*mo.*).
365 days }
52 weeks 1 day } = 1 year (*yr.*).
12 months }
100 years = 1 century (*C.*).

NOTES. — (1.) The exact length of the solar year is 365 days 5 h. 48 46.05 sec.; but, for convenience, the civil year is reckoned 11 min. 5 sec. more than this, or 365 da. 6 h. = 365¼ days. In four years, ¼ day makes 1 day, which, in every fourth year, called bissextile eap year, is added to the shortest month, giving it 29 days. The nbers denoting leap years are exactly divisible by 4; as, 1856, 1860, 4. Years whose number can be divided without a remainder by but not by 400, are not leap years, although divisible by 4.

2.) In most business transactions, 30 days are called a month, and weeks a year.

ORAL PROBLEMS.

l. How many seconds in 2 minutes? In 3 minutes? 4 minutes? In 12 minutes?

2. How many minutes in 120 seconds? In 3 hours? 240 seconds? In 5 hours?

3. How many days in 3 months? In 3 months, 12 's? In 9 months, 7 days?

. How many hours from 6, A.M., to 11, A.M.? From .M., to 12, M.? From 9, A.M., to 3, P.M.?

. How many minutes from 20 minutes past 9 to 10 nutes past 10? Between 13 minutes past 11 o'clock, ., and 12 minutes past 2 o'clock, P.M.?

EVIEW. — *How can bushels be reduced to hectoliters?* (124.) *How can hecto- be reduced to pecks?*

WRITTEN EXERCISES.

146. How many years in 783064 seconds?

147. How many years in 84160 minutes?

148. How many weeks in 71834 seconds?

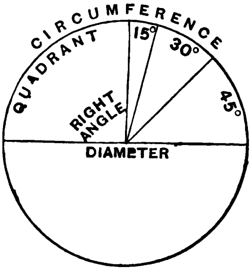
149. How many seconds in 3 years of 365 days?

150. How many seconds in the winter months of a common year?

151. How many in the winter of a leap year?

152. At a speed of 3 rods per second, how long will it take a railroad car to run 138 miles?

CIRCULAR OR ANGULAR MEASURE.



130. *A Circle* is a plane figure bounded by a line, every point of which is equally distant from a point within, called the *centre*.

(*a.*) The *circumference* of a circle is its bounding line. An *arc* is any part of the circumference. A *quadrant* is one-fourth of a circumference.

(*b.*) The *diameter* is the distance across a circle through its centre. A *radius* is one-half of a diameter.

TABLE.

60 seconds (″)	=	1 minute	(′).
60 minutes	=	1 degree	(°).
30 degrees	=	1 sign	(*sig.*).
90 degrees	=	1 quadrant	(*quad.*).
		1 right angle	(*r. a.*).
4 quadrants or 12 signs	=	1 circumference or circle	(*cir.*).

REVIEW.—What is the use of apothecaries' weight? (125.) What of *apothecaries' fluid* measure? How are drugs and medicines bought and sold? *Write the table. Name* and give the number of steps in reduction ascending. Give and *name* the number of steps in reduction descending.

Notes. — (1.) Circular or angular measure is used to measure angles, or the arcs of circles. It is used in astronomy, geography, navigation, and surveying, and for calculating differences of time.

(2.) The greatest circumference of the earth's surface is about 24,930 miles; 1° of that circumference is one 360th of 24,930 miles, which is 69¼ miles.

(3.) Since the sun apparently goes around the earth once in 24 hours, in 1 hour it must pass over *one* 24th of 360°, which is 15°: hence, if, in the sun's apparent motion, 15° of long. require 1 hour, or 60 minutes of time, one deg. will require *one* 15th of 60 minutes, which is 4 minutes.

(4.) Since 1° or 60′ long. requires 4 min., or 240 sec. of time, 1′ long. will require *one* 60th of 240 seconds, which is 4 seconds: therefore, 15° long. require 1 hour of time, 1° long. requires 4 minutes of time, 1′ long. requires 4 seconds of time. Hence the following: —

Rule. — I. *Multiply the difference of longitude by* 4 *to find difference of time.*

2. *To find the time at places east, add the difference to the given time; to find the time at places west, subtract it from the given time.*

WRITTEN EXERCISES.

153. The difference of longitude between two places is 24° 35′: what is the difference of time?

TABLE.

long.	*time.*
1° (*deg.*) =	4 min.
1′ (*min.*) =	4 sec.

PROCESS.

$$\begin{array}{r} 24^\circ\ 35' \\ 4 \\ \hline \text{1 hr. 38 min. 20 sec.} \end{array}$$

Analysis. — Since 1′ of long. requires 4 sec. of time, 35′ will require 35 times 4 sec., or 140 sec; 140 sec. = 2 min. 20 sec.

Since 1° of long. requires 4 min. of time, 24° will require 24 times 4 min., which, with 2 min., = 98 min. = 1 hr. 38 min.

154. What is the difference in time between Greenwich and Washington, the difference of longitude being 77 *degrees?*

Give the differences of time corresponding to the following differences of longitude: —

155. 4° 37′
156. 36° 42′
157. 97° 38′
158. 72° 22′

Give the equivalent in longitude of 17 min. 40 sec.

ANALYSIS. — (1.) Since 4 min. = 1°, 17 min. = 4°, with 1 min. remaining.

(2.) 1 min. = 60 sec., 60 sec. + 40 sec. = 100 sec. Since 4 sec. = 1′, 100 sec. = 25′. Ans. 4° 25′.

Give the differences of longitude corresponding to the following differences of time: —

159. 4 hr. 3 min. 16 sec.
160. 2 hr. 46 min. 21 sec.
161. 1 hr. 2 min. 18 sec.
162. 4 hr. 1 min. 2 sec.

131. STATIONERS' TABLE.

12 units = 1 dozen
12 dozen = 1 gross
12 gross = 1 great gross
20 units = 1 score

24 sheets = 1 quire
20 quires = 1 ream
2 reams = 1 bundle
5 bundles = 1 bale

163. What will 6 dozen penholders cost, at 5 cents apiece? At $4\frac{1}{2}$ cents?

164. Bought 6 gross of pens, at $2.50 per gross, and retailed them at 2 cents apiece: did I make, or lose? and how much?

ENGLISH MONEY.

132. ***English Money*** is the currency of Great Britain.

4 farthings (*far.*) = 1 penny (*d.*)
12 pence = 1 shilling (*s.*)
20 shillings = 1 pound or sovereign (£.)

SILVER, 24 cts.

SOV., GOLD, $4.866.

SILVER, 19.3 cents.

NOTES. — (1.) The gold coins are the sovereign (£) and the half-sovereign (10*s.*). A sovereign is worth $4.866 in gold.

(2.) The silver coins are the crown (5*s.*), the half-crown (2*s.* 6*d.*), the florin (2*s.*), the shilling (12*d.*), sixpenny piece (6*d.*), and the three-penny piece (3*d.*).

(3.) The bronze coins are the penny, the half-penny, and farthing.

(4.) The franc is the unit of the French currency, and is worth $0.193. The denominations are francs and centimes.

(5.) The denominations of the Canadian currency are the same as United States currency.

166. How many shillings in 3 pounds? In 2 pounds? In 327 pounds?

167. How many shillings in £34? How many pence? How many farthings?

ADDITION OF DENOMINATE NUMBERS.

WRITTEN PROBLEMS.

133. What is the sum of 14 bu. 2pks. 4 qts., 29 bu. 11 pks. 8 qts., 23 bu. 5 qts.?

PROCESS.

bu.	pk.	qt.
14	2	4
29	11	8
23	0	5
69	3	1

ANALYSIS. — (1.) For convenience write the numbers so that units of the same denomination will stand in the same column.

(2.) 5 qts. + 8 qts. + 4 qts. = 17 qts. = 2 pks. 1 qt. Write 1 in the place of quarts, and add the 2 pecks to the sum of the pecks.

(3.) 11 pks. + 2pks. + 2 pks. = 15 pks. = 3 bu. 3 pks. Write 3 in the place of pecks, and add the bushels to the sum of the bushels.

(4.) 23 *bu.* + *29 bu.* + 14 bu. + 3 bu. = 69 bu.; which write in the *proper place.*

168. What is the sum of 14 bu. 3 pks. 6 qts., 2 bu. 2 pks. 5 qts., 37 bu. 2 pks. 8 qts., 5 bu. 8 qts., 9 pks. 4 qts.

169. What is the sum of 8 lbs. 5 oz. 11 pwt., 7 lbs. 2 oz. 13 pwt., 27 lbs. 16 pwt., 9 lb. 8 oz.

170. What is the sum of 44 A. 23 p., 10 A. 68 p., 25 A. 105 p., 17 A. 19 p.?

171. What is the sum of 25 rds. 12 ft. 5 in., 28 rds. 9 ft. 10 in., 18 rds. 10 ft., 12 rds. 14 ft. 9 in.?

172. I received at one time £25 6s. 4d.; at another, £36 18s.; at another, £12 10d.: how much was paid me in all?

173. A man owned three farms. The first contained 144 A. 27 P.; the second, 82 A. 16 P.; the third, 152 A. 37 P.: how many acres in all?

SUBTRACTION.

134. From 47 gals. 3 qts. 1 pt. 1 gi. take 29 gals. 2 qts. 1 pt. 3 gi.

PROCESS.

gal.	qt.	pt.	gi.
47	3	1	1
29	2	1	3
18	0	1	2

ANALYSIS. — (1.) For convenience write the subtrahend under the minuend so that units of the same denomination will stand in the same column.

(2.) Since 3 gills cannot be taken from 1 gill, take the 1 pint = 4 gills; 4 gills and 1 gill are 5 gills; 3 gills from 5 gills leave 2 gills, which we write under the gills.

(3.) Since the 1 pint was added to the gills, leaving no pints in the minuend, take 1 quart from 3 quarts, 1 quart = 2 pints, and from *it* subtract 1 pint, leaving 1 pint, which write under pints.

(4.) 2 quarts from 2 quarts leave no quarts: write a cipher under the quarts.

(5.) 29 gallons from 47 gallons leave 18 gallons, which write under gallons.

REVIEW. — What is the use of troy weight? (126.) Write the table. Name *and give the* number of steps in reduction ascending. In reduction descending. *What is the* weight of the carat used in weighing diamonds? What is meant by he *term carat*, in denoting the fineness of gold? What is meant by gold 18 car- *fine?* 14 carats fine? What is alloy? What metals are used to alloy gold?

174. From £78 11s. 5d. 2 far. take £41 13s. 3d. 3 far.

175. From 15 lbs. 3 oz. 12 pwt. 14 gr. take 9 lbs. 11 oz. 17 pwts. 21 gr.

176. From 15 lbs. 7℥ 1ʒ 2℈ 15 gr. take 11 lbs. 9℥ 7ʒ 1℈ 19 gr.

177. From 3 mi. 31 rds. 1 yd, take 2 mi. 9 rds. 8 yds.

178. What is the difference in time between July 3, 1875, and Sept. 1, 1877?

PROCESS.

year	mo.	da.
1877	9	1
1875	7	3
2	1	28

ANALYSIS. — (1.) Sept. 1, 1877, denotes the 1st day of the 9th month of the 1877th year of the Christian era. As it is the later date, it is written as the minuend. July 3, 1875, the earlier date is written as the subtrahend, placing the years, months, days, each in its own column.

(2.) Subtract as in subtraction of denominate numbers.

What is the difference in time between each of the following dates? —

179. Aug. 1, 1872, and Sept. 21, 1875?

180. July 5, 1855, and Aug. 13, 1871?

181. Dec. 3, 1867, and March 5, 1870?

182. Nov. 1, 1859, and April 11, 1869?

183. Sept. 5, 1872, and Nov. 1, 1872?

184. July 5, 1874, and July 1, 1876?

185. Nov. 4, 1875, and July 1, 1877?

186. July 1, 1870, and July 9, 1870?

187. What is the difference in longitude between 132° 3′ 2″ east, and 67° 14′ 38″ east.

188. A ship starting from an island 37° 18′ 10″ west longitude was wrecked in 63° 10′ west. How far had she sailed?

REVIEW. — What is the use of avoirdupois weight? (127.) Write the table. Give the steps. How many grains in 1 lb. troy? In 1 lb. apothecary? In 1 lb. avoirdupois? What is the difference between the weight of a pound avoirdupois and a pound troy? Between a pound troy and a pound apothecary? Which is the heavier, — an ounce troy, or an ounce avoirdupois?

189. What is the difference between 19° 30′ east, and 37° 6′ west?

SUGGESTION.—Add the longitudes of the two places to find the difference.

190. What is the difference between 120° 14′ E., and 36° 46′ W.?

191. What is the difference in time between 44° 8′ E., and 7° 10′ 4″ W.?

192. A grocer bought a hogshead of molasses containing 80 gallons. He sold 54 gals. 3 qts. 1 pt.: how much remained?

193. A firkin of butter contained 133 pounds: how much remained after selling 35 lbs. 7 oz.?

194. From three quarter-sections of land, there were sold 258 A. 34 p.: how much remained?

MULTIPLICATION OF COMPOUND NUMBERS.

WRITTEN PROBLEMS.

135. What will 6 boxes of grain contain, if each hold 11 bu. 3 pks. 5 qts.?

PROCESS.

bu.	pks.	qts.
11	3	5
		6
71	1	6

ANALYSIS.—(1.) For convenience, write the multiplier under the lowest denomination of the multiplicand.

(2.) 6 times 5 quarts = 30 quarts = 3 pecks, 6 quarts; write the quarts in the proper place, and add the pecks to the product of the pecks.

(3.) 6 times 3 pecks = 18 pecks, which with 3 pecks added = 21 pecks, or, 5 bushels, 1 peck; write the peck in the proper place, and add the bushels to the product of bushels.

(4.) 6 times 11 bushels = 66 bushels, which with 5 bushels added = 71 bushels, which write in the proper place.

REVIEW.—How can pounds avoirdupois be reduced to pounds troy? (127.) *ANS.*—*By changing the pounds avoirdupois to grains, then the grains to pounds troy.* How many steps? Which is reduction ascending? Which reduction descending?

195. Multiply £5 7 s. 8 d. 2 far. by 3. By 7.

196. Multiply 2 rds. 4 yds. 2 ft. 6 in. by 6. By 18.

197. Multiply 5 hrs. 42 min. 37 sec. by 13. By 42.

198. How many cranberries in 16 boxes, each containing 2 bu. 2 pks. 6 qts.?

199. If 2 lbs. 5 oz. grapes are put in a box, what will be the weight of a crate containing 24 boxes? 36 boxes?

200. What will be the weight of 2 dozen spoons, each weighing 1 oz., 5 pwt., 12 grs.? What will they cost, at 7 cents per pwt.?

DIVISION OF COMPOUND NUMBERS.

WRITTEN PROBLEMS.

136. Divide 12 lbs. 3 pwt. 11 gr. by 8.

PROCESS.

	lbs.	oz.	pwt.	grs.
8)	12	0	3	11
	1	6	0	$10\frac{3}{8}$

ANALYSIS. — (1.) For convenience, arrange the example as in simple numbers.

(2.) 1 eighth of 12 lbs. is 1 lb. with a remainder of 4 lbs. Write the 1 lb. in the quotient, and reduce the 4 lbs to ounces. 4 lbs = 48 oz.

(3.) 1 eighth of 48 oz. is 6 oz. Write the 6 oz. in the quotient.

(4.) Eight is not contained in 3 pwt. Write a cipher in the quotient, and reduce the pennyweights to grains. 3 pwts. = 72 grs., which, added to 11 grs. = 83 grs.

(5.) 1 eighth of 83 grains is $10\frac{3}{8}$ grains, which write in the quotient.

201. Divide 35 lbs. 8 oz. 4 pwts. by 8. By 5.

202. A teamster drew 450 bu. 16 lbs. wheat at 15 loads, how much did he average per load?

203. A half-dozen spoons weighed 10 oz. 3 pwt., what was the weight of each?

204. 472 gal. 3 qts. ÷ 7 = what?

REVIEW. — Give the number of pounds in a barrel of flour. (128.) Beef. Pork. Fish. *Salt. Give the number of pounds in a bushel of oats. Corn. Rye. Wheat. Beans. Peas. Barley.*

205. 341 A. 24 p. ÷ 4 = what?

206. 243 mi. 43 rds. ÷ 19 = what?

207. 57 bu. 9 qts. ÷ 4 = what?

208. £35 3 s. 4 d. ÷ 14 = what?

209. How many spoons weighing 8 pwts. can be made from 11 oz. of silver?

SUGGESTION. — Reduce both divisor and dividend to the lowest denomination mentioned in either, then divide as in simple numbers.

210. A man feeds his horse five quarts of oats a day, how long will 13 bu. 3 pks. last him?

211. How many bottles, each containing 1 qt. 1 pt., can be filled from 36 gallons of vinegar?

212. How many packages, each containing 3 oz., can be filled from 136 pounds of pepper?

DENOMINATE FRACTIONS.

137. Reduction Descending.

WRITTEN PROBLEMS.

(*a.*) Reduce $\frac{4}{7}$ of a mile to inches.

(*b.*) Reduce $\frac{4}{7}$ of a mile to integral numbers of lower denominations.

(*a.*) PROCESS.

$$\overset{\text{m.}}{\tfrac{4}{7}} \times 320 \times \tfrac{11}{2} \times 3 \times 12 = \overset{\text{in.}}{\tfrac{253440}{7}} = \overset{\text{in.}}{36205\tfrac{5}{7}}$$

(*b.*) PROCESS (1).

$$\begin{array}{rrrrr} \text{m.} & \text{rds.} & \text{yds.} & \text{ft.} & \text{in.} \\ 7)4 & 0 & 0 & 0 & 0 \\ \hline & 182 & 4 & 2 & 1\tfrac{5}{7} \end{array}$$

SUGGESTION. — Since $\frac{4}{7}$ of one mile = $\frac{1}{7}$ of 4 miles, the result may be found by the process used in division of denominate numbers.

REVIEW. — What is time? (129.) How is the year divided? Which are the *winter months?* Which the spring? Which the summer? Which the autumn? *Give the number of days in each.* Write the table.

(*b.*) PROCESS. (2.)

1. 320 rds. × $\frac{4}{7}$ (mi.) = 182 ($\frac{6}{7}$) rds.
2. $\frac{11}{2}$ yds. × $\frac{6}{7}$ (rds.) = 4 ($\frac{5}{7}$) yds.
3. 3 feet × $\frac{5}{7}$ (yd.) = 2 ($\frac{1}{7}$) feet.
4. 12 in. × $\frac{1}{7}$ (ft.) = 1 ($\frac{5}{7}$) in.

ANS. — 182 rds. 4 yds. 2 ft. 1$\frac{5}{7}$ in.

(*b.*) **ANAL. STEPS.** — (1.) *Change the fraction of a mile to rods.*
(2.) *Change the fraction of a rod to yards.*
(3.) *Change the fraction of a yard to feet.*
(4.) *Change the fraction of a foot to inches.*

(*b.*) **ANALYSIS** — (1.) Since in 1 mile there are 320 rods, in $\frac{4}{7}$ of a mile there are $\frac{4}{7}$ of 320 rods; or, 182$\frac{6}{7}$ rods.

(2.) Since in 1 rod there are $\frac{11}{2}$ yards, in $\frac{6}{7}$ of a rod there are $\frac{6}{7}$ of $\frac{11}{2}$ yards; or, 4$\frac{5}{7}$ yards.

(3.) Since in 1 yard there are 3 feet, in $\frac{5}{7}$ of a yard there are $\frac{5}{7}$ of 3 feet; or, 2$\frac{1}{7}$ feet.

(4.) Since in 1 foot there are 12 in., in $\frac{1}{7}$ of a foot there are $\frac{1}{7}$ of 12 in.; or, 1$\frac{5}{7}$ inches.

Reduce .5714 of a mile to integers of lower denominations.

PROCESS.

$$\begin{array}{r l}
.5714 & \text{mi.} \\
320 & \text{(rds.)} \\
\hline
114280 & \\
17142 & \\
\hline
182|.8480 & \text{rds.} \\
5\frac{1}{2} & \text{(yds.)} \\
\hline
424 & \\
4240 & \\
\hline
4|.664 & \text{yds.} \\
3 & \text{(ft.)} \\
\hline
1|.992 & \text{ft.} \\
12 & \text{(in.)} \\
\hline
11.904 & \text{in.}
\end{array}$$

ANS. — 182 rds. 4 yds. 1 ft. 11.904 in.

REVIEW.— What is meant by **leap year?** (129.) **What is meant by July 4, 1876?** *What is meant by the Centennial?*

NOTE. — It will be observed that the analysis of the reduction of denominate fractions is the same as that of denominate numbers. When the multiplier is a common fraction or a decimal, the word "of" is used instead of "times."

213. Change $\frac{3}{5}$ of a mile to integers of lower denominations. In the same way, change .3 of a mile.

214. Reduce $\frac{4}{11}$ of a rod to integers of lower denominations. In the same way, reduce .17 of a rod.

215. How many inches in .5 of a yard? In 2.7 of a foot? In $2\frac{3}{5}$ of a rod?

216. What is the sum of $\frac{5}{9}$ mile and $\frac{3}{7}$ rod?

217. What is the sum of .785 ton and 3.65 cwt.

218. Find the amount of $\frac{3}{7}$ lb. + $2\frac{3}{5}$ oz. + $\frac{3}{7}$ pwt.; of .375 mi. + 3.625 rd. — .275 yds.

219. What is the cost of $3\frac{2}{3}$ lbs. of gold, at $16 per oz.?

220. Bought 37.85 lbs. of gold-dust, at $0.80 per pwt.: what did it cost? What will $\frac{3}{11}$ lb. cost?

138. Reduction Ascending.

(*a.*) Reduce 3 inches to the fraction of a rod. (*b.*) In the same way, reduce 3 inches to the decimal of a rod.

(*a.*) PROCESS.	(*b.*) PROCESS.
(1.) 3 in. ÷ 12 in. = $\frac{1}{4}$ (ft.)	(1.) 3 in. ÷ 12 in. = .25 (ft.)
(2.) $\frac{1}{4}$ ft. ÷ 3 ft. = $\frac{1}{12}$ (yd.)	(2.) .25 ft. ÷ 3 ft. = .083+ (yd.)
(3.) $\frac{1}{12}$ yd. ÷ $\frac{11}{2}$ yd. = $\frac{1}{66}$ (rd.)	(3.) .083 yd. ÷ $\frac{11}{2}$ yd. = .0150+ (rd.)

ANAL. STEPS. — (1.) *Reduce inches to the fraction of a foot.*
(2.) *Reduce fraction of a foot to fraction of a yard.*
(3.) *Reduce fraction of a yard to fraction of a rod.*

ANALYSIS (Process *a*). — (1.) Since in 12 inches there is 1 foot, in 3 inches there are as many feet as 12 inches are contained times in 3 inches; or, $\frac{1}{4}$ of a foot.

REVIEW. — *What is a circle?* (130.) *Where is the center of a circle? What is meant by the circumference of a circle? By an arc? By a quadrant? By a diameter? Write the table. What is the use of angular measure?*

(2.) Since in 3 feet there is 1 yard, in $\frac{1}{4}$ of a foot there are as many yards as 3 feet are contained times in $\frac{1}{4}$ of a foot; or, $\frac{1}{12}$ of a foot.

(3.) Since in $\frac{11}{2}$ yards ($5\frac{1}{2}$ yards) there is 1 rod, in $\frac{1}{12}$ of a yard there are as many rods as $\frac{11}{2}$ yards are contained times in $\frac{1}{12}$ of a yard; or, $\frac{1}{66}$ of a rod.

(*a.*) What part of a rod is 4 yds. 2 ft. 5 in.?

(*b.*) Reduce 4 yds. 2 ft. 5 in. to decimal of a rod.

CONTRACTED PROCESSES.

(*a.*)		(*b.*)	
12 in.)5	(in.)	12 in.)5.000	(in.)
3 ft.)$2\frac{5}{12}$	(ft.)	3 ft.)2.416+	(ft.)
$\frac{11}{2}$yd.)$4\frac{29}{36}$	(yd.)	$\frac{11}{2}$yd.)4.805+	(yd.)
$\frac{173}{198}$	(rd.)	.873+	(rd.)

NOTE.—Since the analytical steps and the analysis are essentially the same as in the previous processes, they are not repeated.

221. Reduce 2 ft. 10 in. to the fraction of a yard. To the decimal of a yard.

222. Change 7 rds. 2 yds. 1 ft. to the fraction of a mile. To the decimal of a mile.

223. What part of a gallon is 3 qts. 2 pts. 3 gi.? Reduce 3 qts. 2 pts. 3 gi. to the decimal of a gallon.

224. What is the cost of 2 pks. 2 qts. of apples, at $1.60 per bushel?

SUGGESTION.—2 pks. 2 qts. = $\frac{9}{16}$ bush. $1.60 × $\frac{9}{16}$ (bush.) = ? or, 2 pks. 2 qts. = .5625 bush. $1.60 × .5625 bush. = ?

225. What will 3 qts. 1 pt. 2 gills of vinegar cost, at 33 cts. per gallon?

226. Change .3 in. to the decimal of a rod. $\frac{7}{8}$ yd. 4 ft. to the fraction of a rod.

227. Change 7 yds. 2 ft. 8 in. to the decimal of a rod, to the fraction of a rod.

REVIEW.—What is meant by currency? What is English money? (132.) Recite the table. Give the coins and the value of each. What is the value of a franc in U. S. currency? *How does the Canadian currency differ from U. S. currency? How many units make a dozen?* (131.) Recite the entire table.

228. What is the cost of 7 yds. 2 ft. 8 in. of picket fence, at $24.20 per rod?

229. What is the value of 3 gals. 3 qts. ½ pt. of domestic wine, at $3.75 per gallon?

DRILL CARD EXERCISE. — TABLE VI.

DENOMINATE FRACTIONS.

Exercise I. — Reduce **Z** mi. to its value in lower denominations.
II. — Reduce **Z** sq. mi. to integers of lower denominations.
III. — What is the value of **Z** cords?
IV. — Place a decimal point before each number in **W**, and, calling it the decimal of a mile, change it to integers of lower denominations.
V. — In the same manner, make each number in **W** the decimal of a square mile, and give its value in lower denominations.
VI. — In the same way, change **W** to decimals of a ton, and reduce to ounces.
VII. — What is the value of **Z** cwt. avoirdupois?
VIII. — What is the value of **Z** months?
IX. — Reduce **V** feet to the fraction of a rod.
X. — Reduce **W** square yards to a fraction of an acre.
XI. — Reduce **W** cubic feet to the decimal of a cord.
XII. — Change **W** pints to gallons, and decimals of a gal.
XIII. — Change **W** quarts to bushels.
XIV. — Change **W** pennyweights to pounds.
XV. — Change **W** oz. avoirdupois to hundred-weights.
XVI. — Change **W** days to the fraction of a year.

REVIEW. — Write the analysis of a short example in addition of denominate numbers. (133.) Write the analysis of a short example in the subtraction of denominate numbers. (134.) Write the analysis of a short example in the multiplication of denominate numbers. (135.) Write the analysis of a short example in the division of denominate numbers. (136.)

REVIEW. — BUSINESS PROBLEMS.

WRITTEN EXERCISES.

230. What must I give for 26 acres of land, at £5 2 s. 6 d. per acre? How many dollars U. S. currency will that amount to?

PROCESS.

$$\begin{array}{r} 12\underline{)6.00} \\ 20\underline{)2.50} \\ £5.125 \\ \underline{26\ (\text{acres})} \\ £133.250 \end{array}$$

$4.866 × 133.25 = $648.39

or,

PROCESS.

(1.) £5 2 *s.* 6 *d.* = £$5\frac{1}{8}$
(2.) £$5\frac{1}{8}$ × 26 = £$133\frac{1}{4}$
(3.) $4.866 × $133\frac{1}{4}$ = $648.39

ANAL. STEPS. — (1.) *Change 2 s. 6 d. to the decimal of a pound.*
(2.) *Find cost of 26 acres.*
(3.) *Find the value in U. S. currency.*

231. What is the cost of 48 bushels of wheat, at 8s. 5 d. per bushel? What is the cost in U. S. currency?

232. Bought 148 bushels of wheat at 48 d. per bushel: what is the cost in U. S. currency?

233. Bought 978 pounds of pork at 4 d. per pound: what is the cost in U. S. currency?

234. What is the value of a silver vessel weighing 1 lb. 7 oz. 3 pwt., at $9 per lb.?

235. What is the value of a gold chain weighing 1 oz. 3 pwt., at $18.37 per oz.?

236. What will 6 silver spoons cost, at 75 cents per oz., if each spoon weighs 12 pwt. 11 gr.?

237. How many silver spoons can be made from 4 oz. of silver, and how much silver will remain, if each spoon weighs 8 pwt. 7 gr.?

238. What will $5\frac{3}{4}$ tons of coal cost, at $5.37 per ton?

239. What will 3 cwt. 39 lbs. of coal cost, at $7.35 per ton? At $9.57 per ton?

3 cwt. 39 lbs. = .1695 ton.

240. What must I give for 87 lbs. 6 oz. flour, at $3 per cwt.?

241. What must be given for 16 lbs. 8 oz. butter, at $37\frac{1}{2}$ cents per lb.? At 45 cents per lb.?

242. If 7 pounds of sugar are worth $1.20, what is 1 lb. worth? What are $5\frac{3}{8}$ pounds worth? What are 9 lbs. 9 oz. worth?

243. What is the value of $15\frac{1}{2}$ bbls. flour, at $2.75 per cwt.?

244. What is the value of 15 lbs. pork, at $2.55 per cwt.?

245. How many doses in 7℥ 6ʒ 2℈ of tartar-emetic, allowing 20 gr. for each dose?

246. How many pills can be made from 9℥ 4ʒ 2℈ 8 gr. of calomel, allowing 12 gr. for each pill?

247. At $1.75 per yard, what cost 4 and 3 quarter yards of cloth? $3\frac{5}{8}$ yards? $2\frac{7}{16}$ yards?

248. What will it cost to lay 3 miles 120 rds. of railroad track, at $1500 per mile?

249. How many yards of carpeting 1 yard wide will it take to cover the floor of a room 30 feet long and 18 feet wide? 50 feet long and 24 feet wide?

250. How many square feet in the ceiling and walls of a room 38 feet by 67, and 11 feet high?

251. What will it cost to paper the walls of a room 36 feet by 24, and 11 feet high, at $\$1.37\frac{1}{2}$ per square yard?

252. What will it cost to paint the walls of a room 36 feet by 54, and 10 feet high, at $87\frac{1}{2}$ cents per square yard?

REVIEW — What is a composite number? (70.) What is a prime number? (71.) What is meant by a factor or divisor? (72.) Find the prime factors of 24. (73) What is the greatest common divisor of two or more numbers? (74.) What is a common multiple of two or more numbers? (75.) What is the least common multiple of two or more numbers? (76.) Illustrate the method of cancellation. (77.) Give the principles upon which it is based. (78.)

253. How many square feet in a board 9 feet long and 1 foot wide? How much will it cost, at 2 cents per square foot?

254. What will it cost to slate a roof 44 feet by 63, at \$3.80 per square of 100 square feet each.

PROCESS.

44 ft. × 63 ft. = 2772 sq. ft.
2772 sq. ft. ÷ 100 = 27.72 squares
\$3.80 × 27.72 = \$105.34

ANAL. STEPS. — (1.) *Find number of square feet.*
(2.) *Find number of squares of* 100 *square feet.*
(3.) *Find cost of* 27.72 *squares.*

255. What will it cost to have a tin roof put on a house, each slope being 60 feet by 37, at \$2.27 per square of 100 feet?

256. What will it cost to pave a walk 300 feet long, 8 feet wide, at \$3 per 100 square feet?

257. What will it cost to pave a street 80 rods long, and 79 feet between curb-stones, at \$7.25 per 100 square feet?

258. How much will it cost to plaster the walls and ceiling of a room 17 feet long, 13 feet wide, and 9 feet high, at \$7.11 per 100 square feet?

259. 1 cubic yard is considered a load: how many loads of earth must be removed to dig a cellar 40 feet long, 18 feet wide, and 9 feet deep?

260. What will it cost to dig a cellar 27 by 60 feet, 10 feet deep, at \$.75 per load?

261. How many cubic feet in a bin 8¼ feet long, 3 feet wide, and 4 feet deep?

REVIEW. — What is meant by a fraction? (79.) What does the denominator show? (80.) What does the numerator show? (81.) What is a proper fraction? (82.) An improper fraction? (83.) A mixed number? (84.) Name the principles affecting the *terms of a fraction*. (85.) How does multiplying the terms of a *fraction by the same number* affect the value? (86.) How does dividing the *terms by the same number* affect the value? (88.)

262. Making no allowance for thickness of boards or corners, how many square feet of boards will it take to make a box and cover 15 feet long, 8 feet wide, and 4 feet high? What will they cost, at $2.50 per hundred?

263. Making no allowance for corners, how many perches of stone are required in the walls of a cellar 37 feet long, 18 feet wide, 6 feet high, and 3 feet thick?

264. What will 24 gallons 1 quart of domestic wine cost, at $2.75 per gallon?

265. What will 25 gallons 3 quarts of molasses cost, at 60 cents per gallon?

266. What should I give for a pint of domestic wine, at $2 per gallon?

267. Bought 14 barrels of cider, holding 40 gallons each, at 18 cents a gallon, to be paid for in potatoes, at $1.80 per bushel: how many bushels must be given in payment?

268. How many bushels are contained in a bin 4 feet long, 3 feet wide, and 2 feet deep?

269. How much sugar at 12 cents per pound must be given for 359 pounds of pork at 7 cents per pound?

270. What is .32 of a ton of iron worth, at 2 cents a pound?

271. At 11 cents a pound, what is the value of $\frac{3}{5}$ of a hundred-weight of rice? Of .024 of a cwt. of sugar?

272. How many loads of corn, 22 bushels each, can

Review. — When is a fraction said to be at its lowest terms? (87.) Reduce $2\frac{3}{4}$ to an improper fraction, and write an analysis. (89.) Reduce 305 eighths to a mixed number, and write an analysis. (90.) When are fractions said to have a common denominator? (92.) Upon what principle does the addition of fractions depend? (93.) Add $\frac{2}{3}$ and $\frac{3}{4}$, and write the analysis. Find the difference of $\frac{7}{8}$ and $\frac{3}{4}$, and write the analysis. (94.) Give the rule for multiplying a fraction by an integer. (95.) For multiplying an integer by a fraction. (96.) For multiplying a fraction by a fraction. (97.)

be taken from a bin 8 feet wide, 6 feet deep, and 9 feet long?

273. A farmer has 1000 bushels of potatoes: how many barrels are required to hold them, if each barrel holds 2 bushels, 3 pks?

274. Bought 200 gallons of molasses, paid $0.45 per gallon, and sold at the rate of 3 gallons for $2: what was the total gain?

275. How many rods of fence will enclose a farm $\frac{1}{3}$ of a mile square?

ANALYSIS OF SECTION X.

I. Compound Denominate Numbers.

1. Illustration.
2. Definitions.
3. Reduction.
    - (*a.*) *Descending.*
        - Analysis.
    - (*b.*) *Ascending.*
        - Analysis.
4. Space.
    - (*a.*) *Length.*
        - Long Measure.
        - Surveyors' Measure.
        - Analytical Steps.
    - (*b.*) *Area.*
        - Square Measure.
        - Surveyors' Square Measure.
        - Analytical Steps.
    - (*c.*) *Volume.*
        - Cubic Measure.
        - Liquid Measure.
        - Dry Measure.
        - Apothecaries' Fluid Meas.
        - Analytical Steps.
5. Weight.
    - (*a.*) *Apothecaries' Weight.*
    - (*b.*) *Troy Weight.*
    - (*c.*) *Avoirdupois Weight.*
    - *Analytical Steps.*
6. Duration.
    - *Time.*
    - Analytical Steps.
7. Geometrical Measure.
    - *Circular Measure*, or *Angular Measure.*
    - Definitions.
    - Analytical Steps.
8. Currency.
    - (*a.*) *U. S. Currency.*
    - (*b.*) *English Currency.*
    - (*c.*) *French Currency.*
    - Analytical Steps.
9. Addition Denominate Numbers.
    - *Analytical Steps.*
10. Subtraction Denominate Numbers.
    - *Analytical Steps.*
11. Multiplication Denom. Numbers.
    - *Analytical Steps.*
12. Division Denominate Numbers.
    - *Analytical Steps.*
13. Processes.
    - *Analysis.*
14. Application.
    - *Analysis.*

II. Denominate Fractions.

1. COMMON.
2. DECIMAL.
3. REDUCTION.
    - (*a.*) *Descending.*
        - Analytical Steps.
    - (*b.*) *Ascending.*
        - Analytical Steps.
4. PROCESSES.
    - *Analysis.*
5. APPLICATIONS.
    - *Analysis.*

TEST QUESTIONS FOR EXAMINATION:

113. What is a denominate number?

114. What is the reduction of a denominate number? How is the value affected by reduction? Give an illustration.

115. What is meant by reduction descending? How is the value of the number affected? How is the number of units affected? Give an illustration.

116. What is meant by reduction ascending? How is the value of the number affected? How is the number of units affected? Which of the fundamental operations are used in reduction descending? Which in reduction ascending?

117. What is meant by space? Give an illustration. What is meant by a solid? Give an example of a solid. Illustrate the meaning of surface. Of a line. Is the space of a room a solid? With what is it filled? Give examples of surfaces. Can a solid exist without a surface? What is a line? What dimension has it? Give examples of a line. Is a spider's web a line? ANS. — *It is not: it is a solid.* Why? Can a line exist without a surface? Can a surface be found that is not bounded by a line?

118. What are the measures of length? What is an inch? Make a mark an inch in length on the blackboard, or on paper. How many inches in length make a foot? Recite or write the table of long measure. Give the denominations. Give the analytical steps for reducing miles to inches. Inches to miles. Recite or write the table of surveyors' measure. For what is it used? What is the length of a surveyor's chain? How are surveyors' measurements recorded? How long is a link of a surveyor's chain? How long is a link of an engineer's chain? How do engineers record their measurements? How is cloth measured? Which is the unit? How many halves make a yard? Quarters? Eighths? Sixteenths? What is the difference between a statute mile and a geographical or nautical mile? Which is the longer? What is meant by saying that a ship sails ten knots an hour?

119. For what is square measure used? What is a surface? What is a square? What is a parallelogram? What is a square foot? Make a square inch on paper. For what is surveyors' square measure used? What are the dimensions of a square foot? Of a square rod? Of a square chain? Of an acre? Of a section? What are the dimensions of a square yard? How many square feet does it contain? *How many* times is the square foot repeated along one side? How many times *are the rows re*[illegible]? (See ill[illegible].)

120. What is the common rule for finding the area? What work is estimated by the square foot? What by the square yard? By the 100 square feet?

121. What is a cube? What are the dimensions of a cubic foot? Of a cubic yard? How many cubic feet in a cubic yard? Show how cubic feet are arranged to make a cubic yard. How many cubic feet in each row? How many rows in each tier? How many tiers make up the cube? Give the table of cubic measure. For what is it used?

122. What is the rule for finding the solid contents of a body? What is the weight of a cubic yard of earth? What is the weight of a cubic foot of water? What are the dimensions of a cord of wood? What is meant by a cord foot? What is a perch of masonry? What are the usual dimensions of a brick? What is meant by board measure? How is sawed lumber sold?

123. For what is liquid measure used? Recite the table. In calculating the capacity of cisterns, how much is a hogshead estimated to hold?

124. For what is dry measure used? Recite the table. How many cubic inches in a gallon? In a bushel? In a liter? How can gallons be reduced to bushels? Bushels to gallons? Bushels to liters? Liters to gallons? What are the dimensions of a bushel? When is even measure used? When is heaped measure used? How many gills liquid measure make a quart dry measure? How can cubic feet be changed to gallons? To bushels? Bushels to cubic feet? Gallons to cubic feet?

125. For what are apothecaries' weight and fluid measure used? How are drugs and medicines bought and sold? Recite the tables.

126. For what is Troy weight used? Recite the table. What is the weight of a carat? What is meant by saying gold is twenty carats fine?

127. For what is avoirdupois weight used? Recite the table. What is the number of grains in a pound Troy? In a pound apothecary? In a pound avoirdupois? How can Troy pounds be reduced to avoirdupois pounds? How can avoirdupois pounds be reduced to pounds apothecary? How can bushels of corn be reduced to pounds? Bushels of oats to pounds? Pounds of clover-seed to bushels?

129. What is time? Write the table. Write the table of the divisions of the year. Give the abbreviations for the months. Give the number of days in each. When has February 29 days? What is the exact length of a year? What is the cause of leap year? How often does it occur? Why? The usual business month contains how many days? What is meant by July 4, 1876, 9 o'clock, A.M.?

130. What is a circle? What is the center of a circle? What the circumference? An arc? A quadrant? A diameter? A radius? What is the difference between a diameter and a radius? Which radius of a circle is the longest? Which diameter is the least? How is the angle measured? What is the measure of a right angle? What is a degree? A minute? Are the degrees of one circle equal to those of another circle? How can degrees of different circles be used to measure the same angle? Write the table. What is the greatest circumference of the earth? One degree of longitude is equal to how many minutes of the apparent motion of the sun? Give the rule for finding the difference of time between places, the difference of longitude being known.

130. What apparent daily motion has the sun? What causes this apparent motion? How often does the earth rotate? In what direction does the earth turn? In what direction does the sun seem to move? When is it noon at any place? Does noon come to all places on the earth at the same instant? How does the time noon differ in two places? How far does the sun seem to move in 24 hours? In 1 hour? One degree of motion requires how much time? One minute of motion? Of two places, which has the faster time? Which has the faster time, — Chicago, or New York? Chicago, or San Francisco? New Orleans, or Cincinnati? London, or New York?

132. What is money? What is meant by currency? What is used by savages as money? By civilized nations? What is a coin? What is paper money? What kinds of paper money are now used in the United States? What are United States legal tender notes? National bank notes? What is the currency of Great Britain? Recite the table. Name the gold coins. Silver coins. The bronze coins. What is the unit of the French currency? What is its value? How does the Canadian currency differ from Federal money?

133, 134. How are denominate numbers written for addition and subtraction?

135. What is the difference between the multiplication of denominate numbers and simple numbers?

136. Write the analysis of an example in the division of denominate numbers.

137. Write the analysis of an example reducing a common fraction of a higher denomination to integers of lower denominations. In the same manner, reduce a decimal fraction.

138. Write the analysis of an example reducing a denominate number of several terms to a common fraction of a higher denomination. To a decimal of a higher denomination.

ANSWERS.

ADDITION.

Page 18.

2. 334166.
3. 2326369.
4. 3021197.
5. 2870561.

Page 19.

6. 2550245.
7. 1988881.
8. 52596760.
9. 107533540.
10. 113300644.
11. 175304305.
12. 3202103538.
13. 4601005537.
14. 4100384269.
15. 4901478277.

Page 20.

16. 1031875.
17. 504268991.
18. 1307048626.
19. 311803918.
20. 901041406.
21. 1306088136.
22. *804125325.*
23. *619927113.*

Page 21.

25. $25.481.
26. $212.752.
27. $54.902.
28. $44.943.
29. $222.134.
30. $146.816.
31. $60.545.
32. $35.485.

Page 23.

34. $2.98.

Page 24.

35. $4.06.
36. $2695.
37. $109.
38. 394 barrels.
39. $2.22.
40. 1682 sq. miles.

Page 25.

41. $2684.
42. $836.
43. 868 sheep.
44. 659 peach-trees.
45. $777.
46. $987.
47. 320 sheep.
48. $6125.

Page 26.

49. $40772.
50. 986 miles.
51. 1045 pupils.
52. 7732 pounds.
53. $1085.
54. 4888 sheep.
55. 5408 sheep.
56. 4585 sheep.
57. 1821 sheep.
58. 348 melons.
59. $2295.

Page 27.

60. $94.34.

Page 28.

61. 100 acres.
62. 108 acres.
63. 62½ acres.
25 acres.
62½ acres.
64. 37 acres.

5

65. 121½ acres W.
113½ acres E.
66. 62 acres.
67. 7½ acres.
68. 300 rods.
200 rods.
69. 350 rods.
450 rods.
70. 300 rods.
360 rods.
71. 200 rods.
300 rods.

Page 29.

72. 180 rods.
300 rods.
73. $559.02.
74. $126.90.
217 bushels.
75. 188 bushels.
$293.43.
76. 115 tons.
$250.50.
77. 749952 feet.
78. 160 acres.
79. 298 acres.
80. 3526 eggs.
81. 632 barrels.

Page 30.

82. $3.15.
83. 908 trees.
84. $716.
85. 1483 pounds.
86. 107 yards.
87. $85.
88. 256 marbles.
89. *$8348.*
90. *$2360.*
11379 bl'kbirds.

Page 31.

92. $1708.
93. $1401.
94. 48501.
95. $4791.
96. 1796.
97. 2071.
98. $742.
$740.
$1482.
99. $745.
$1355.
100. 278.

Page 32.

101. $9213.
102. $1273.
103. $5.66.
104. 201 rods.
105. 52 rods north.
47 rods south.
99 rods apart.
106. 40 rods.
107. 78 rods.

Page 33.

108. $2538.
109. 96 years.
110. $923.
111. 60 dahlias.
112. 423 cents.
113. $1.31.
114. $16.52.
115. $5.87.
116. $122.815.
117. $86.315.
118. $21.253.
119. $70.506.
120. $2.342.
121. $53.66.

Page 34.

122. $6.322.
123. $0.548.
124. $6.048.
125. $90.607.
126. $2007.02.
127. $5.555.
128. $60.603.
129. $50.361.
130. $11.111.
131. $1339.35.
132. $31.11.
133. $219.31.
134. $64.865.

Page 35.

135. $43.99.
136. $24.10.
137. $4370.
138. $30.74.
139. $4.774.
140. $448.416.
141. $27.28.
142. $251.443.
520 bushels.
143. 817 cents.
$8.17.
67 pounds.
144. 170 cents.
56 in all.
37 oranges & lemons.

Page 36.

145. $891.74.
146. 2931.
147. $71.636.
148. $13.799.
149. $3946.433.
150. $8015.002.

SUBTRACTION.

Page 44.

1. 437 acres.
2. 92 children.
3. 111 barrels.
4. 268 men.
5. $1596.
6. $285.
7. $45.
8. 3 marbles.
9. 8.
10. 63.
11. 6263 lambs.

Page 45.

12. 300 cents.
13. 9279.
14. 1282.
15. 10269.
16. 39 years.
17. 71 years.
18. 128 years.
19. 1302.
20. 1769.
21. 85 years.
22. 111 years.
23. 1182.

Page 46.

24. 44.
25. 1774.
26. 797.
27. 3322.
28. 18 years.
29. 848.
30. 1060.
31. 286.
32. 3504.
33. 4580.
34. 34984.
35. 2464.
36. 42593.
37. 21997.
38. 116787.
39. 302122.
40. 434288.

Page 47.

41. $85470.
42. $99.43.
43. $388.357.
44. $51.366.
45. $305.66.
46. $282.87.
47. $94.05.
48. $99.
49 $9.90.
50. $0.09.
51. $0.009.
52. $3.189.
53. $2.211.
54. $2.764.
55. $56.666.
56. $75.787.
57. $8.287.
58. $51.50.
59. $2.50.
60. $944.961.
61. $61.213.
62. $24.625 lost.

Page 48.

63. $12.125.
64. $59.827.
65. $63.
66. $15 gained.
67. $5.
68. $53.
69. 82 miles.
70. 93 miles.

Page 49.

71. 53 years.
72. $0.03.
73. 72 nuts.
74. $0.14.
75. 135 sheep.
76. 20 oranges.
77. $46.
78. $11.
79. $0.14.
80. $195.

Page 50.

81. Nothing due.
82. $0.87.
83. 29 marbles.
84. $20.
85. $3.
86. $30.
87. $7.
88. $81.
89. $128.

Page 51.

90. 211 cents.
91. $5968.
92. $0.07.
93. $5.11.
94. 136 yards.

Page 52.

95. 88 nuts.
96. 1374 sheep.
97. $1227.
98. $4.17.
99. $3159.
100. Given.
101. $574.
102. 1282 sheep.

Page 53.

103. 2275 men.
104. 94 bushels.
105. 250 miles.
106. 120 minutes.
107. 131 minutes.
108. 252 minutes.
109. 58 minutes.
110. 259 minutes.
111. 639 minutes.
112. 366 minutes.

Page 54.

113. $87.37.
114. $19.81.
115. $5.36 lost.
116. 280 barrels.
$1880.
117. $4078.29.
118. $184.54 gain.
119. $4227.10.
120. $32.48.
121. $63.38.

Page 55.

122. $0.06.
123. $16.
124. 250 barrels.
125. 16 days.
126. 19 days.
127. $2379.74.
128. $0.86 lost.
129. 25 miles.
130. $18.352, amt. of bill.
$9.599.
131. $24.15, amt. of bill.
$15.85 change.

Page 56.

132. $54.23 amt. of bill.
$45.77 lacking.

MULTIPLICATION.

Page 65.

1. Given.
2. $22.75.
3. $69.60.
4. $2.91.
5. $29.70.

Page 66.

6. $2.48.
$7.28.
7. 94 days.
8. 1341 days.
9. 1500 days.
10. 407 days.
11. 7398 horses.
12. *$39.*
13. *$2.97.*
14. *$1.71.*
15. $3.56.
16. $6.45.

Page 67.

17. $8.89.
18. $10.00.
19. $21.98.
20. 7616.
21. 1584 men.
22. 560 cows.
23. Given.
24. $61.

Page 68.

25. $100.
26. $48.
27. 127 miles.
28. 427 shillings.
29. 5 yards.
30. 4 yards.
31. $1.94.
32. 264 cents.
33. $97.
34. $129.
35. $105.

Page 69.

36. 188 pounds.
37. 23 gallons.
38. $3.12.
39. $4.08.
40. 1443 shillings.
41. $71.40.
42. 960 pounds.
43. $9.
44. 13 cents.

45. 182 trees.
46. 1000 trees.
47. 64 bushels.

Page 70.

49. 4200 dimes.
350 dimes.
50. 20000 mills.
75000 mills.

Page 71.

51. 2500 mills.
1830 mills.
52. Given.
53. $2.61.
$338.43.
54. $3.60 for hose.
$17.50 for boots
$6.00 for shoes.
$1.52 " gloves.
$7.00 " kid "
Bill $35.62.
55. $1.20 1st r'ders.
$2.50 2d "
$2.10 3d "
$7.20 4th "
$8.00 5th "
Bill $21.00.
56. $0.55 for sugar.
$2.70 " tea.
$0.88 for coffee.
$2.48 " raisins.
$0.64 " rice.
Bill $7.25.
57. $0.02 change.

Page 72.

58. $0.54 fr. $2 bill.
$3.54 " $5 "
$8.54 " $10 "
59. $0.65 cost.
$0.35 change.
60. $0.77.
61. $5.10.
62. $2.28.
63. $43.80.
64. $105.
$1277.50.
65. $31.25.
$2975.

Page 73.

66. 956 pounds.
2390 pounds.
183552 pounds.
67. $239.
$18355.20.
68. 376 gallons.
31576 gallons.
69. 736 pounds.
55648 pounds.
70. $4.41.
$59.19.
71. $13.75.
$459.25.
72. $1.44.
$5.04.
73. $4.80.
74. $1.78.
$8.22.
75. $1.20.
$3.84.
76. $9.63.
$11.50.
$8.46.
77. $6.23.
$100.90.

Page 74.

78. $97.25.
79. $294.90.
80. $6.20.
81. $15.375.
82. $69.12.
83. $129.235.
84. $28.233.
85. $297.30.
86. $0.453.
87. $302.945.
88. $651.03.

DIVISION.

Page 85.

9. $148.
10. $3776.
11. $2401.
12. 3924 miles.

Page 86.

13. *3420 cents.*
14. 680 trees.
15. 288 miles.
16. 450 bushels.
17. 2520 cents.
18. $2702.
19. 1351 boxes.
20. 72 years.
21. 347 pencils.

Page 87.

22. 89 pounds.
23. 974 barrels.
24. 30 men.
25. 14 men.
26. 50 persons.
27. 128 coats.
28. 123 hours.

29. 49 pounds.
30. $3082\frac{2}{3}$ oranges.
31. 30 pounds.
32. 140 rods.

Page 88.

33. 12 rods.
34. 9 days.
35. Given.
36. 63360 inches.
37. 2016 gills.
38. 960 farthings.
39. 3000 families.
40. 1120 feet.
41. 1150 men.
42. 192 hours.
43. $\$774\frac{4}{5}$.

Page 89.

44. $449.
45. $36835\frac{2}{4}$.
46. $8264\frac{4}{5}$.
47. Given.
48. $476\frac{8}{10}$ pounds.

Page 91.

49. $144.
50. 60 pounds.
300 pounds.
51. 84 bushels.
52. 1000 gallons.
53. 25 barrels.
54. 40 barrels.

Page 92.

55. 640 rods.
13440 rods.
56. 2 miles.
$22\frac{124}{320}$ miles.
57. 2.
2.
$52\frac{5}{9}$.
$198\frac{28}{46}$.
58. $10\frac{1}{46}$.
59. 5.
60. 36 sum.
61. 52 rem.
62. 1918 prod.
63. $6\frac{19}{92}$ quo.
64. 195 prod.
65. 378 sum.
66. $43\frac{10}{21}$ quo.
67. 399 rem.
68. 69.
69. 3.
70. 40.
71. 363.
72. 35.
73. $8\frac{2}{4}$.
74. $8\frac{2}{4}$.
75. $19\frac{2}{4}$.

Page 93.

76. 11.
77. 11.
78. 132.
79. 38.
80. 6.
81. $2\frac{9}{11}$.
82. 455.
83. $(408 \div 12) - 3 = 31$.
84. $\$.20 + \$1.30 + \$.875 + \$2.30 = \$4.675$.
85. $\$.80 + \$.40 - \$1.00 = \$.20$.
86. $\$0.22 \times 8 = \$1.76$.
87. $\$72 \div 12 \times 5 = \$30$.
88. $(\$54 \div 18) - (\$25 \div 12) = \$0.91\frac{2}{3}$.

Page 96.

89. $89.
90. $257.
$243.
91. $1.16.
$8.84.
92. 230 sheep.
193 sheep.
93. Given.
94. 18 sheep.
210 sheep.
95. 20 pounds.

Page 97.

96. $107.
97. Nothing.
98. 82 cents.
$4.18.
99. $23.47.
100. $2.06.
$24\frac{6}{12}$ pounds.
101. 38 yards.
102. 11 miles.
103. 9 miles.
48 miles.

Page 98.

104. 72 miles.
80 miles.
105. 95 miles.
245 miles.
106. Given.
107. 1143 cents.
1341 cents.
108. 360 cents.
1038 cents.

109. $2568.
$59896.
110. 760 problems.
2000 "
111. $1372.
65289.
112. 972 miles.
6696 miles.
113. 648 miles.
64152 miles.

Page 99.

114. $37544.
115. $25311.
116. $228.
$60.
117. $137.30.
$54.92.
118. $1368.
119. Given.
120. 32 men.
121. 8 days.
30 days.
122. 14 hours.
742 hours.
123. 110 lemons.
124. 28 barrels.

Page 100.

125. 18 dozen.
72 dozen.
126. 26 eggs.
3704 eggs.
127. 60 gallons.
57668$\frac{22}{35}$ galls.
128. $4.60.
129. $20.30 lost.
130. $3.
$4.
131. $2 loss for ea.
132. 2 cwt.
4 cwt.
133. $20 ea. for 2.
$60 each for others.

Page 101.

134. $6 each for 3.
$30 for the other.

U. S. CURRENCY.

Page 103.

1. 100 times.
111 times.
2. 100 times.
370 times.

Page 104.

3. 100 times.
10 times.
26 times.
4. 9 times.
11 times.
5. 110 times.
11 times.
80 times.
6. 27 times.
135 times.
75 times.
7. *$1.625.*
8. $2.125.
9. $1.625.
10. $1.05.
11. $8.875.
12. $1.25.
13. $2.80.
14. $2.40.
$4.20.
$3.00.

Page 105.

15. $20.82.
$16.56.
16. $2970.08.
17. $5809.125.
18. $6989.375.
$8387.25.
19. $4382.15.
$7887.87.
20. Given.
21. $0.472 +.
$0.638 +.
$1.208 +.
22. $2.75.
$37.125.
$1377.75.
23. $0.495.
$50.655.
$660.495.
24. $0.167 +.
$0.171 +.
$0.178 +.
25. $0.093 +.
$0.156 +.
26. 20 oranges.
40 oranges.
27. 733 pounds.
28. 296 oranges.
592 oranges.
29. 108 pounds.
54 pounds.

Page 106.

30. 24 yards.
1632 yards.
18437 yards.
31. $121.875.
$140.625.
$67.875.
32. $19.305.
$14.625.
$18.72.
33. $1.071 +.
$1.964 +.
$1.410 +.
34. $64.17.
$85.002 +.
$93.
35. $3.40.
$3.23.
$2.55.
$2.992.
36. $0.793 +.
$0.555 +.
37. $3.875.
38. $1.10.
$4.125.
$20.625.
39. $0.225.
$2.25.
40. $0.225.
$150.225.
41. $29.435.
42. $6355.045.
43. $32943.725.

Page 107.

44. $191.17.

Page 108.

45. $399.32.
46. $10933.32.
47. $93.25.
48. $412.74.

Page 109.

49. $6938.71.
50. $88204.10.

PRIME FACTORS.

Page 114.

1. 2, 2, 2, 3.
2. 2, 2, 3, 3.
3. 2, 2, 2, 2, 3.
4. 2, 2, 2, 2, 3, 3.
5. 2, 2, 2, 3, 5.
6. 2, 2, 2, 17.
7. 5, 29.
8. 3, 3, 5, 5.
9. 2, 2, 199.
10. 2, 2, 2, 2, 2, 2, 3, 3.
11. 2, 2, 2, 2, 2, 2, 2, 2.
12. 2, 2, 2, 43.
13. 2, 2, 2, 2, 2, 3, 3, 3.
14. 3, 3, 3, 5, 7.
15. 2, 2, 2, 2, 2, 2, 5.
16. 2, 2, 5, 5, 5.
17. 5, 193.
18. 7, 7, 143.
19. 2, 2, 3, 7, 97.
20. 719.
21. 11, 83.
22. 2, 2, 2, 3, 1001
23. 3, 5, 43.
24. 2, 2, 2, 3, 5, 7

GREATEST COMMON DIVISOR.

Page 115.

25. 6, 4, 6.
26. 3, 4, 2.
27. 9, 13, 24.
28. 78, 4.

Page 116.

29. 7.
30. 22.
31. 17.
32. 17.

LEAST COMMON MULTIPLE.

Page 117.

33. 90.
34. 240.
35. 12600.
36. 504.
37. 1134.
38. 144.
39. 2520.
40. 194040.
41. 156240.
42. 144.
43. 19530.
44. 4095.
45. 6384.
46. 336.

CANCELLATION.

Page 118.

47. $2\frac{2}{35}$.
48. 20.
49. $6\frac{89}{96}$.
50. 63.
51. $34\frac{2}{3}$.
52. 36.
53. $6\frac{3}{4}$.

Page 119.

54. $18\frac{2}{3}$.
55. 6.
56. 3.
57. 16.
58. 90 cents.
59. 6 tons.
60. 75 cents.

FRACTIONS.

Page 126.

1. $\frac{1}{2}$.
3. $\frac{1}{3}$.

Page 127.

5. $\frac{2}{5}$, $\frac{2}{3}$.
6. $\frac{1}{7}$, $\frac{2}{7}$.

Page 128.

7. $\frac{8}{9}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{1}{9}$.
8. $\frac{3}{5}$, $\frac{11}{12}$, $\frac{5}{6}$.
9. $\frac{7}{11}$, $\frac{11}{20}$, $\frac{75}{89}$.
10. $\frac{3}{10}$, $\frac{7}{20}$, $\frac{11}{20}$, $\frac{22}{123}$.
11. $\frac{9}{50}$, $\frac{22}{25}$, $\frac{11}{40}$, $\frac{2}{7}$, $\frac{3}{4}$, $\frac{226}{107}$.
12. $\frac{3}{4}$, $\frac{22}{25}$, $\frac{19}{20}$, $\frac{3}{4}$, $\frac{134}{269}$. $\frac{279}{178}$, $\frac{3}{5}$.

Page 129.

13. $\frac{171}{4}$.
14. $\frac{201}{7}$.
15. $\frac{275}{3}$.
16. $\frac{607}{9}$.
17. $\frac{538}{9}$.
18. $\frac{187}{4}$.
19. $\frac{4384}{5}$.
20. $\frac{777}{9}$.
21. $\frac{827}{2}$.
22. $\frac{5461}{8}$.
23. $\frac{752}{9}$.
24. $\frac{5373}{13}$.
25. $\frac{10092}{11}$.
26. $\frac{12980}{17}$,
27. $\frac{32567}{79}$.

Page 130.

28. $1703\frac{3}{4}$, $53\frac{1}{4}$.
29. $169\frac{1}{2}$, $224\frac{1}{4}$.
30. $179\frac{1}{5}$, $74\frac{1}{5}$.
31. 136, $82\frac{3}{5}$.
32. $1118\frac{5}{6}$, 78.
33. $701\frac{6}{7}$, $63\frac{2}{5}$.
34. $1229\frac{1}{2}$.
35. $5203\frac{3}{4}$.
36. $717\frac{1}{11}$.
37. $8376\frac{2}{5}$.
38. $304\frac{9}{16}$.
39. $129\frac{1}{22}$.
40. $1658\frac{35}{37}$.
41. $31\frac{57}{67}$.
42. $10\frac{157}{191}$.
43. $159\frac{5}{6}$.
44. $1\frac{158}{631}$.
45. 1.
46. $1\frac{300}{397}$.
47. 1.
48. $35\frac{208}{241}$.
49. $7\frac{665}{874}$.
50. $7\frac{56}{83}$.
51. $75\frac{13}{92}$.

Page 106.

30. 24 yards.
1632 yards.
18437 yards.
31. $121.875.
$140.625.
$67.875.
32. $19.305.
$14.625.
$18.72.
33. $1.071 +.
$1.964 +.
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7. 5, 29.
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9. 2, 2, 199.
10. 2, 2, 2, 2, 2, 2, 3, 3.
11. 2, 2, 2, 2, 2, 2, 2, 2.
12. 2, 2, 2, 43.
13. 2, 2, 2, 2, 2, 3, 3, 3.
14. 3, 3, 3, 5, 7.
15. 2,2,2,2,2,2,5.
16. 2, 2, 5, 5, 5.
17. 5, 193.
18. 7, 7,143.
19. 2, 2, 3, 7, 97.
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22. 2, 2, 2, 3, 1001.
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48. 20.
49. $6\frac{89}{96}$.
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51. $34\frac{2}{3}$.
52. 36.
53. $6\frac{3}{4}$.

Page 119.

54. $18\frac{2}{3}$.
55. 6.
56. 3.
57. 16.
58. 90 cents.
59. 6 tons.
60. 75 cents.

FRACTIONS.

Page 126.

1. $\frac{1}{2}$.
3. $\frac{1}{3}$.

Page 127.

5. $\frac{2}{5}$, $\frac{2}{3}$.
6. $\frac{1}{7}$, $\frac{2}{7}$.

Page 128.

7. $\frac{8}{9}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{1}{9}$.
8. $\frac{3}{5}$, $\frac{11}{12}$, $\frac{5}{6}$.
9. $\frac{7}{11}$, $\frac{11}{20}$, $\frac{75}{89}$.
10. $\frac{3}{10}$, $\frac{7}{20}$; $\frac{11}{20}$; $\frac{22}{123}$.
11. $\frac{9}{50}$, $\frac{22}{25}$. $\frac{11}{40}$. $\frac{4}{7}$. $\frac{3}{4}$. $\frac{226}{107}$.
12. $\frac{3}{4}$. $\frac{22}{25}$. $\frac{19}{20}$. $\frac{3}{4}$. $\frac{134}{283}$. $\frac{279}{175}$. $\frac{3}{5}$.

Page 129.

13. $\frac{171}{4}$.
14. $\frac{201}{7}$.
15. $\frac{275}{3}$.
16. $\frac{601}{9}$.
17. $\frac{538}{9}$.
18. $\frac{187}{4}$.
19. $\frac{4384}{5}$.
20. $\frac{177}{9}$.
21. $\frac{821}{2}$.
22. $\frac{5461}{8}$.
23. $\frac{152}{9}$.
24. $\frac{5373}{14}$.
25. $\frac{10092}{11}$.
26. $\frac{12982}{17}$.
27. $\frac{22561}{73}$.

Page 130.

28. $170\frac{3}{4}$. $53\frac{1}{4}$.
29. $169\frac{1}{2}$. $221\frac{1}{4}$.
30. $179\frac{1}{5}$. $74\frac{1}{5}$.
31. 136, $82\frac{1}{6}$.
32. $1118\frac{5}{8}$, 78.
33. $701\frac{6}{7}$, $63\frac{7}{8}$.
34. $1229\frac{1}{2}$.
35. $520\frac{3}{8}$.
36. $717\frac{1}{11}$.
37. $8376\frac{4}{5}$.
38. $304\frac{9}{16}$.
39. $129\frac{1}{22}$.
40. $1658\frac{34}{37}$.
41. $31\frac{47}{87}$.
42. $10\frac{87}{131}$.
43. $159\frac{3}{8}$.
44. $1\frac{15}{31}$.
45. 1.
46. $1\frac{22}{37}$.
47. 1.
48. $35\frac{47}{211}$.
49. $7\frac{22}{24}$.
50. $7\frac{2}{5}$.
51. $75\frac{1}{12}$.

36. $\$3.75$.
$\$14.87\frac{1}{2}$.
37. $\$1.44\frac{9}{10}$.
$\$4.34\frac{7}{10}$.
38. $9\frac{19}{24}$ shillings.
$26\frac{7}{8}$ shillings.
39. $\$0.40\frac{8}{27}$.
$\$0.83\frac{19}{63}$.
$\$8.59\frac{1}{45}$.
40. $\$0.65\frac{25}{64}$.
$\$2.11\frac{13}{21}$.
41. $12\frac{19}{36}$ yds.
42. $\$0.15\frac{5}{18}$.
$\$0.76\frac{7}{18}$.
$\$1.01\frac{23}{27}$.
43. $73\frac{13}{14}$ lbs.
44. $\$4.31\frac{117}{128}$.
$\$4.35\frac{75}{112}$.

Page 157.

45. $114\frac{1}{3}$.
$89\frac{5}{6}$.
$134\frac{3}{4}$.
$70\frac{7}{9}$.
46. $106\frac{2}{7}$.
$70\frac{16}{29}$.
$21\frac{9}{35}$.
$19\frac{3}{8}$.
47. $24\frac{7}{11}$.
$15\frac{18}{53}$.
$6\frac{241}{276}$.
$18\frac{97}{106}$.
48. $130\frac{27}{338}$.
$50\frac{417}{871}$.
$40\frac{807}{1075}$.
$179\frac{16}{35}$.

Page 159.

50. *$12\frac{7}{9}$ persons.*
$13\frac{4}{5}$ persons.
$12\frac{1}{21}$ persons.
51. $2\frac{39}{200}$ persons.
$1\frac{47}{70}$ persons.
$1\frac{11}{40}$ persons.
52. $11\frac{1}{2}$ times.
$20\frac{4}{7}$ times.
$185\frac{1}{22}$ times.
53. $656\frac{1}{2}$ oranges.
1023 oranges.
$680\frac{7}{16}$ oranges.
54. $\$0.45\frac{5}{11}$.
$\$0.77\frac{7}{9}$.
$\$0.47\frac{1}{17}$.
$\$7\frac{9}{13}$.
55. $\$0.80$.
$\$0.857\frac{1}{7}$.
$\$0.923\frac{1}{13}$.
$\$0.285\frac{5}{7}$.

Page 161.

57. $388\frac{1}{3}$ tons.
58. $12\frac{5}{14}$ bbls.
59. $150\frac{1}{16}$ lbs.
$166\frac{2}{35}$ lbs.
$140\frac{1}{30}$ lbs.
60. $\$44\frac{211}{435}$.

Page 162.

62. $\$0.64\frac{1}{6}$.
$\$0.825$.
63. $\$15\frac{61}{119}$.
$\$12\frac{103}{153}$.
64. $72\frac{15}{16}$ lbs.
$\$22.36\frac{3}{4}$.
65. $\$1.375$.
66. $\$5.13\frac{21}{58}$.
67. $\$1.37\frac{2}{9}$.
$\$0.19\frac{38}{63}$.
$\$0.37\frac{14}{33}$.
$\$3.88\frac{43}{54}$.

Page 163.

69. $\$0.21\frac{2}{3}$.
70. $\$2.66\frac{497}{1112}$.
71. $\$0.25\frac{265}{782}$.
72. $\$\frac{49}{340}$.
73. $100\frac{49}{110}$ bushels.
74. $\$5.25$.
75. $\$34.28\frac{27}{32}$.
76. $\$14.50$.
77. $\$15.55\frac{5}{8}$.
78. $\$11.50\frac{1}{2}$.
79. $\$5.95$.
80. $\$1.89\frac{7}{8}$.
81. 60 cords.

Page 164.

82. $80\frac{5}{8}$ miles.
83. $\$24.87\frac{3}{8}$.
84. $77\frac{38}{81}$ feet.
86. $6\frac{10}{13}$ yds.
$1\frac{7}{48}$ yds.
87. $1\frac{7}{293}$ yds.
$5\frac{110}{293}$ yds.
$6\frac{267}{293}$ yds.
88. $162\frac{8}{11}$ qts.
$276\frac{1}{4}$ qts.

Page 165.

89. $7\frac{24}{33}$ yds.
$83\frac{5}{81}$.
90. $\$0.14$.
91. $\$0.26\frac{1}{3}$.
$\$0.22\frac{74}{185}$.
$\$0.38\frac{18}{47}$.
92. $\$1.92\frac{31}{45}$.
$\$2.75\frac{25}{48}$.
$\$2.21\frac{1037}{1140}$.
93. $\$0.50\frac{2}{5}$.
$\$0.72\frac{9}{20}$.
$\$7.16\frac{2}{3}$.
94. $7\frac{17}{24}$ shillings.
$80\frac{5}{8}$ shillings.
95. $\$0.04\frac{5}{28}$.
$\$0.06$.

96. $\$0.05\frac{11}{48}$.
$\$0.04\frac{5}{12}$.
$\$0.18\frac{145}{261}$.

Page 166.

98. $\frac{21}{22}$.
99. $2\frac{4}{9}$.
100. $\frac{31}{40}$.
101. $8\frac{3}{11}$.
104. $1\frac{5}{9}$ shillings.
$18\frac{11}{20}$ shillings.
$402\frac{2}{5}$ shillings.
105. $\$1.57\frac{1}{2}$.
$24.30.
106. $\$1\frac{1}{6}$.
$\$3\frac{1}{2}$.
$\$34\frac{11}{18}$.

Page 167.

107. $8.
$\$40\frac{4}{11}$.
$\$504\frac{6}{11}$.
108. $\$12\frac{1}{2}$.
$\$62\frac{1}{2}$.
$\$91\frac{2}{3}$.
109. 75 lbs.
$55\frac{5}{9}$ lbs.
$85\frac{5}{7}$ lbs.
110. $122\frac{1}{2}$ lbs.
3136 lbs.
$4573\frac{1}{3}$ lbs.
111. 40 lbs.
2796 lbs.
112. $87.71.
$313.60.
$627.20.
$19913.60.
113. $\$20\frac{4}{9}$.
$\$116\frac{8}{15}$.
$\$19626\frac{2}{3}$.
$\$107946\frac{2}{3}$.

114. $\$0.41\frac{2}{3}$.
$\$0.61\frac{7}{8}$.
$3.20.
$\$11.73\frac{1}{3}$.
115. $29\frac{11}{16}$ years.
116. $180.15 in safe.
$480.40 in both.

Page 168.

117. 18.
$70\frac{7}{8}$.
$3526\frac{1}{4}$.
118. 50.
$13\frac{5}{7}$.
119. $\frac{5}{32}$ bushel.
$\frac{9}{40}$ bushel.
120. $\$\frac{1}{12}$ more.
$\$1\frac{7}{12}$ both.
$\$8\frac{5}{12}$ left.
121. $\$3\frac{5}{8}$.
$\$27\frac{47}{56}$.
122. $\$14\frac{1}{4}$ both.
$\$5\frac{3}{4}$ lacking.

Page 169.

126. $\frac{11}{12}$.
$\frac{1}{12}$.
$1503.
127. $\$30\frac{10}{147}$.
128. $\$43\frac{4}{5}$.
129. $\$6\frac{1}{24}$.
130. $\$1\frac{14}{25}$.
$\$8\frac{1}{25}$.
132. $\$9\frac{29}{60}$.
133. $\frac{15}{56}$ too large.
$\frac{12}{35}$ too large.
1 too small.
$1\frac{4}{77}$ too large.
134. $\$\frac{1}{56}$ gained.
135. $\$\frac{41}{110}$.

Page 170.

136. $\$5\frac{31}{40}$.
$\$6\frac{3}{10}$.
$\$12\frac{3}{5}$.
$\$36\frac{3}{4}$.
137. $\$72\frac{2}{9}$.
$91\frac{13}{27}$.
138. $\$\frac{2}{5}$.
$\$\frac{3}{44}$.
$\$\frac{42}{715}$.
$\$27\frac{3}{11}$.
140. $1\frac{31}{54}$.
$\$38\frac{6}{7}$.

Page 171.

141. $\$0.05\frac{5}{14}$.
$\$1.33\frac{13}{14}$.
$\$4.30\frac{15}{56}$.
$\$21.54\frac{1}{56}$.
142. $\$15.91\frac{2}{3}$.
$79.58\frac{1}{3}$.
143. $\$5.68\frac{3}{4}$.
$250.
145. 46 bushels.
$8\frac{3}{8}$ bushels.
86 bushels.
$64\frac{1}{2}$ bushels.
$67\frac{4}{7}$ bushels.
146. $\$94\frac{2}{7}$.
147. $10\frac{18}{55}$.
$\frac{8}{27}$.
$\frac{33}{58}$.
148. $\$51.428\frac{4}{7}$.
$\$83.636\frac{4}{11}$.

Page 172.

149. $128\frac{2}{25}$ acres.
150. $\$24.17\frac{1}{7}$.
151. $750.
152. $\$1166.66\frac{2}{3}$.
153. $\$10\frac{8}{23}$.
154. $8\frac{3}{8}$ bu

ADDITION OF DECIMALS.

Page 177.

1. 24.7.
2. 39.31.
3. 57.99.
4. 97.17.
5. 88.41.
6. 93.492.
7. 81.807.
8. 43.762.

Page 178.

9. .6367.
10. 622.6363.
11. 6.00186.
12. 108.33033.
13. 11.88237.
14. 18.27005.

SUBTRACTION OF DECIMALS.

15. 58.74.
16. 19.97.
17. .05023.
18. .494541.
19. .995299.
20. 19.997.
21. .297.
22. 2999.997.
23. 100999999.999899.
24. 78999.9921.

Page 179.

25. 58.41.
3.33.
36.963.
26. 59.34 years.
2.54 years.
27. $311.50.
236.64.
28. 12.03 gal.
97.5 gal.
29. 54.50 rods.
9 rods more.
30. $108.93.
$22.13.
31. 1.60.
32. 1.81.
33. 13.64.
34. 4.91.
35. 4.859.
36. 11.834.

Page 181.

37. 4.492.
38. 5.738.
39. 6.158.
40. 7.474.
41. 6.567.
42. 12.872.
43. 5.502.
44. .91021.
45. 3.54476.

MULTIPLICATION OF DECIMALS.

Page 184.

46. 12.9.
75.09.
47. 2.952.
60.66.
48. .003784.
.01624.
49. *.0162.*
.000618.
50. .4824.
.032272.
51. 240.024.
2.40024.
3.60036.
52. .06934.
.0080006.
.00014088.
53. $197.75.
$226.59.
54. $2.46.
$2.55.
$2.314.
55. $84.
$0.75.
56. $5.925.
$2.025.

Page 185.
57. $14.90 lost.
58. 1.841\frac{2}{3}$ gained.
59. $0.50 gained.
60. $1.50 lost.
61. 8.944\frac{4}{9}$.
62. 6.077$\frac{1}{2}$.
63. 68.38.
64. 3.51225.
65. 13.
66. 0.
67. 0.

DIVISION OF DECIMALS.

Page 188.
68. .8525.
69. 33.5.
70. 13.05714 +.
71. 167.5.
72. 223.333 +.
73. .25.
74. 87.5.
75. 16.6666 +.
76. 571.42857.
77. 4000.
78. 5.
79. 10.
80. .01.
81. 1.
82. 10.
83. 1.6666 +.
84. .6666 +.
85. 2.
86. .1.
87. 1000.

Page 189.
88. 108.1 cords.
282.9 cords.
89. $69.60.
164.40
90. $62.10.
$6630.90.
91. $13.35.
$3.3642.
$228.1782.
92. 114.5 gal.
123.675 gal.

Page 190.
93. $2.60.
$3.785.
94. 38.1904 lbs.
286.428 + lbs.
95. 562.5 lbs.
2238.75 lbs.

COMMON FRACTIONS TO DECIMALS.

Page 191.
97. .625.
.54545 +.
.571428 +.
.81818 +.
(*1.*) $.1666 +.
$.1428 +.
$.2857 +.
$.375.
(*2.*) .5714 +.
.3333 +.
.3636 +.
.8571 +.
(*3.*) .8888 +.
.0945 +.
.2162 +.
.0947 +.
(*4.*) .0740 +.
.1052 +.
.0061 +.
.0683 +.
(*5.*) .0364 +.
.0133 +.
.0146 +.
.0019 +.

Page 192.
98. $\frac{1}{2}$.
$\frac{1}{20}$.
99. $\frac{1}{4}$.
$\frac{3}{4}$.
$\frac{11}{20}$.
$\frac{1}{8}$.
$\frac{37}{200}$.

Page 193.
100. (*1.*) $\frac{3}{11}$.
$\frac{8}{9}$.
$\frac{5}{9}$.
$\frac{5}{11}$.
(*2.*) $\frac{7}{9}$.
$\frac{7}{11}$.
$\frac{7}{30}$.
$\frac{4}{11}$.
(*3.*) $\frac{13}{30}$.
$\frac{29}{30}$.
$\frac{1}{9}$.
$\frac{7}{9}$.
(*4.*) $\frac{1}{7}$.
$\frac{2}{7}$.
$\frac{4543}{11000}$.
$\frac{1}{6}$.

102. (*1*.) $\frac{7}{8}$.
$\frac{21}{50}$.
$\frac{11}{20}$.
$\frac{3}{4}$.
$\frac{9}{20}$.
$\frac{151}{200}$.
$\frac{1}{8}$.
(*2*.) $\frac{3}{20}$.
$\frac{3}{200}$.
$\frac{1}{40}$.
$\frac{1}{5}$.
$\frac{4}{5}$.
$\frac{7}{25}$.
$\frac{3}{8}$.
(*3*.) $\frac{1}{3}$.
$\frac{2}{3}$.
$\frac{7}{9}$.
$\frac{8}{9}$.
$\frac{6}{25}$.
$\frac{63}{100}$.
$\frac{4}{9}$.
(*4*.) $\frac{6}{25}$.
$\frac{7}{20}$.
$\frac{1}{250}$.
$\frac{5}{11}$.
$\frac{19}{75}$.
$\frac{37}{50}$.
$\frac{7}{25}$.

Page 194.

103. 8.0714.
104. 1.15625 lbs.
105. $0.921.
106. $0.0557.
107. 5.94 bbls.
108. 57.8687.
109. .675.
110. 5.5027 +.
111. 15.7527 +.
112. 18.5279 +.

Page 195.

113. $33.87.
114. $457.76.
115. $76.45.
$23.62.
$2.58.
116. $17.85.
$2.47.
$0.47.
117. $1.46.
$0.16.
$190.78.
118. $11.47.
$7.98.
119. $4.41.
120. $2.07.

EXAMPLES FOR REVIEW.

121. $2.493.
122. $13.482.
123. $6.842.
124. $6.593.
125. $160.714.

Page 196.

126. $274.288.
127. $11.342.
128. $1.086.
129. $17.192.
130. $2.925.
131. $78.187.
132. $58.65.
133. $21.511.
134. $11.226.
135. $63.685.
136. $929.88.
137. $16.056.
138. $8.875.
139. $2084.197.
140. $183.174.
141. $114.858.
142. $1131.212.
143. $65.38 +.
144. $13.375.
145. $68.45.
146. $183.19.
147. $1001.10 received.
$58.97 paid out.
$942.13 less.
148. $29.725 bo't.
$9.632 sold.
$20.093 bo't more.

DENOMINATE NUMBERS.

Page 206.

2. *$412\frac{1}{2}$ yards.*
$1237\frac{1}{2}$ feet.
3. $11104\frac{1}{2}$ feet.
2349 feet.
4. 7716 inches.
1872 inches.
5. 231 feet.
$397\frac{1}{2}$ feet.

6. 5280 feet.
36960 feet.
15856½ feet.

Page 207.
7. 42240 ties.
8. 156 yards.
$273.
9. 378400 ties.
$30272.
10. 132 feet.
198 feet.
660 feet.
11. 396 feet.
96 boards.
$11.52.
12. 78 feet.
26 yards.
$0.78.
13. 310 panels.
$579.70.

Page 208.
14. 517 steps.

Page 210.
15. 30 ft. 7 in.
72 ft. 8 in.
16. 782 yds. 1 ft.
268 yds. 2 ft. 1 in.
17. 4 rds. 5 yds.
149 rds. 4 yds. 2 ft. 6 in.
25 rds. 1 ft. 6 in.
18. $12.30.
$16.40.
$46.33.
19. $284.24.
$222.64.
$167.20.
20. $119.
$153.

Page 211.
22. $3.18.
23. $2.58.
$5.34.
$4.92.
24. $13.25.
$8.51.
25. $260.66.

Page 216.
27. 234 sq. in.
520 sq. in.
28. 7500 sq. ft.
646` sq. ft.
5762 sq. ft.
29. 1296 sq. in.
9 sq. ft.
30. 1248 sq. ft.
138⅔ sq. yds.
31. 120 sq. ft.
13⅓ sq. yds.
32. 648 sq. ft.
72 sq. yds.
33. 1458 sq. ft.
162 sq. yds.
34. 400 sq. ft.
44 4/9 sq. yds.
35. 151 5/9 sq. yds.
36. 169⅓ sq. yds.
37. $28.
38. 180 sq. yds.
$32.40.

Page 217.
39. $63.84.
40. 640 acres.
$21333.33.
41. 40 chains.
$11264.
42. 40 chains.
80 acres.
400 acres.
$6300.

Page 218.
43. 7 acres.
1280 acres.
1273 acres.
$12730.
44. $14836.80.
45. 160 acres.
$3573.33.
46. $556.80.
47. $393.60.
$18653.25.
48. 64 rods.
64 acres.
$1088.
$2707.20.
$896.
$1248.
$5939.20.
$3520.
$6459.20 more.
$6459.20 gain.

Page 219.
49. $1008 each.
50. 7680 acres.
51. $53096.33.
52. $6.91.
54. $106.17.
$61.10.

Page 220.
55. $28.
$18.89.
56. $120.
57. $266.66.
58. 224 sq. ft.
425 sq. ft.

59. 168 sq. ft.
154 sq. ft.
60. 240 sq. ft.
198 sq. ft.
61. 306 sq. ft.
$373\frac{1}{3}$ sq. ft.
62. $94.40.
63. $24.
64. $389.33.

Page 224.

66. 320 c. ft.
67. 48 c. ft.
68 c. ft.
68. 676 c. ft.
69. 1280 c. ft.
70. 24 c. ft.
71. 128 c. ft.
221184 c. in.
72. 2688 bricks.
$13.44.
73. 14196 bricks.
$103.63.
74. 26880 bricks.
$301.06.

Page 225.

75. 51 pchs. $17\frac{3}{4}$ c. ft.
76. 62 pchs. $1\frac{1}{2}$ c. ft.
77. 1280 c. ft.
78. 2211840 blocks.
79. $9575\frac{15}{231}$ gals.

Page 226.

80. 9 cords.
81. $3\frac{3}{8}$ cords.
$12.66.
82. $11.81.
83. $22.22.
84. 2400 feet.

85. $60.
86. 1600 feet.
87. $30.80.
88. 2100 feet.
$48.99.
89. 330 feet.
$8.25.

Page 227.

90. $67.20.
91. $215.24.
93. 115 gals. 3 qts.
14 gals. 1 qt. 1 pt. 3 gills.
95. 172 qts.
$15.48.

Page 228.

96. 84 gals.
$26.88.

Page 229.

97. 1200 pints.
3008 pints.
99. $3.84.
$529.92.
100. $6.40.
101. $6.60.
102. Gained $2.25.
Lost $0.42.
103. $26\frac{2}{3}$ bottles.
80 pt. bottles.
64 cups each.
104. $4.77.
$5.36.
105. $8.70.
$18.90.
106. 1 bu. 1 pk. 7 qts.
107. $2.50.
$2.97.

Page 230.

109. $24.38.
$33.44.
110. $41.21.
$53.97.
111. $25.52.
35.56.

Page 231.

113. $36\frac{36}{215}$ bush.
$77\frac{169}{1075}$ bush.
$1\frac{1}{215}$ bush.
114. $1671\frac{159}{215}$ bush.
115. $97\frac{269}{1075}$ bush.

Page 232.

117. 151984 c. in.
$657\frac{147}{231}$ gals.
118. $375\frac{223}{231}$ gals.
$2631\frac{175}{231}$ gals.
119. $11913\frac{93}{231}$ gals.

Page 233.

120. 160 pills.
120 pills.
121. $4.20.
122. $9.60.
$8.04.
123. $20.25.

Page 234.

124. 1200.
10352.
125. 1346 lbs.
1237 lbs.
126. 107 cwt.
293 cwt.
127. 444 oz.
128. 3 cwt. 47 lbs.
4 cwt. 76 lbs.
1 cwt. 54 lbs.
3 oz.
820 cwt.

Page 235.

130. $49.28.
$55.09.
131. $9.73.
$7.94.
$10.38.
132. $118.97.
$79.38.
$97.16.
133. 8$\frac{1}{28}$ bushels.
$15.36.
134. 108$\frac{22}{32}$ bushels oats.
62$\frac{6}{56}$ bushels corn.
57$\frac{58}{60}$ bushels wheat.
72$\frac{22}{48}$ bushels barley.
57$\frac{58}{60}$ bushels potatoes.
61$\frac{1}{57}$ bushels onions.
69$\frac{28}{56}$ bushels salt.
135. 7$\frac{47}{56}$ bushels rye.
7$\frac{19}{60}$ bushels beans.
7$\frac{19}{60}$ bushels clover.
6$\frac{19}{70}$ bushels ears of corn.
9$\frac{43}{44}$ bushels hempseed.
136. $0.59.
137. $0.94.
138. $1.62.
139. $5.12.
140. $4.88.

Page 236.

141. $17.45 +.
142. $15.
143. $135.06.
144. $33.36.
145. $29.74.

Page 238.

146. 9 d. 1 hr. 31 m. 4 sec.
147. 58 d. 10 hr. 40 min.
148. 19 hr. 57 min. 14 sec.
149. 94608000 seconds.
150. 7776000 seconds.
151. 7862400 seconds.
152. 4 hr. 5 min. 20 sec.

Page 239.

154. 5 hr. 8 min.

Page 240.

155. 18 min. 28 sec.
156. 2 hr. 26 min. 48 sec.
157. 6 hr. 30 min. 32 sec.
158. 4 hr. 49 min. 28 sec.
159. 60° 49′.
160. 41° 35′ 15″.
161. 15° 34′ 30″.
162. 60° 15′ 30″.
163. $3.60.
$3.24.
164. Gained $2.28.

Page 241.

166. 60 shillings.
40 shillings.
6540 shillings.
167. 680 shillings.
8160 pence.
32640 farthings.

Page 242.

168. 62 bu. 3 pks. 7 qts.
169. 52 lb. 5 oz.
170. 97 A. 55 p.
171. 85 rds. 14 ft.
172. £74 5 s. 2 d.
173. 378 A. 80 p.

Page 243.

174. £36 18 s. 1 d. 3 far.
175. 5 lb. 3 oz. 14 pwt. 17 grs.
176. 3 lb. 9℥ 2ʒ 16 grs.
177. 1 m. 20 rds. 4 yds.
179. 3 yrs. 1 mo. 20 days

180. 16 yrs. 1 mo. 8 days.
181. 2 yrs. 3 mo. 2 days.
182. 9 yrs. 5 mo. 10 days.
183. 1 mo. 26 days.
184. 1 yr. 11 mo. 26 days.
185. 1 yr. 7 mo. 27 days.
186. 8 days.
187. 64° 48′ 24″.
188. 25° 51′ 50″.

Page 244.

189. 56° 36′.
190. 157°.
191. 51° 18′ 4″.
192. 25 gallons, 1 pint.
193. 97 pounds, 9 ounces.
194. 221 A. 126 perches.

Page 245.

195. £16 3 s. 1 d. 2 far.
£37 13 s. 11 d. 2 far.
196. 17 rds. 1 yd. 1 ft. 6 in.
51 rds. 4 yds. 1 ft. 6 in.
197. 3 d. 2 hr. 14 min. 1 sec.
9 d. 23 hr. 49 min. 54 s.
198. 43 bushels.
199. 55 pounds, 8 ounces.
83 pounds, 4 ounces.
200. 2 lb. 6 oz. 12 pwt.
$42.84.
201. 4 lb. 5 oz. 10 pwt. 12 gr.
7 lb. 1 oz. 12 pwt. 19$\frac{1}{5}$ gr.
202. 30 bu. 1 lb. 1$\frac{1}{15}$ oz. rem.
203. 1 oz. 13 pwt. 20 gr.
204. 67 gal. 2 qt. 1$\frac{1}{7}$ gills.

Page 246.

205. 85 A. 46 p.
206. 12 m. 254 rds. 4 yds. 2 ft. 9$\frac{3}{19}$ in.
207. 14 bu. 1 pk. 2 qt. 2 gi.
208. £2 10 s. 2 d. 3$\frac{3}{7}$ far.
209. 27$\frac{1}{2}$ spoons.
210. 88 days.
211. 96 bottles.
212. 725$\frac{1}{3}$ packages.

Page 248.

213. 192 rods.
96 rods.
214. 2 yards.
2 feet 9.66 inches.
215. 18 inches.
32.4 inches.
514.8 inches.
216. 178 rds. 1 yd. 4$\frac{6}{7}$ in.
217. 19 cwt. 35 lbs.
218. 7 oz. 15 pwt. 6$\frac{6}{7}$ grs.
123 rds. 3 yds. 5.85 in.
219. $704.
220. $7267.20.
52.36\frac{4}{11}$.

Page 249.

221. $\frac{17}{18}$ yards.
.94 yards.
222. $\frac{49}{2112}$ miles.
.0232 + miles.
223. 1$\frac{3}{32}$ gallons.
1.09375 gallons.
224. $0.90.
225. $0.309.
226. .0015 rods.
$\frac{53}{132}$ rods.
227. 1.43 rods.
1$\frac{43}{99}$ rods.

Page 250.

228. $34.71.
229. $14.30.

Page 251.

231. £20 4 s.
$98.29 +.
232. $144.03.
233. $79.32.
234. $14.36.
235. $21.13.
236. $2.80.
237. 9 spoons: 5 pwt. 9 grs. rem.
238. $30.87 +.
239. $1.25.
$1.62.

Page 252.

240. $2.62.
241. $6.19.
$7.43.
242. $0.17.
$0.92.
$1.64.
243. $83.55.
244. $0.38.
245. 188 doses.
246. 384 pills.
247. $8.31.
$6.34.
$4.27.
248. $5062.50.
249. 60 yards.
$133\frac{1}{3}$ yards.
250. 4856 square feet.
251. $201.67.
252. $175.

Page 253.

253. 9 square feet.
$0.18.
255. $100.78.
256. $72.
257. $7560.30.
258. $54.11.
259. 240 loads.
260. $450.
261. 99 cubic feet.

Page 254.

262. 424 square feet.
$10.60.
263. 80 perches.
264. $66.69.
265. $15.45.
266. $0.25.
267. 56 bushels.
268. 19.285 + bushels.
269. $209\frac{5}{12}$ pounds.
270. $12.80
271. $6.60.
$0.26.

Page 255.

272. 15 loads: $17\frac{223}{1075}$ bush. rem.
273. $363\frac{7}{11}$ barrels.
274. $43.33.
275. $426\frac{2}{3}$ rods.

JUL 14 1945